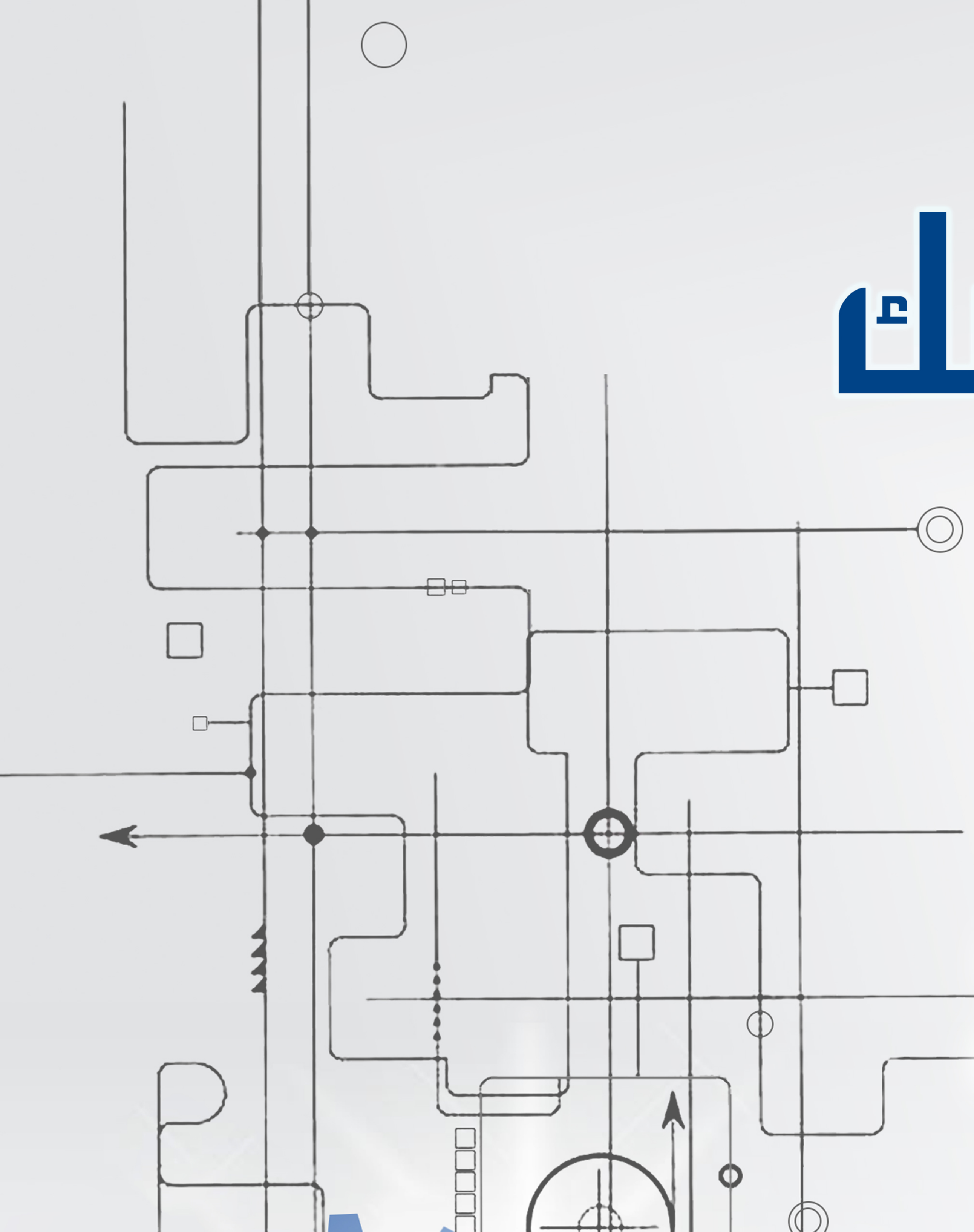
ملخص فيديوهات مادة..

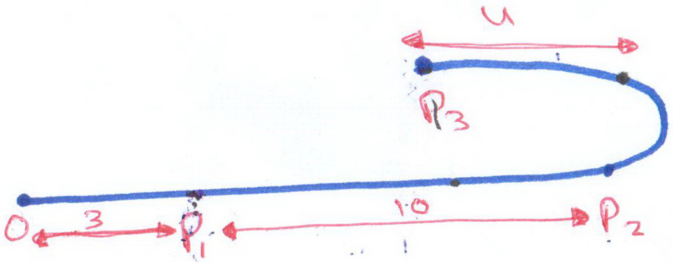
الداينمك



لجنة الميكانيك

Polytechnic

Ch 12 Section 2 :- Kinematics of partic



- **Position** : Distence from the origin

$P_1 = 3$ $P_2 = 13$

- **Displacement** : change in position

$S_{12} = 10$

$S_{13} = 10 + (-4) = 6$

- **Distence** : length of the path

$d_{13} = 14$ $d_{12} = 10$

- **Average Velocity** : Rate of Displacement

* $\Delta t = 4$ sec

$V_{avg} = \frac{\Delta S}{\Delta t} = \frac{6}{4} = 1.5 m/s$

- **Average Speed** : Rate of Distance

$S_{avg} = \frac{\sum|S|}{\Delta t} = \frac{14}{4} = 3.5$

* $S_{avg} \neq V_{avg}$

- **Istentanous Velocity**

$S = 5t^2 - 4t + 9$

$\boxed{V = \frac{\partial S}{\partial t}} = 10t - 4$

- // acceloration

$a = \frac{\partial V}{\partial t}$

- **Average acceleration**

$a_{avg} = \frac{\Delta V}{\Delta t}$

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• Constant accelaration

1) $v_2 = v_1 + at$

2) $DS = v_1 t + 1/2 a t^2$

3) $v_2^2 = v_1^2 + 2a DS$

× 4) $DS = \left(\frac{v_1 + v_2}{2}\right) * t$

× • Constant velocity

(*) $S_1 = S_0 + v \cdot t$

ex 12.1 | $v = 3t^2 + 2t$

P & a ∴ t=3 sec

① $v = \frac{ds}{dt}$ $\quad ds = v \cdot dt$

$$\int_{s_1}^{s_2} ds = \int_0^3 3t^2+2t \cdot dt$$

$$ds = t^3 + t^2 = \boxed{36}$$

$$a = \frac{dv}{dt} = 6t+2 = \boxed{20}$$

P 12.2 | $v = 2$ m/s , $a = 60 v^{-4}$

(P & v) ∴ t = 3 sec

$$a = \frac{60}{v^4} \qquad \frac{dv}{dt} = \frac{60}{v^4}$$

$$\int_2^v v^4 \, dv = \int_0^t 60 \, dt$$

$$\frac{v^5}{5}\Big]_2^v = 60t\Big]_0^t \longrightarrow \frac{v^5}{5} - \frac{32}{5} = 60t$$

$$v = (300t + 32)^{1/5}$$

$$\boxed{v = 3.93 \text{ m/s}}$$

$$\frac{ds}{dt} = (300t + 32)^{1/5}$$

$$s = \frac{(300t+32)^{1.2}}{1.2(300)}$$

$$\boxed{\therefore S = 10}$$

P 12.17

$a = 2t - 1$ $\therefore S = 1$ $v = 2$ @ $t = 0$

find the total disp~~lp~~ p @ t = 6 sec

$a = 2t - 1 \rightarrow \frac{dv}{dt} = 2t - 1$

$\int_2^v dv = \int_0^t (2t - 1) . dt$

$v - 2 = t^2 - t \rightarrow \boxed{v = t^2 - t + 2}$

$\frac{ds}{dt} = t^2 - t + 2 \rightarrow ds = t^2 - t + 2 \, dt$

$\int_1^s ds = \frac{t^3}{3} + \frac{t^2}{2} + 2t$

$\boxed{s = \frac{t^3}{3} - \frac{t^2}{2} + 2t + 1}$

$v(6) = 6^2 - 6 + 2 = 32$ m/s

$s(6) = 67$ m

$s(0) = 1$ m

$d = P(6) - P(0) = \boxed{66 \text{ m}}$

P12.6

$Dy = v . t + 1/2 \, at^2$

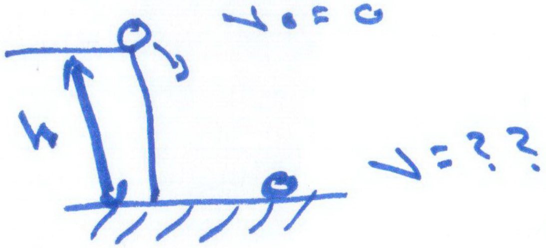
$Dy_A = 0 + 1/2 (9.81)(2)^2$
$= 18.62$ m

$Dy_B = 0 + 1/2 (9.81)(1)^2$
$= 4.91$ m

$Dy = Dy_A - Dy_B$

$= \boxed{13.71 \text{ m}}$

P12.9

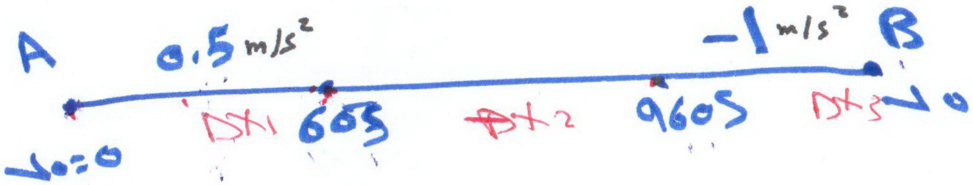


$\therefore t = 3sec$

$$Dy = \overset{0}{\cancel{v_0}}t + 1/2\, at^2$$
$$= 0 + 1/2\,(9.81)(3)^2$$
$$= \boxed{44.15\,m}$$

$$v_2 = v_1 + at$$
$$= 0 + (9.81)(3)$$
$$= 29.43\ m/s \downarrow$$

P12.15



$$DX_1 = \overset{0}{\cancel{v_0}}t + 1/2\, at^2$$
$$= 0 + 1/2\,(0.5)(60)^2 = \boxed{900\,m}$$

$$v = v_0 + at$$
$$= 0 + 0.5 \times 60 = \boxed{30\,m/s}$$

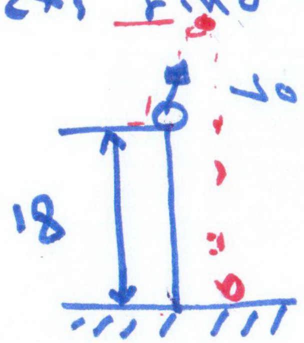
$$DX_2 = v_1 \cdot t = 30 \times 900 = \boxed{27000\,m}$$

