# Chapter 5: Kinetic Energy, Work, and Power

## Concept Checks

**5.1.** c **5.2.** b **5.3.** d **5.4.** a) True b) False c) True

## Multiple-Choice Questions

**5.1.** c **5.2.** d **5.3.** a **5.4.** d **5.5.** e **5.6.** e **5.7.** c **5.8.** b **5.9.** b **5.10.** c **5.11.** e **5.12.** d **5.13.** b **5.14.** a

## Conceptual Questions

**5.15.** If the net work done on a particle is zero, then the net force on the particle must also be zero. If the net force is zero, then the acceleration is also zero. Hence, the particle's speed is constant.

**5.16.** If Paul and Kathleen both start from rest at a height $h$, then, by conservation of energy, they will have the same speed when they reach the bottom. That is, their initial energy is pure potential energy $mgh$ and their final energy is pure kinetic energy $(1/2)mv^2$. Since energy is conserved (if we neglect friction!) then $mgh = (1/2)mv^2 \Rightarrow v = \sqrt{2gh}$. Their final velocity is independent of both their mass and the shape of their respective slides! They will in general not reach the bottom at the same time. From the figure, Kathleen will likely reach the bottom first since she will accelerate faster initially and will attain a larger speed sooner. Paul will start off much slower, and will acquire the bulk of his acceleration towards the end of his slide.

**5.17.** No. The gravitational force that the Earth exerts on the Moon is perpendicular to the Moon's displacement and so no work is done.



**5.18.** When the car is travelling at speed $v_1$, its kinetic energy is $(1/2)mv_1^2$. The brakes do work on the car causing it to stop over a distance $d_1$. The final velocity is zero, so the work $Fd_1$ is given by the initial kinetic energy: $(1/2)mv_1^2 = Fd_1$. Similarly, when the car is travelling at speed $v_2$, the brakes cause the car to stop over a distance $d_2$, so we have $(1/2)mv_2^2 = Fd_2$. Taking the ratio of the two equations, we have

$$\frac{(1/2)mv_2^2 = Fd_2}{(1/2)mv_1^2 = Fd_1} \rightarrow d_2 = d_1\frac{v_2^2}{v_1^2} = d_1\frac{(2v_1)^2}{v_1^2} = 4d_1.$$

Thus the braking distance increases by a factor of 4 when the initial speed is increased by a factor of 2.

## Exercises

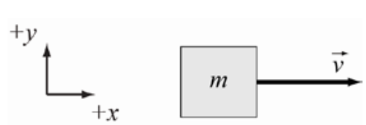
**5.19.** **THINK:** Kinetic energy is proportional to the mass and to the square of the speed. $m$ and $v$ are known for all the objects:

(a) $m = 10.0$ kg, $v = 30.0$ m/s

(b) $m = 100.0$ g, $v = 60.0$ m/s

(c) $m = 20.0$ g, $v = 300.$ m/s

**SKETCH:**




**RESEARCH:** $K = \frac{1}{2}mv^2$

**SIMPLIFY:** $K = \frac{1}{2}mv^2$ is already in the right form.

**CALCULATE:**

(a) $K = \frac{1}{2}(10.0 \text{ kg})(30.0 \text{ m/s})^2 = 4500 \text{ J}$

(b) $K = \frac{1}{2}(100.0 \cdot 10^{-3} \text{ kg})(60.0 \text{ m/s})^2 = 180 \text{ J}$

(c) $K = \frac{1}{2}(20.0 \cdot 10^{-3} \text{ kg})(300. \text{ m/s})^2 = 900 \text{ J}$

**ROUND:**

(a) 3 significant figures: $K = 4.50 \cdot 10^3 \text{ J}$

(b) 3 significant figures: $K = 1.80 \cdot 10^2 \text{ J}$

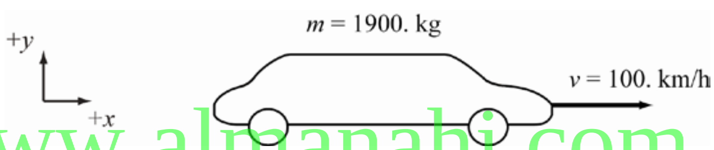
(c) 3 significant figures: $K = 9.00 \cdot 10^2 \text{ J}$

**DOUBLE-CHECK:** The stone is much heavier so it has the greatest kinetic energy even though it is the slowest. The bullet has larger kinetic energy than the baseball since it moves at a much greater speed.