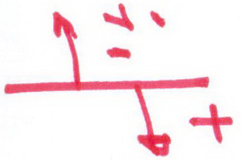
$$\overset{0}{\cancel{v_2}}^2 = \overset{30}{\cancel{v_1}}^2 + 2\overset{\boxed{-1}}{a} DX_3$$

$$\boxed{DX_3 = 450\,m}$$

$$\boxed{DX\ total = 28.4\ Km}$$

Ex, find $v(t)$ & $y(t)$, y_{max} & the time when the ball hits the ground & v





① $v(t) = v_0 + at$

$= -12 + 9.81\,t$

② $y(t) = vt + \frac{1}{2}at^2$

$= -12t + \frac{1}{2}(9.81)\,t^2$

③ $v_2^2 = v_1^2 + 2a\Delta s$

$0 = (-12)^2 + 2(9.81)\,\Delta s$

$\Delta s = \ominus 7.33$ m

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$\therefore y_{max} = 7.33 + 18 = \boxed{25.33\text{ m}}$

$y = v_1 t + \frac{1}{2}at^2$

$18 = -12t + \frac{1}{2}(9.81)\,t^2$

$\boxed{t = 3.55\text{ sec}}$

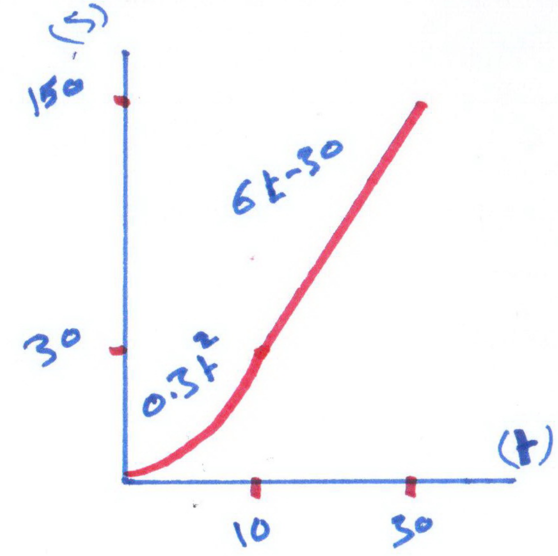
$v_f^2 = v_i^2 + 2ad$

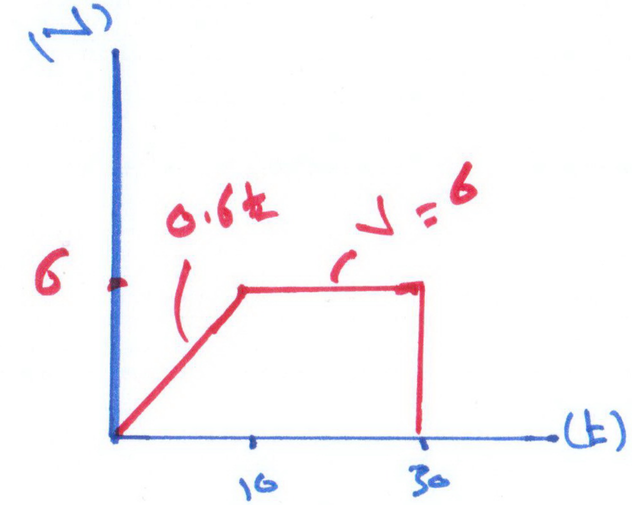
$= (-12)^2 + 2(9.81)(18)$

$\boxed{v_f = 22.3\text{ m/s}}$

Ch12 Section 3: Erratic Motion

Ex 12.61

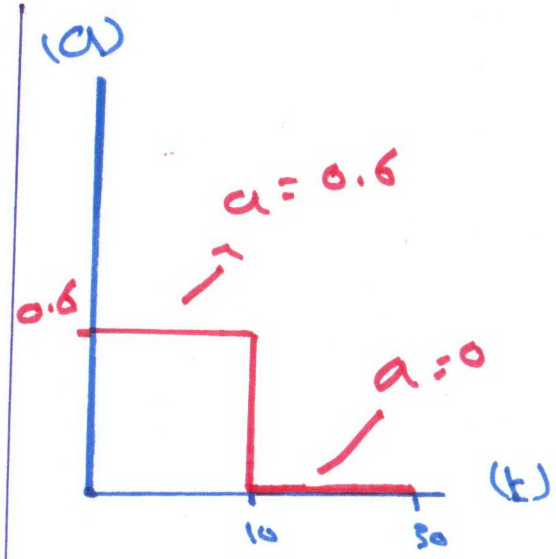




① (v-t) (0 - 10)

$$v = \frac{ds}{dt} = 0.6t$$

$$v = \frac{ds}{dt} = 6$$

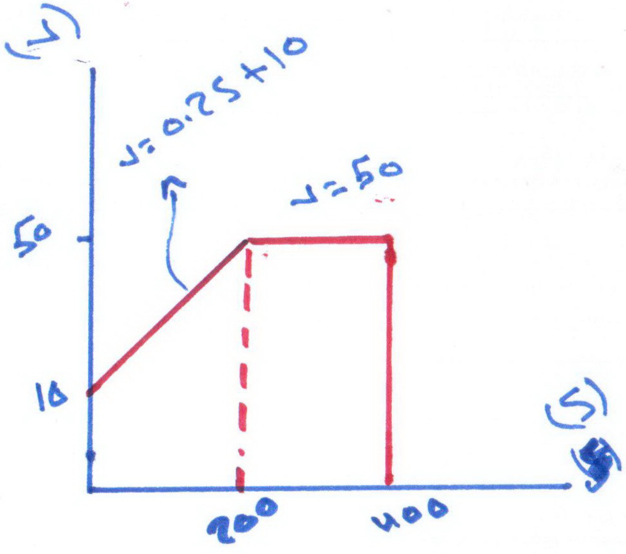


$$a = \frac{dv}{dt} = 0.6$$

$$a = \frac{dv}{dt} = \text{zero}$$

Ex 12.8

$$\boxed{\therefore (ax+b) \rightarrow \left(\frac{b}{a}\right) \div b}$$



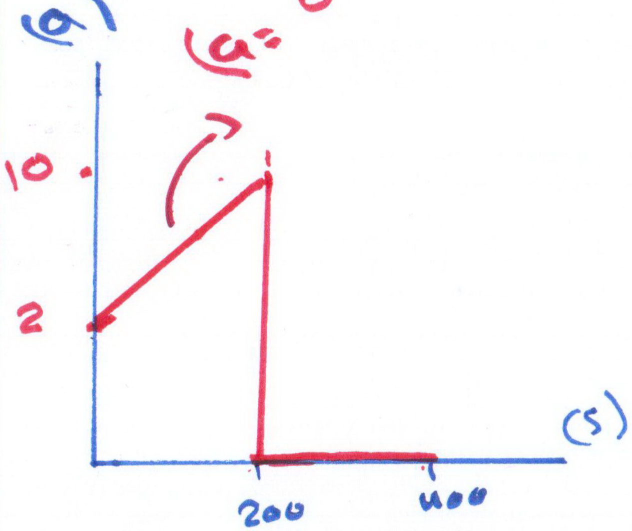
$$\text{" } a\,ds = v\,dv \text{ "}$$

$$a = v\frac{dv}{ds}$$

$$\boxed{a = (0.2s+10)*0.2}$$

$$\text{' } a = 0.04s + 2 \text{ '}$$

$$(a = 0.04s + 2)$$



$s=0 \Longrightarrow a=2$

$s=200 \longrightarrow a=10$

$$\frac{ds}{dt} = v \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{0.2s+10}$$

$$t = (5\ln(0.2s+10) - 5\ln 10)$$

when $s=200$

$$\underline{t = 8.05 \text{ sec}} \rightarrow \boxed{4 \text{ sec}}$$

$$\frac{ds}{dt} = v \xrightarrow{400} dt = \frac{ds}{v}$$

$$\int_{8.05}^{t} dt = \int_{200} \frac{ds}{50}$$

$$\boxed{t = 12 \text{ sec}}$$

Ch12 Section 5 : Rectangular components

$$\vec{r} = (x i + y j + z K)\ m$$

$$\vec{v} = \frac{\vec{dr}}{dt} \qquad \vec{a} = \frac{\vec{dv}}{dt}$$

$$= (\dot{x} i + \dot{y} j + \dot{z} K)\ m/s \qquad = (\ddot{x} i + \ddot{y} j + \ddot{z} K)$$

$$= (v_x i + v_y j + v_z K)\ m/s \qquad = (\dot{v}_x i + \dot{v}_y j + \dot{v}_z K)$$

$$= (a_x i + a_y j + a_z K)\ m/s^2$$

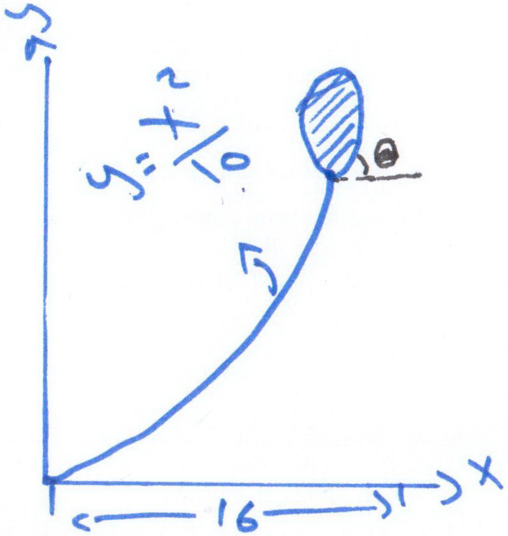
$$|v| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Ex 12.9

$x = 8t$

$\therefore t = 2$



$v_x = \dot{x} = 8$

$v_y = \frac{d}{dt} \frac{x^2}{10} \Big|_{t=2}$

$= \frac{d}{dt} \frac{(8t)^2}{10} \Big|_{t=2} = \frac{2x\dot{x}}{10} = \frac{2 \cdot 16 \cdot 8}{16} = \boxed{25.6}$

$v| = \sqrt{25.6^2 + 8^2} = 26.8 \text{ m/s}$

* $\theta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{25.6}{8} = \boxed{72.6^\circ}$

$a_x = \frac{d^2}{dt^2} (8t) = \text{zero}$

$a_y = \frac{d^2}{dt^2} \frac{(8t)^2}{10} \Big|_{t=2} = 12.8$

$|a| = \sqrt{12.8^2 + 0^2} = \boxed{12.8}$

P 12.74

$v = 3i + (6 - 2t) j$

$\therefore t_1 = 1$ to $t_2 = 3$

find dis?

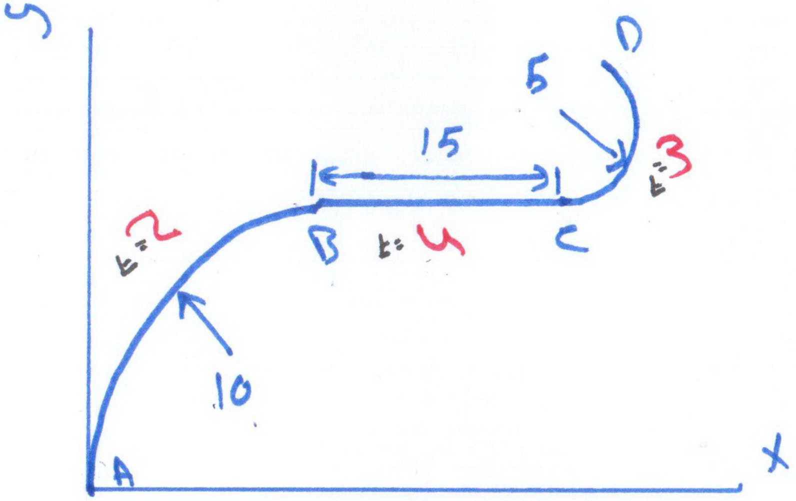
$\bar{r} = 3t i + \left(6t - \frac{2t^2}{2}\right) j$

$\bar{r}_1 = (3i + 5j) \text{ m}$

$\bar{r}_3 = (9i + 9j) \text{ m}$

$\boxed{\Delta\bar{r} = (6i + 4j) \text{ m}}$

P12.80



Find S_{avg} ...?

$$S_{avg} = \frac{\sum d}{\sum t}$$

$$= \frac{\frac{(2\pi(10))}{4} + 15 + \frac{(2\pi(5))}{4}}{2+4+3}$$

$$= \boxed{4.28 m/s}$$

P12.81

$x = 0.25t^3$, $y = 1.5t^2$, $z = 6 - 0.75t^{5/2}$

∴ $t = 2$

$$\bar{r} = 0.25t^3 i + 1.5t^2 j + (6 - 0.75)t^{5/2} k$$

$$\bar{v} = 0.75t^2 i + 3tj - 1.875t^{1.5} k$$

$$\bar{a} = 1.5tj + 3j - 2.8125t^{0.5} k$$

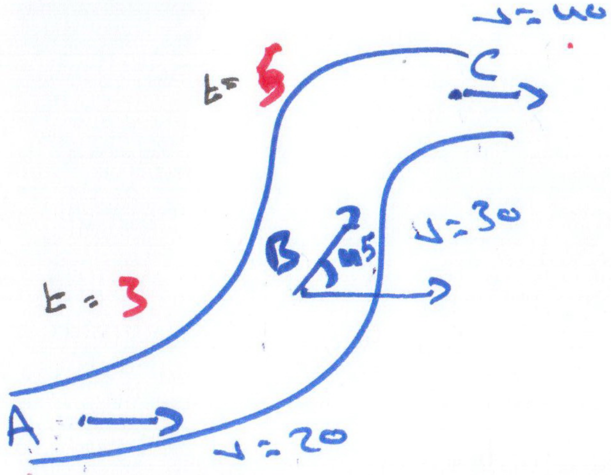
$$|v| = \sqrt{\quad + \quad + \quad}$$

$$|a| = \sqrt{\quad + \quad + \quad}$$

$|v| =$

$|a| =$

P 12.79|



$(\bar{a}_{avg} = \frac{\Delta v}{\Delta t})$

$$\bar{a}_{AB} = \frac{(30\cos 45 i + 30\sin 45 j) - (20 i)}{3}$$

$$= (0.404 i + 7.07 j) \; m/s^2$$

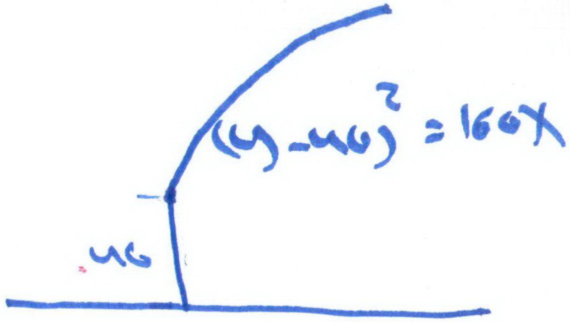
$$\bar{a}_{Ac} = \frac{(40)i - (20)i}{3+5}$$

$$= \boxed{2.5 \; m/s^2}$$

P 12.86|

y = 80 m

v_y = 180 m/s



$$(y-40)^2 = 160x$$

$$2(y-40)v_y = 160 v_x$$

$$2(y-40)\overset{0}{a_y} + 2v_y(v_y) = 160 a_x$$

@ y = 80 v_y = 180

$$2(80-40)180 = 160 v_x$$

$$\therefore v_x = 90 \; m/s$$

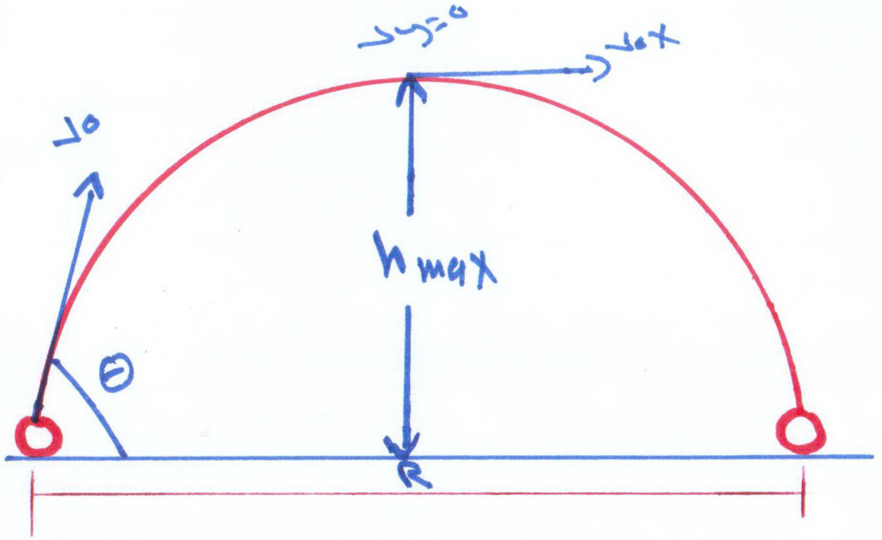
$$|v| = \sqrt{180^2 + 90^2} = \boxed{201 \, m}$$

$$2 * 180^2 = 160 a_x$$

$$a_x = 405 \; m/s^2$$

$$|a| = a_x = 405 \; m/s^2$$

Ch12 Section 6: Motion of a Projectile...



$a_y = -9.81$

x - dir

$v_{ox} = v_o \cos\theta$

$a_x = 0$

$\Delta x = v_{ox} \cdot t$

$$R = \frac{v_o^2 \sin 2\theta}{g}$$

$$h_{max} = \frac{v_o^2 \sin^2\theta}{2g}$$

y - dir

$v_{oy} = v_o \sin\theta$

$v_y = v_{oy} + a_y t$

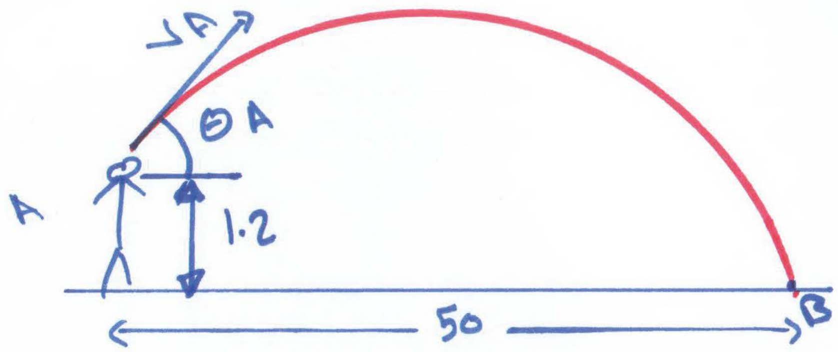
$v_y^2 = v_{oy}^2 + 2a_y \Delta y$

$\Delta y = v_{oy} \cdot t + 1/2\, a_y t^2$

$$\Delta y = \left(\frac{v_{oy} + v_y}{2}\right) \cdot t$$

P12.89

$\therefore t = 2.5$ sec



x-dir

$\Delta x = v_{ox} \cdot t$

$50 = v_A \cos\theta_A \cdot (2.5)$

$v_A \cos\theta_A = 20$

$$\boxed{v_A = \frac{20}{\cos\theta_A}}$$

y-dir

$$\Delta y = v_{oy} t + \frac{1}{2} \overset{-9.81}{a_y} t^2$$

$-1.2 = v_A \sin\theta (2.5 - 4.9 (2.5)^2$

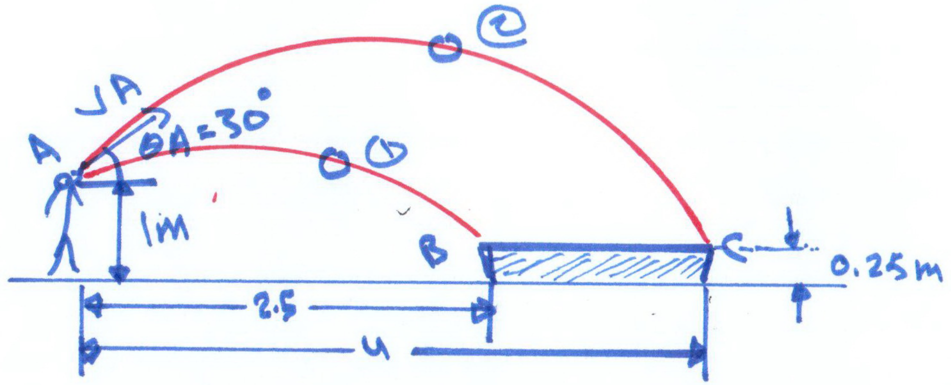
$29.425 = 2.5 \left(\frac{20}{\cos\theta_A}\right) * \sin\theta$