**5.20.** **THINK:** I want to compute kinetic energy, given the mass ( $m = 1900. \text{ kg}$ ) and the speed ( $v = 100. \text{ km/h}$ ). I must first convert the speed to $\text{m/s}$ .

$$100. \frac{\text{km}}{\text{h}} \cdot 10^3 \frac{\text{m}}{\text{km}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = \frac{100. \cdot 10^3 \text{ m}}{3600 \text{ s}} = 27.778 \text{ m/s}$$

**SKETCH:**






**RESEARCH:** $K = mv^2 / 2$

**SIMPLIFY:** No simplification needed.

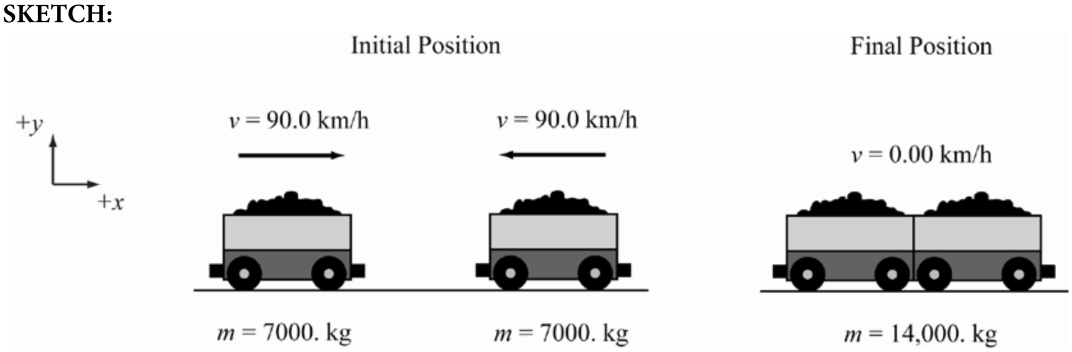
**CALCULATE:** $K = (1/2)mv^2 = (1/2)(1900. \text{ kg})(27.778 \text{ m/s})^2 = 7.3302 \cdot 10^5 \text{ J}$

**ROUND:** $100. \text{ km/h}$ has three significant figures. Round the result to three significant figures: $K = 7.33 \cdot 10^5 \text{ J}$.

**DOUBLE-CHECK:** This is a very large energy. The limo is heavy and is moving quickly.

**5.21.** **THINK:** Since both cars come to rest, the final kinetic energy of the system is zero. All the initial kinetic energy of the two cars is lost in the collision. The mass ( $m = 7000. \text{ kg}$ ) and the speed $v = 90.0 \frac{\text{km}}{\text{h}} \cdot \frac{10^3 \text{ m}}{\text{km}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} = 25.0 \text{ m/s}$ of each car is known. The total energy lost is the total initial kinetic energy.

**SKETCH:**



**RESEARCH:** $K_{lost} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

**SIMPLIFY:** $K_{lost} = 2\left(\frac{1}{2}mv^2\right) = mv^2$

**CALCULATE:** $K_{lost} = 7000.\text{ kg}(25.0\text{ m/s})^2 = 4.375 \cdot 10^6\text{ J}$

**ROUND:** To three significant figures: $K_{lost} = 4.38 \cdot 10^6\text{ J}$.

**DOUBLE-CHECK:** Such a large amount of energy is appropriate for two colliding railroad cars.

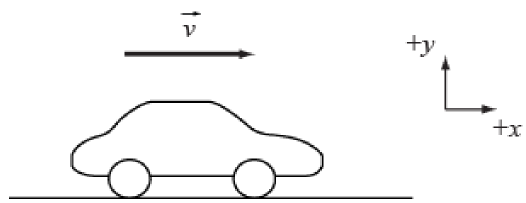
**5.22.** **THINK:** The mass and the speed are given:

(a) $m = 1500.\text{ kg}, \quad v = 15.0\text{ m/s}$

(b) $m = 1500.\text{ kg}, \quad v = 30.0\text{ m/s}$

With this information, I can compute the kinetic energy and compare the results.