$\tan\theta = 0.5885$

$\therefore \boxed{\theta = 30.5^\circ}$

$$\boxed{\therefore v_A = 23.2 \text{ m/s}}$$

P12.92|



① toy 1

x-dir

$\Delta x = v_\circ \cos\theta \cdot t$

$2.5 = v_\circ \cos 30 * t \rightarrow \boxed{v_\circ = \frac{2.5}{\cos 30 \cdot t}}$

y-dir

$Dy = v_\circ \sin\theta t - 4.9 t^2$

$-1 = \left(\frac{2.5}{\cos 30 \cdot \cancel{t}}\right) * \sin\theta \cancel{t} - 4.9 t^2$ ∴

$\boxed{t_1 = 0.706 s}$

$\boxed{v_{o,1} = 4.1 m/s}$

② toy 2

x-dir

$4 = v_\circ \cos 30 \cdot t$

y-dir

$-1 = v_\circ \sin 30 t - 4.9 t^2$

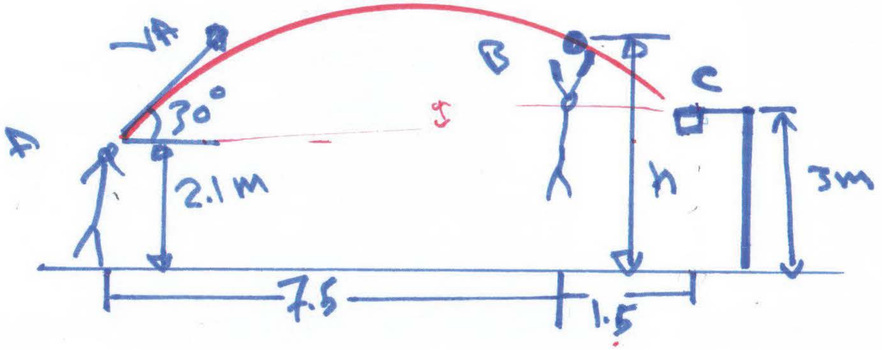
$\boxed{t_2 = 0.822 sec}$

$\boxed{v_{o,2} = 5.62 m/s}$

∴ interval $= t_2 - t_1$

$\boxed{= 0.116 s}$

P 12.99



x-dir

$Dx = V_A \cos 30\, t$

$9 = V_A\, t \cos 30$

$\boxed{t = \frac{9}{V_A \cos 30}}$

y-dir

$Dy = V_A \sin 30\, t - 4.9 t^2$

$0.9 = \cancel{V_A} \sin 30 \left(\frac{9}{\cancel{V_A} \cos 30}\right) - 4.9 \left(\frac{9}{V_A \cos 30}\right)^2$

$\therefore V_A = 11.1$ m/s

x-dir

$Dx = V_A \cos 30\, t$

$7.5 \quad 11.1$

$\boxed{t = 0.78\, s}$

y-dir

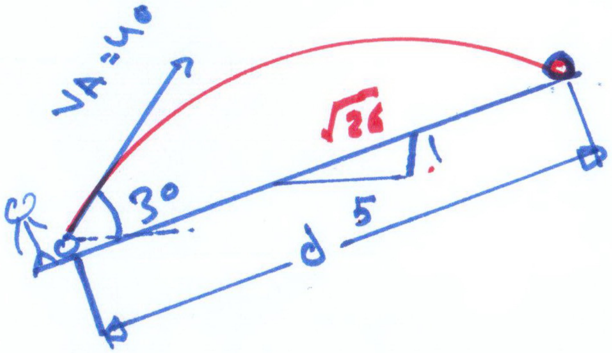
$Dy = V_A \sin 30\, t - 4.9 t^2$

$\boxed{Dy = 1.35\, m}$

$\therefore h = 2.1 + 1.35$

$\boxed{= 3.45\ m}$

P12.107
or 102



x-dir

$\Delta x = V_A \cos\theta\, t$

$\frac{5}{\sqrt{26}} * d = 40 \cos 30\, t$

$\boxed{t = 0.0283\, d}$

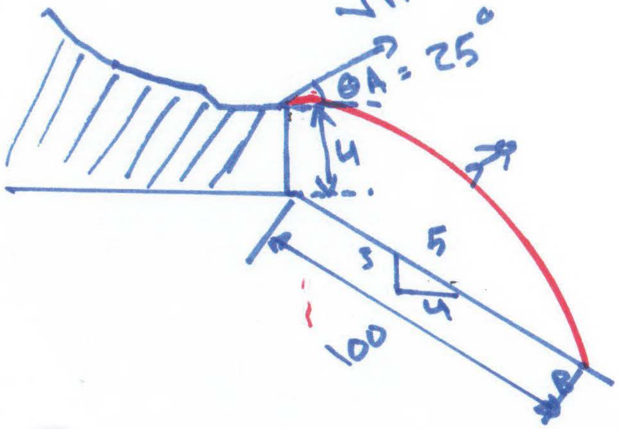
y-dir

$\Delta y = V_{0y} t + \frac{1}{2} a_y t^2$

$\frac{1}{\sqrt{26}} . d = 40 \sin 30 - 4.9 \times (0.0285 d)^2$

$\boxed{\therefore d = 94.4\ m}$

P12.101



x-dir

$\Delta x = V_A \cos\theta\, t$

$80 = V_A \cos 25\, t$

$t = \left(\frac{80}{V_A \cos 25}\right)$

y-dir

$\Delta y = v_{oy} t - \frac{1}{2} a_y t^2$

$-64 = \cancel{V_A} \sin 25 \left(\frac{80}{\cancel{V_A} \cos 25}\right) - 4.9 \left(\frac{80}{V_A \cos 25}\right)^2$

$V_A = 19.4 \text{ m/s}$

$V_{Ax} = 19.4 \cos 25 = 17.6$

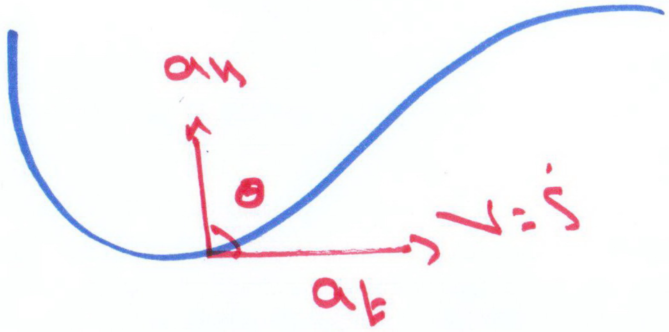
$V_y = V_A \sin 25 - 9.8 \left(\frac{80}{19.4 \cos 25}\right)$

$V_y = -36.4 \text{ m/s}$

$|V| = \sqrt{\quad} = \boxed{40.4 \text{ m/s}}$

Ch 12

Section 7: Normal & Tangential Components...



- $v = \dot{s}$
- $a_t = \dot{v} = \ddot{s}$
- $a_n = \dfrac{v^2}{\rho}$
- $a = \sqrt{a_t^2 + a_n^2}$
- $\theta = \tan^{-1}\left(\dfrac{a_n}{a_t}\right)$
- $\rho = \dfrac{\left(1+(y')^2\right)^{3/2}}{y''}$

P12.115

$a_n = a = 7.5\ m/s^2$

$\rho = 200$

@ max Speep $(a_t = 0)$

$\overset{7.5}{a_n} = \frac{v^2}{\rho_{200}}$?!

$$\boxed{v = 38.7\ m/s}$$

P12.117

$a_t = 2000\ Km/h^2,\ v = 60\ Km/h$

$R = 600m$

$a_t = \frac{2000 * 1000}{(3600)^2} = 0.154\ m/s$

$v = \frac{60 * 1000}{3600} = 16.67\ m/s$

$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{600}$

$= 0.463\ m/s^2$

$a = \sqrt{a_n^2 + a_t^2}$

$$\boxed{= 0.488\ m/s^2}$$

P12.118 $R = 50$ $(v = 4 \quad s = 0)$

$a_t = \dot{v} = 0.05s$ $\quad v = ? @ s = 10$

(a)

$$\int_0^{10} a\,ds = \int_4^{v} v\,dv$$

$$\left[0.05\frac{s^2}{2}\right]_0^{10} = \frac{v^2}{2}\Big|_4^{v}$$

$$\therefore \boxed{v = 4.58\,m/s}$$

$$a_t = 0.05\,\overset{10}{s}$$

$$= 0.5\,m/s^2$$

$$a_c = a_n = \frac{v^2}{\rho} = \frac{4.58^2}{50} = 0.42\,m/s^2$$

$$a = \sqrt{a_n^2 + a_t^2}$$

$$\boxed{= 0.653\,m/s^2}$$

P12.121 $v = 25\,m/s$ $\quad , a_t = 3\,m/s^2$

$y = \frac{1}{100}x^2$ $\quad x = 30\,m$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{\rho}$$

$$\rho = \frac{(1 + y'^2)^{3/2}}{y''}$$

$$\boxed{\rho = 79.3\,m}$$

$$y' = \frac{2 * 30}{100} = 0.6$$

$$y'' = \frac{2}{100} = 0.02$$

$$a_c = a_n = \frac{25^2}{79.3} = 7.88\,m/s^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$\boxed{= 8.43\,m/s^2}$$

P12.124) $a_t = 0.5e^t$ $S = 18$

$\rho = 30$

$$\frac{dv}{dt} = 0.5e^t$$

$v = 0.5(e^t - 1)$

$S = 0.5(e^t - t - 1)$

$* \; 18 = 0.5(e^t - t - 1)$ $\boxed{\therefore t = 3.71\,s}$

$* \; v = 0.5(e^{3.71} - 1) = 19.8\,m/s$

$a_t = 0.5e^{3.71} = 20.43$

$$a_n = \frac{19.8^2}{30} = 13.13\,m/s^2$$

$$a = \sqrt{a_n^2 + a_t^2}$$

$$\boxed{= 24.3\,m/s^2}$$

P12.131) $v = 20$

$a_t = 14\cos 75$

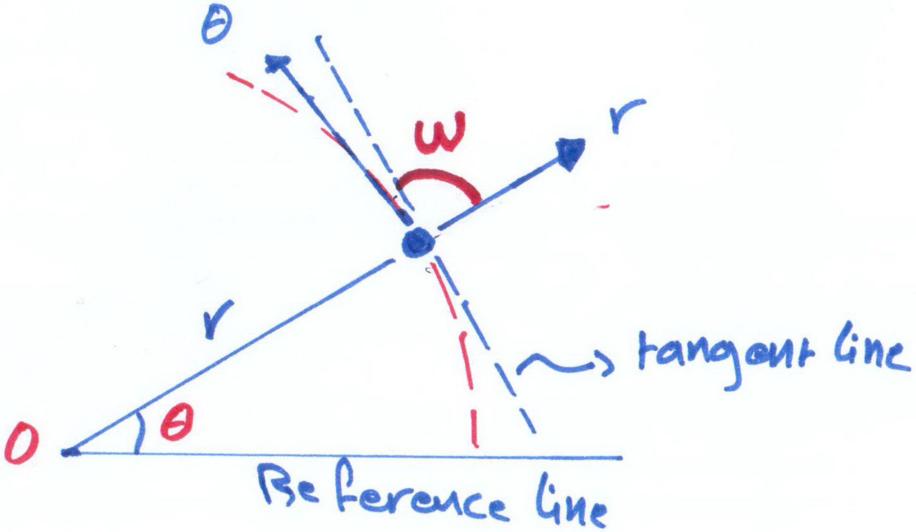
$a_n = 14\sin 75$

$* \; a_t = 3.62\,m/s^2$

$$\overset{14\sin 75}{\widehat{a_n}} = \frac{\cancel{v}^2 \rightarrow 20}{\rho}$$

$$\boxed{\rho = 29.6\,m}$$

Ch12 Section 12.8: Cylindrical components..



- $\Sigma F = ma$
- $\Sigma F_r = ma_r$
- $\Sigma F_\theta = ma_\theta$
- $\omega = \tan^{-1} \frac{r}{dr/d\theta}$

- $v_r = \dot{r} \quad , \quad v_\theta = r\dot{\theta}$

$$|v| = \sqrt{v_r^2 + v_\theta^2}$$

- $a_r = \ddot{r} - r\dot{\theta}^2 \quad , \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$$|a| = \sqrt{a_r^2 + a_\theta^2}$$

P12.163)

$\dot{r} = 3\,m/s \quad \ddot{r} = 0 \quad r = .2$

$\dot{\theta} = 2\,rad/s \quad \ddot{\theta} = 3\,rad/s^2$

$v_r = \dot{r} = 3\,m/s$

$v_\theta = r\dot{\theta} = 2 * 0.2 = 0.4\,m/s$

$|v| = \sqrt{v_r^2 + v_\theta^2}$
$= 3.03\,m/s$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.2(2)^2$
$= -0.8$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.2(3) + 2(3)(2)$
$= 12.6$

$|a| = \sqrt{a_r^2 + a_\theta^2}$
$= 12.63\,m/s^2$

P12.173)

$\dot{\theta} = 3, \quad r = 0.4\theta, \quad r = 0.5$

$\ddot{\theta} = 0$

$\dot{r} = 0.4\dot{\theta} = 0.4(3) = 1.2\,m/s$

$\ddot{r} = 0.4\ddot{\theta} = 0.4(0) = 0$

$v_r = \dot{r} = 1.2 \quad v_\theta = r\dot{\theta} = 0.5(3)$
$= 1.5$

$|v| = \sqrt{1.5^2 + 1.2^2}$
$= \boxed{1.9\,m/s}$

$a_r = \ddot{r} - r\dot{\theta}^2 = -4.5\,m/s^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.2\,m/s^2$

$|a| = \sqrt{a_r^2 + a_\theta^2}$
$= \boxed{8.49\,m/s^2}$

$\boxed{P12.177}$ $r=(100\cos 2\theta)$

$\theta = 15$ $\quad v = 50$

$\dot{r} = -200\dot{\theta}\sin 2\theta$

$v_r = \dot{r} = -100\dot{\theta}$

$v_\theta = r\dot{\theta} = 86.6\dot{\theta}$

$\cancelto{50}{v} = \sqrt{(-100\dot{\theta})^2 + (86.6\dot{\theta})^2}$

$\boxed{\dot{\theta} = 0.378}$

$\boxed{P12.197}$ $r = 40e^{0.05\theta}$, $\dot{\theta} = 4 \rightarrow \ddot{\theta} = 0$

$\theta = 30$

$\theta = \frac{30\pi}{180} = \boxed{\frac{\pi}{6}}$ rad

$\dot{r} = 40e^{0.05\theta} \cdot 0.05\dot{\theta}$

$\ddot{r} = 2(e^{0.05\theta}\underset{0}{\ddot{\theta}} + \dot{\theta}e^{0.05\theta} \cdot 0.05\dot{\theta}]$

$= 0.1\dot{\theta}^2 e^{0.05\theta}$

$r = 41.1$, $\dot{r} = 8.21$, $\ddot{r} = 1.64$

$v_r = \dot{r} = 8.21$, $v_\theta = r\dot{\theta} = 164.4$

$|v| = \sqrt{v_r^2 + v_\theta^2} = \boxed{164.6}$

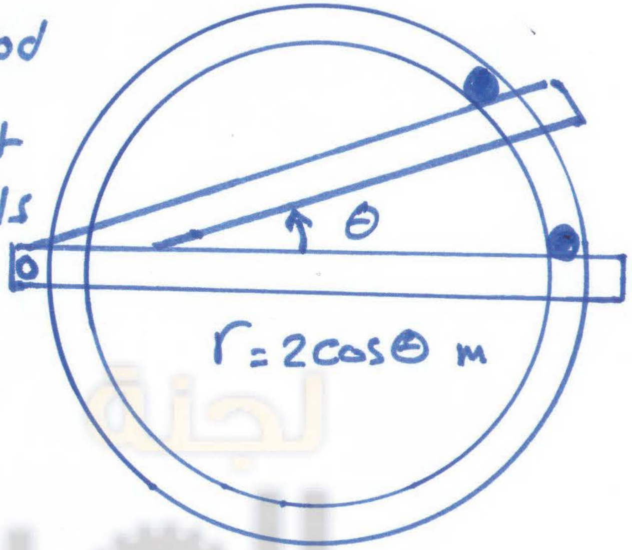
4

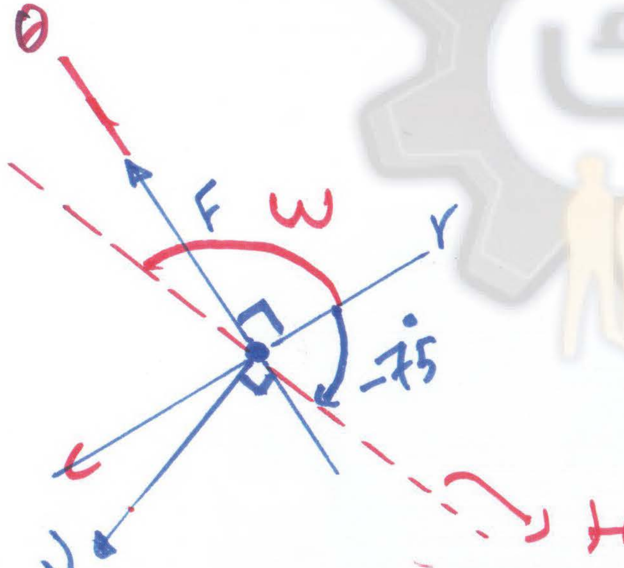
:

:

↓

ex: A 1 kg-ball is being pushed by a rod to move in a horizontal grooved, smooth slot. it starts from $\theta = 0$. Determine the force the rod exerts on the ball at $\theta = 15°$ if at this instant the rod moves at angular speed $\dot{\theta} = 1$ rad/s with angular acceleration $\ddot{\theta} = 2$ rad/s². The ball is only in contact with outer side of the slot...





tangent line

F.B.D

K.D

$$w = \tan^{-1} \frac{r}{dr/d\theta}$$

$$\frac{dr}{d\theta} = -2\sin\theta \quad \text{at } \theta = 15°$$

$$= \tan^{-1} \frac{2\cos 15°}{-2\sin 15} = \boxed{-75°}$$

$$\overset{+}{\nearrow} \quad \sum F_r = -N\sin 75^\circ = m a_r$$

$$\nwarrow + \quad \sum F_\theta = F - N\cos 75 = m a_\theta$$

$$r = 2\cos\theta$$

$$\dot{r} = \frac{\partial r}{\partial t} = \frac{\partial r}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} = -2\sin\theta\,\dot{\theta}$$

$$\ddot{r} = \frac{\partial \dot{r}}{\partial t} = -2\sin\theta \frac{\partial \theta}{\partial t} - 2\frac{\partial \sin\theta}{\partial t}\dot{\theta}$$

$$= -2\sin\theta\,\dot{\theta} - 2\frac{\partial \sin\theta}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} \cdot \dot{\theta}$$

$$\boxed{= -2\sin\theta\,\ddot{\theta} - 2\cos\theta \cdot \dot{\theta}^2}$$

@ $\theta = 15$, $\dot{\theta} = 1$, $\ddot{\theta} = 2$

$r = 2\cos\theta = \underline{1.932}$

$\dot{r} = -2\sin\theta\,\dot{\theta} = \underline{-0.5176}$

$\ddot{r} = -2\sin\theta\,\ddot{\theta} - 2\cos\theta \cdot \dot{\theta}^2 = \underline{-2.967}$

$\underline{a_r} = \ddot{r} - r\dot{\theta}^2 = -4.899$

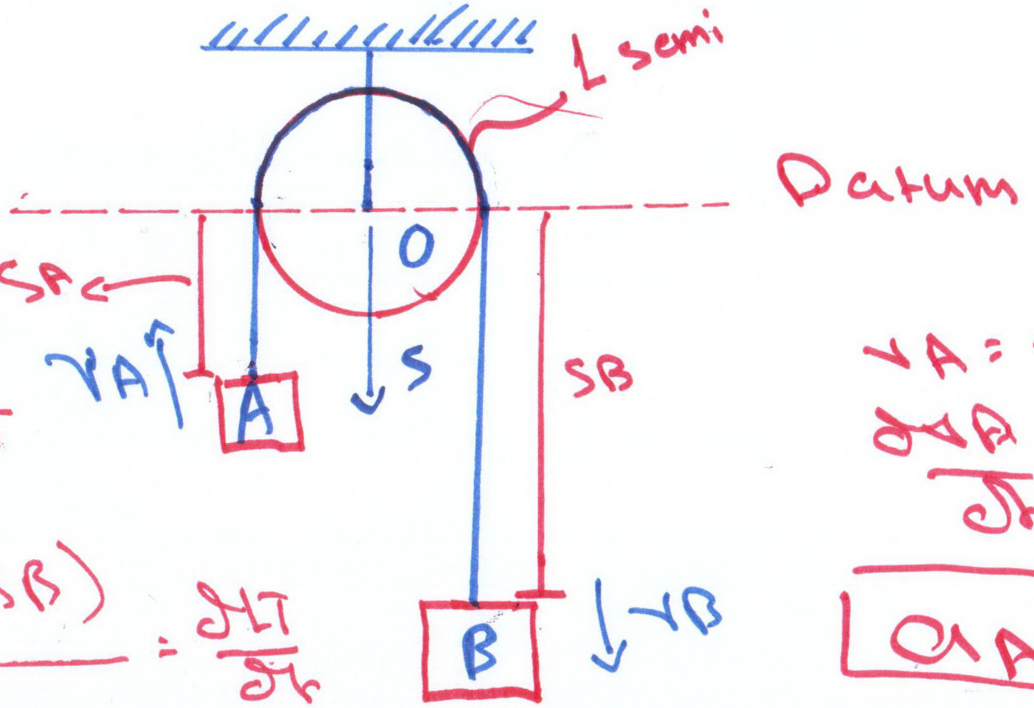
$\underline{a_\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.828$