**SKETCH:**




$m = 1500.\text{ kg}$

**RESEARCH:** $K = \frac{1}{2}mv^2$

**SIMPLIFY:** b) The change in kinetic energy is the difference of the kinetic energies.



**CALCULATE:**

(a) $K = \frac{1}{2}(1500.\text{ kg})(15.0\text{ m/s})^2 = 1.688 \cdot 10^5\text{ J}$

(b) $K = \frac{1}{2}(1500.\text{ kg})(30.0\text{ m/s})^2 = 6.750 \cdot 10^5\text{ J}$, so the change is $6.750 \cdot 10^5\text{ J} - 1.688 \cdot 10^5\text{ J} = 5.062 \cdot 10^5\text{ J}$.

**ROUND:** Three significant figures:

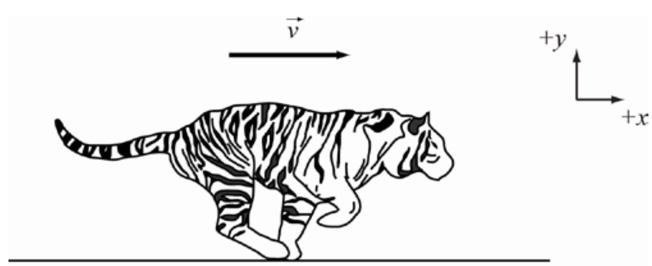
(a) $K = 1.69 \cdot 10^5\text{ J}$

(b) $\Delta K = 5.06 \cdot 10^5\text{ J}$

**DOUBLE-CHECK:** Such large energies are reasonable for a car. Also, when the speed doubles, the kinetic energy increases by a factor of 4, as it should since $K \propto v^2$.

**5.23.** **THINK:** Given the tiger's mass, $m = 200.\text{ kg}$, and energy, $K = 14400\text{ J}$, I want to determine its speed. I can rearrange the equation for kinetic energy to obtain the tiger's speed.

**SKETCH:**




$m = 200.\text{ kg}$

$K = 14400\text{ J}$

**RESEARCH:** $K = \frac{1}{2}mv^2$

**SIMPLIFY:** $v = \sqrt{\frac{2K}{m}}$

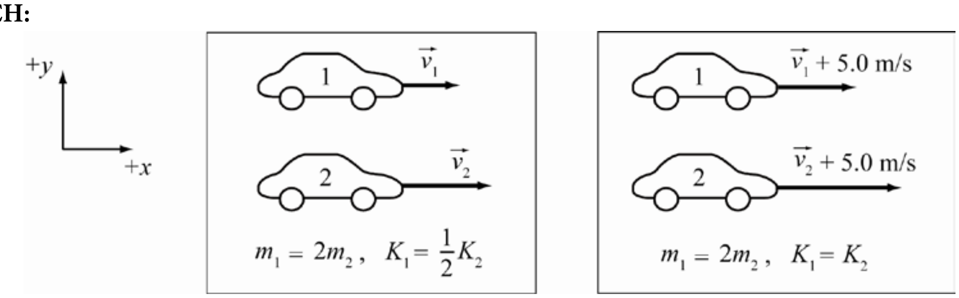
**CALCULATE:** $v = \sqrt{\frac{2(14400 \text{ J})}{200. \text{ kg}}} = 12.0 \text{ m/s}$

**ROUND:** Three significant figures: $v = 12.0$ m/s.

**DOUBLE-CHECK:** $(10 \text{ m/s}) \cdot \frac{10^{-3} \text{ km}}{1 \text{ m}} \cdot \frac{3600 \text{ s}}{1 \text{ h}} = 36 \text{ km/h}$. This is a reasonable speed for a tiger.