$\boxed{\sum F_r} = -N\sin 75 = m a_r$

$\boxed{\sum F_\theta} = F - N\cos 75 = m a_\theta$

$$\boxed{\begin{array}{l} N = 5.07N \\ F = \cancel{F}\,4.1MN \end{array}}$$

Ch 12 Section 9: Absolute dependen motion analysis



$$S_A + L_{semi} + S_B = L_T$$

$$\frac{d(S_A + L_{semi} + S_B)}{dt} = \frac{dL_T}{dt}$$

$$\frac{dS_A}{dt} + 0 + \frac{dS_B}{dt} = 0$$

$$v_A + v_B = 0$$

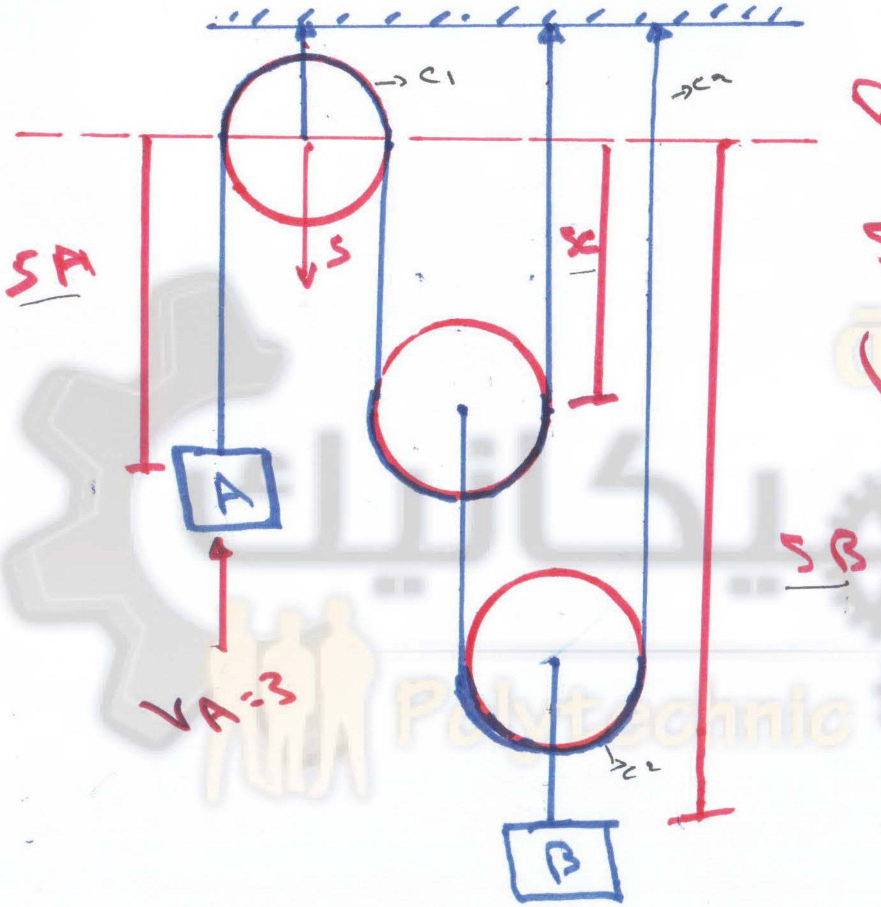
$$\boxed{v_A = -v_B} \text{ — ①}$$

$$v_A = -v_B$$

$$\frac{dv_A}{dt} = -\frac{dv_B}{dt}$$

$$\boxed{a_A = -a_B} \text{ — ②}$$

Ex:



$$S_A + S_c + S_c + c_1 = L_{T_1}$$

$$(S_B - S_c) + S_B + c_2 = L_{T_2}$$

$$S_A + 4S_B = L_{const}$$

$$V_A + 4V_B = 0$$

$$V_B = -\frac{V_A}{4} = \boxed{0.75\,m/s}$$

Ch12 Section 10 : Relative-Motion

* Relative position:

$r_{B/A} = r_B - r_A$ or $r_B = r_A + r_{B/A}$

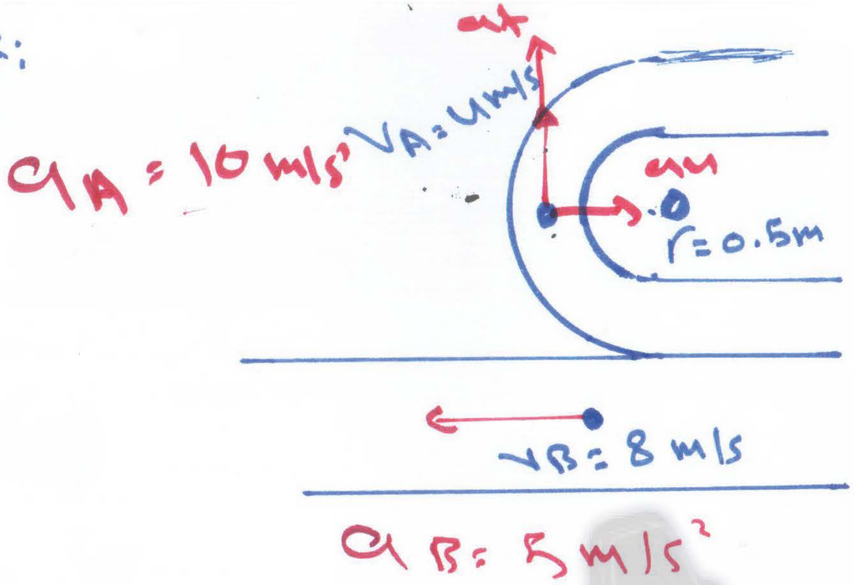
* Relative velocity:

$v_{B/A} = v_B - v_A$ or $v_B = v_A + v_{B/A}$

* Relative acceleration:

$a_{B/A} = a_B - a_A$ or $a_B = a_A + a_{B/A}$

ex:



$v_A = 4j$ m/s

$v_B = -8i$ m/s

$v_{A/B} = v_A - v_B$

$= (8i + 4j)$ m/s

$v_{A/B} = 8.94$ m/s

$(a_A)_t = 10j\ m/s^2$

$(a_A)_n = \left(\frac{v_A^2}{\rho}\right) i = 32i\ m/s^2$

$a_A = (32i + 10j)$

$a_B = -5i$

$a_{A/B} = a_A - a_B$

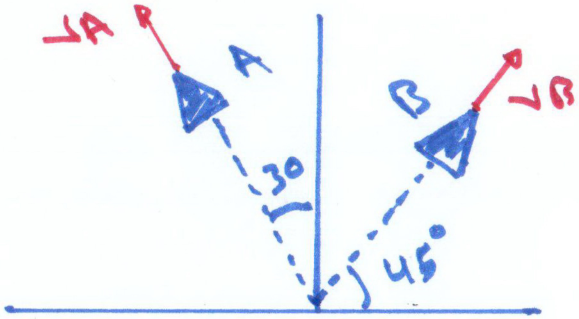
$= 27i + 10j$

$| \xrightarrow{} | = 28.8\ m/s^2$

P 12.227s

$v_A = 20$

$v_B = 15$



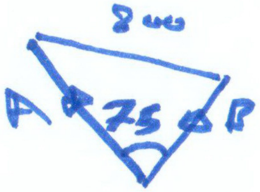
$v_{A/B}$

$$v_A = v_B + v_{A/B}$$

$$-20\sin 30^\circ i + 20\cos 30^\circ j = 15\cos 45\, i + 15\sin 45^\circ j + v_{A/B}$$

$$v_{A/B} = (-20.61\, i + 6.714\, j)$$

$$|\ \ | = 21.7 \text{ m/s}$$



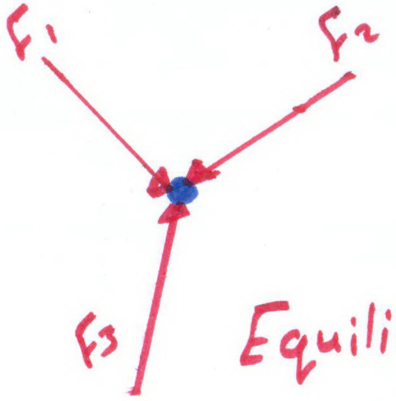
① $$(800)^2 = (20t)^2 + (15t)^2 - 2(20t)(15t)\cos 75$$

$$\boxed{t = 36.9\ s}$$

② $$t = \frac{800}{v_{A/B}} = \frac{800}{21.7} = \boxed{36.9\ s}$$

Ch 13 Equation's of Motion

1) Newton's first law

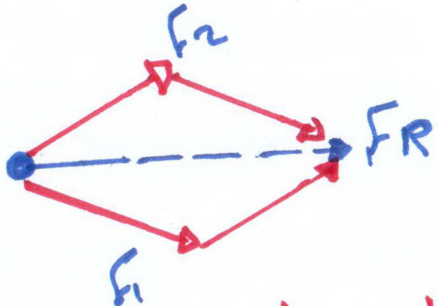


Equilibrium

$$\sum F = 0$$

$$a = \frac{\sum F}{m} = 0$$

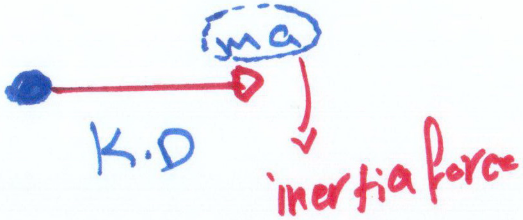
2) Newton's Second law



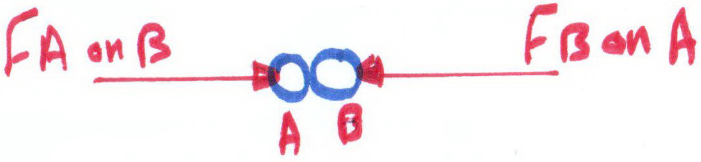
Accelerated Motion

$$\sum F = ma$$

$$a = \frac{\sum F}{m}$$



3) Newton's third law



Action & reaction

* Procedure for problem solving

1] Select the inertial coordinate Sys.
(rectangular, normal/tangential, or cylindrical)

2] Draw F.B.D

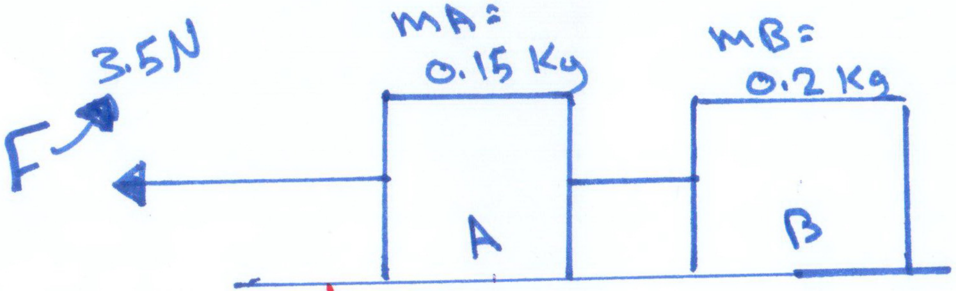
3] Draw K.D

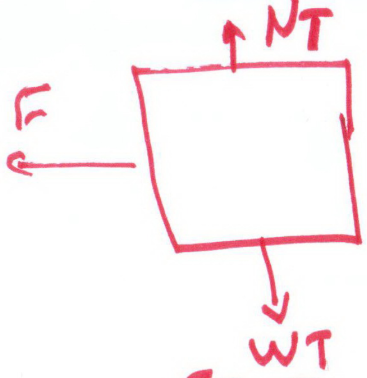
4] Apply the eq. of. Motion

5] Identify the Unknowns & solve it

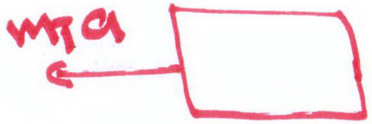
6] $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$, $a\,ds = v\,dv$

Ex: (Rectangular) (Neglect friction) Determine the acceleration? & FAB?





F.B.D

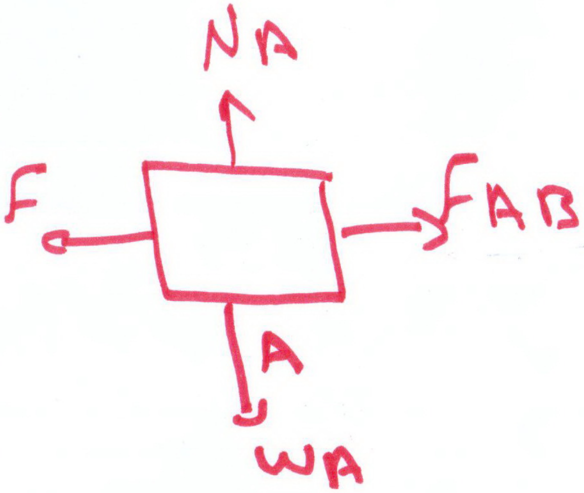


K.D

$$\xleftarrow{+} \sum F_x = F = m_T a_x$$

$$3.5 = (0.15 + 0.2) a_x$$

$$\boxed{a_x = 10 \text{ m/s}^2 \leftarrow}$$



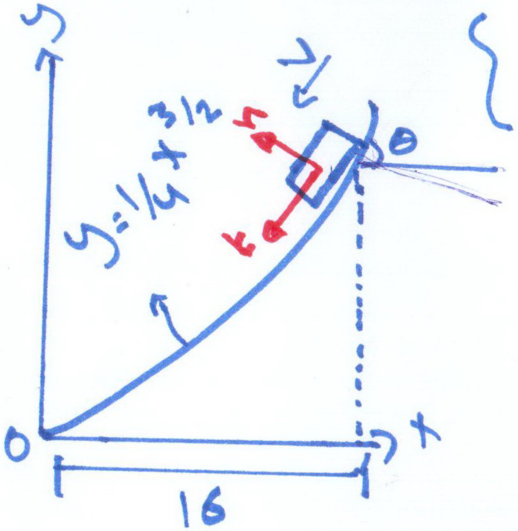
$$\xleftarrow{+} \sum F_x = F - F_{AB} = m_A a_x$$

$$(3.5 - F_{AB}) = (0.15 * 10)$$

$$\boxed{F_{AB} = 2 N}$$

Ex: (Normal, tangential)

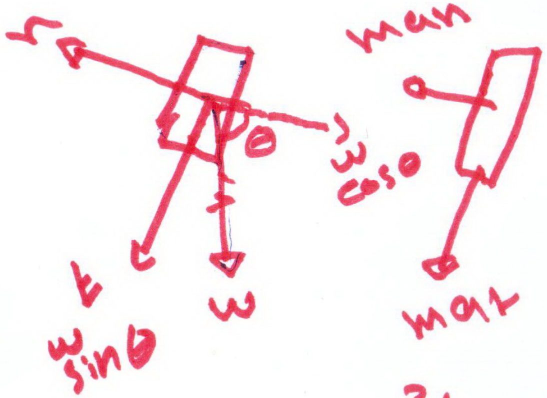
determine the Normal force exerted by the slope? And the acceleration?



$\left\{ \begin{array}{l} m = 10\ kg \\ v = 20\ m/s \end{array} \right.$

↖+ $\sum F_n = N - \overset{(m \cdot g)}{W}\cos\theta = ma_n$

+↙ $\sum F_t = W \sin\theta = ma_t$



$\underline{a_n} = \frac{v^2}{\rho}$

$= \frac{20^2}{125} = \boxed{3.2\ m/s^2}$

$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$

@ $x = 16$ $\frac{dy}{dx} = 1.5$, $\frac{d^2y}{dx^2} = \frac{3}{6}$

$\boxed{\rho = 125 m}$

$\therefore y = \frac{1}{4}x^{3/2}$

slope $= \frac{dy}{dx} = \frac{3}{8}x^{1/2}$

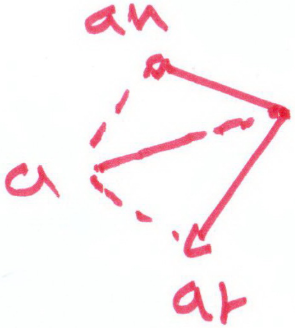
@ $x = 16$ → $\boxed{s = 1.5} = \tan\theta$

$\theta = \tan^{-1} 1.5 = \boxed{56.3^\circ}$

* $\underline{N} - (10 \cdot 9.81)\cos 56.3^\circ = 10 \times 3.2$

* $(10 \times 9.81) \sin 56.3 = 10 \cdot \underline{a_t}$

$\left\{ \begin{array}{l} N = 86.4\ N \\ a_t = 8.16\ m/s^2 \end{array} \right.$

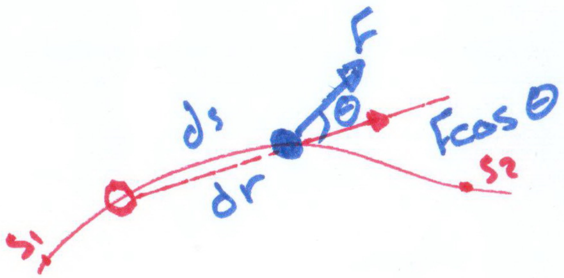


$$a = \sqrt{at^2 + an^2}$$

$$= \boxed{8.77\ m/s^2}$$

ch 14 ⊛ the work of a force

1) work of force



$$dU = F\cos\theta \cdot ds = F \cdot dr$$

$$U_{1-2} = \int_{s_1}^{s_2} F\cos\theta \, ds$$

$$U_{1-2} = \int_{r_1}^{r_2} F \cdot dr$$

$$[N \cdot m] = [J]$$

* Scalar

2) work of weight

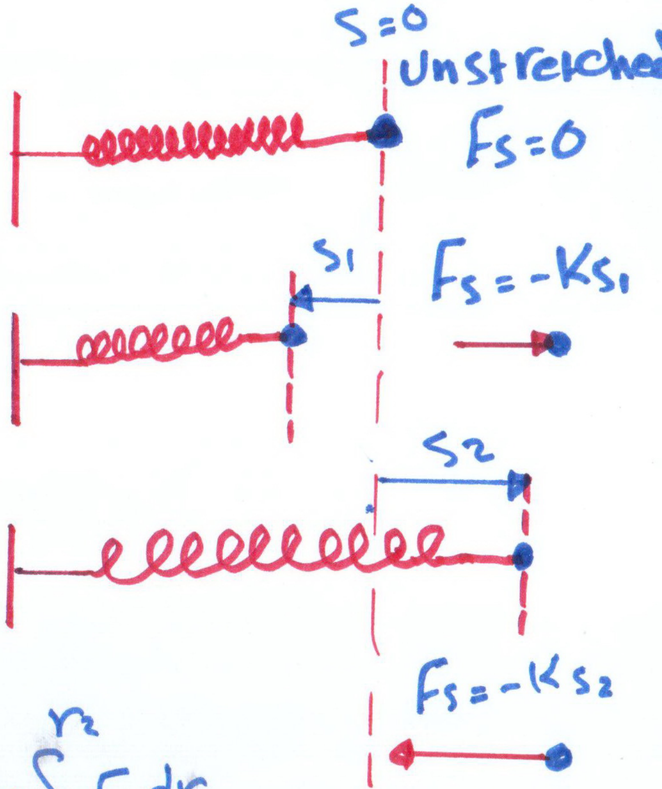
$$U_{w,1-2} = -W(z_2 - z_1)$$

$$= -W(\Delta z)$$

Positive ↓

Negative ↑

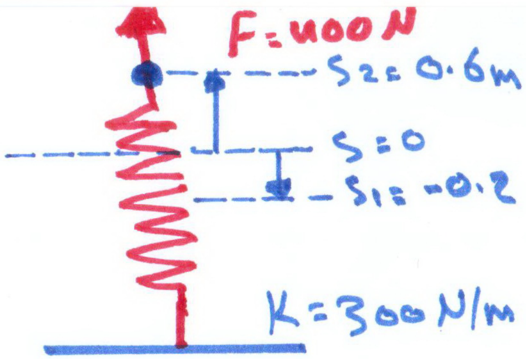
3) work of spring



$$U_{1-2} = \int_{r_1}^{r_2} F \cdot dr$$

$$= \int_{s_1}^{s_2} (-Ks)\, ds = -\frac{1}{2}K(S_2^2 - S_1^2$$

Ex:- The 8 Kg ball is conceected with a Spring Initially the spring is compressed by 0.2m , then Under the applied force F, the spring is stretched to 0.6m Determine the total work done to the ball?



weight . $U_{w,1-2} = -W(S_2 - S_1)$

$= (-8)(9.81)(0.6 - (-0.2))$

$= -62.8 J$

spring $U_{sp,1-2} = -1/2 K (S_2^2 - S_1^2)$

$= -1/2 (300)(0.6^2 - (-0.2)^2)$

$= -48 J$

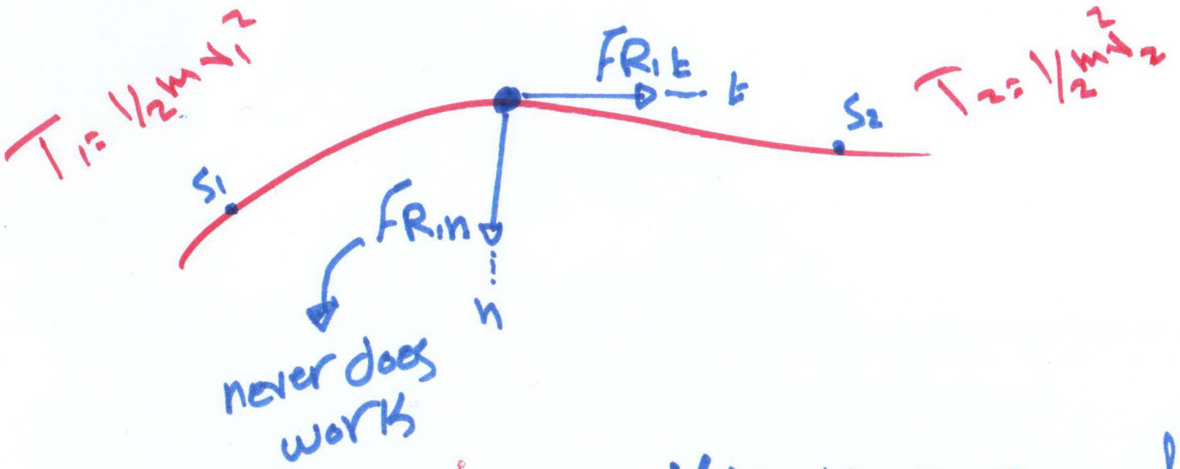
force $U_{f,1-2} = F(S_2 - S_1)$

$= 400(0.6 - (-0.2))$

$= 320 J$

Total W = 209 J

Ch14 * principle of work & energy



$$\begin{Bmatrix} F_{R,t} = m a_t \\ a_t\,ds = v\,dv \end{Bmatrix}$$

$$F_{R,t}\,ds = m v\,dv$$

$$\int_{s_1}^{s_2} F_{R,t}\,ds = m \int_{v_1}^{v_2} v\,dv$$

$$\boxed{\Sigma U_{1-2} = 1/2 m v_2^2 - 1/2 m v_1^2}$$

Kinetic energy of the particle:

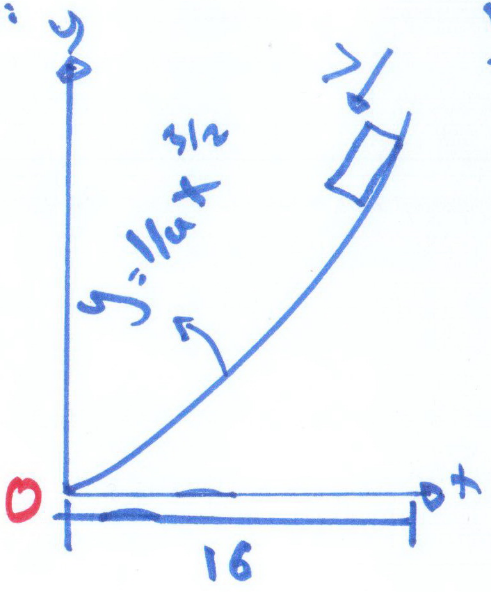
$$\boxed{T = 1/2 m v^2}$$

$$\boxed{\Sigma U_{1-2} = T_2 - T_1}$$

or

$$\boxed{T_1 + \Sigma U_{1-2} = T_2}$$

Ex:



m = 10 Kg
v = 20 m/s

, determine its speed when it gets to the Bottom of the slope (x=0)?

@ $x = 16$, $y_1 = 1/4\,(16)^{3/2} = 16\text{m}$

@ $x = 0$, $y_2 = 0$

$U_{w,1-2} = -W\Delta y = (-10)(9.81) * (0-16) = 1569.6\text{J}$

$T_1 = 1/2\, m_1 v_1^2 = 1/2\,(10)(20)^2 = 2000\text{J}$

$T_2 = 1/2\, m v_2^2 = 5 v_2^2$

$T_1 + \sum U_{1-2} = T_2$

$$\boxed{v_2 = 26.7 \text{ m/s}}$$

Ch.14 ⊛ power & efficiency

power

$$P = \frac{dU}{dt}$$

$$dU = F \cdot dr$$

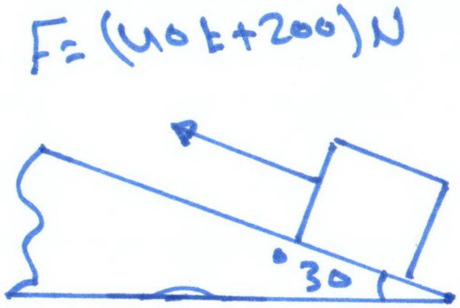
$$P = F \cdot \frac{dr}{dt} = F \cdot v$$ scalar

[J/S] [W]

efficiency

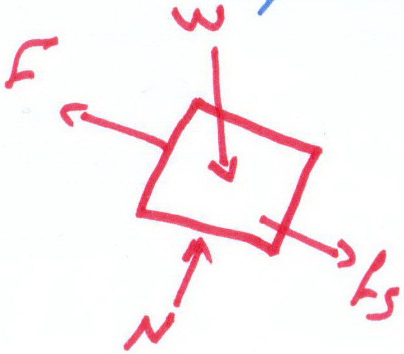
$$\varepsilon = \frac{\text{power output}}{\text{power input}}$$

Ex: The 50 kg, uniform crate initially rests on the inclined surface, if a force as shown starts to pull the crate, determine the power output developed by the force at t = 6 s, the coefficients of static & kinetic friction between the crate & the surface $\mu_s = 0.2$ and $\mu_k = 0.15$.



@ t = 6 s

$P = F.V$



B.M.S

$\Sigma F_x = F - W\sin 30 - f_s = 0$

$\Sigma F_y = N - W\cos 30 = 0$

$\therefore N = 424.8\,N$

$\therefore f_s = \mu_s . N = 0.2 * 424.8$
$= 84.96\,N$

$\therefore F = 330.2\,N = (40t+200)$

$\therefore t = 3.255\,s$

A.M.S

$\Sigma F_x = F - W\sin 30 - f_s = m a_x$

~~ΣF~~ $(40t+200) - 245.3 - (0.15) * (424.8)$

$= 50\, a_x$

$\therefore a_x = 0.8t - 2.18 = \frac{dv_x}{dt}$

$v_x = \int_{3.255}^{6} (0.8t - 2.18)\, dt$

$\therefore v_x = 4.178\,m/s$

$P = F.V_x$

$\boxed{= 1.84\,KW}$

Ch.14 * Conservation of energy

Conservative force

- A force with the property that the work done to the object only depends on it's initial and final positions, but is independent of the path taken

Ex: gravitational force
spring force

Ex: of Non-Conservative force
frictional force

Potential energy

- The ability (the potential) for a force to do work depending on it's position.
- the change in potential energies

ex: $\therefore V_g = Wz$

$\therefore V_e = \frac{1}{2} K s^2$ (always positive)

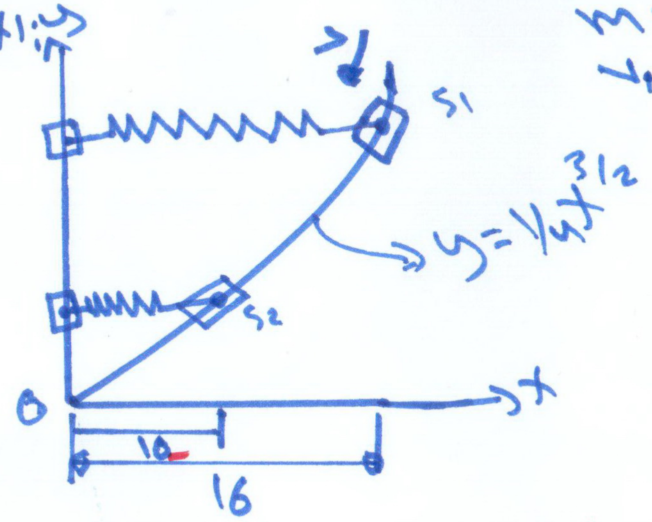
Conservation of energy

$$\underset{\text{Kinetic energy}}{T_1} + \underset{\text{potential energy} - V = V_g + V_e}{V_1} = T_2 + V_2$$

for a system of particles:

$$\boxed{\sum T_1 + \sum V_1 = \sum T_2 + \sum V_2}$$

Ex1:



m = 10 Kg   a smooth slope

v_1 = 20 m/s , determine the speed at S2 , the spring remains horizontal , It has an original length 8m & K = 40 N/m ...?

State $\boxed{1}$

$K.E = T_1 = 1/2\, m v_1^2 = 1/2 (10)(20)^2 = 2000\, J$

$g.P.E = V_{g,1} = W y_1 = (10)(9.81)(1/4 \times 16^{3/2})$

$= 1569.6\, J$

$E.P.E = V_{e,1} = 1/2\, K s_1^2 = 1/2 (40)(16-8)^2$

$= 1280\, J$

State ②

$K.E = T_2 = 1/2 (10) v_2^2 = 5 v_2^2\, J$

$g.P.E = (10)(9.81)(1/4 (10)^{3/2})$

$= 775.5\, J$

$E.P.E = 1/2 (40)(10-8)^2$

$= 80\, J$

$\boxed{\text{Cons. of energy}}$

$T_1 + V_{g_1} + V_{e_1} = T_2 + V_{g_2} + V_{e_2}$

$\boxed{v = 28.3\, m/s}$