**5.24.** **THINK:** I want to calculate the original speeds of both cars. I know only the ratios of their masses and kinetic energies. For speeds $v_1$ and $v_2$: $m_1 = 2m_2$, $K_1 = (1/2)K_2$. For speeds $(v_1 + 5.0 \text{ m/s})$ and $(v_2 + 5.0 \text{ m/s})$: $m_1 = 2m_2$, $K_1 = K_2$.

**SKETCH:**




**RESEARCH:** $K = \frac{1}{2}mv^2$; $m_1 = 2m_2$; $K_1 = \frac{1}{2}K_2$



**SIMPLIFY:** $K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}\left(\frac{1}{2}m_2v_2^2\right) \Rightarrow \frac{1}{2}\left(2m_2v_1^2\right) = \frac{1}{4}m_2v_2^2 \Rightarrow v_1 = \frac{1}{2}v_2$

When the speeds are increased by $\Delta v$, the kinetic energies are equal:

$$K_1 = \frac{1}{2}m_1\left(v_1 + \Delta v\right)^2 = K_2 = \frac{1}{2}m_2\left(v_2 + \Delta v\right)^2$$

$$\frac{1}{2}(2m_2)\left(\frac{1}{2}v_2 + \Delta v\right)^2 = \frac{1}{2}m_2\left(v_2 + \Delta v\right)^2 \Rightarrow (2)\left(\frac{1}{2}v_2 + \Delta v\right)^2 = \left(v_2 + \Delta v\right)^2$$

$$\Rightarrow v_2\left(1 - \frac{\sqrt{2}}{2}\right) = \left(\sqrt{2} - 1\right)\Delta v \Rightarrow v_2 = \Delta v \frac{\left(\sqrt{2} - 1\right)}{\left(1 - \frac{\sqrt{2}}{2}\right)}$$

$$\Rightarrow \sqrt{2}\left(\frac{1}{2}v_2 + \Delta v\right) = \left(v_2 + \Delta v\right) \Rightarrow \frac{\sqrt{2}}{2}v_2 + \sqrt{2}\Delta v = v_2 + \Delta v$$

**CALCULATE:** $v_2 = (5.0 \text{ m/s})\frac{\left(\sqrt{2} - 1\right)}{\left(1 - \frac{\sqrt{2}}{2}\right)} = 7.0711 \text{ m/s}$, $v_1 = \frac{7.0711 \text{ m/s}}{2} \Rightarrow v_1 = 3.5355 \text{ m/s}$

**ROUND:** Two significant figures: $v_1 = 3.5$ m/s, and $v_2 = 7.1$ m/s.

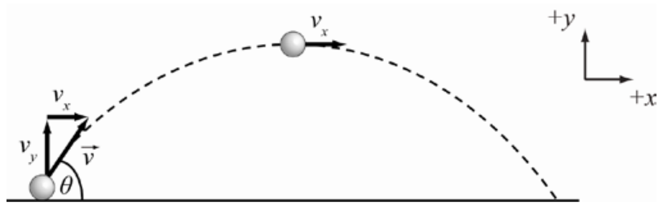
**DOUBLE-CHECK:** If $m_1 = 2m_2$ and $v_1 = (1/2)v_2$, then $K_1 = (1/2)K_2$:

$$K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}(2m_2)(\frac{1}{2}v_2)^2 = \frac{1}{4}m_2v_2^2 = \frac{1}{2}\left(\frac{1}{2}m_2v_2^2\right) = \frac{1}{2}K_2$$

The results are consistent.

**5.25.** **THINK:** At the apex of the projectile's trajectory, the only velocity is in the horizontal direction. The only force is gravity, which acts in the vertical direction. Hence the horizontal velocity is constant. This velocity is simply the horizontal component of the initial velocity or $27.3 \text{ m/s}$ at an angle of $46.9°$. The mass of the projectile is $20.1 \text{ kg}$.

**SKETCH:**




**RESEARCH:** $E = \frac{1}{2}mv_x^2$ (at the apex, $v_y = 0$); $v_x = v\cos\theta$

**SIMPLIFY:** $K = \frac{1}{2}mv^2\cos^2\theta$

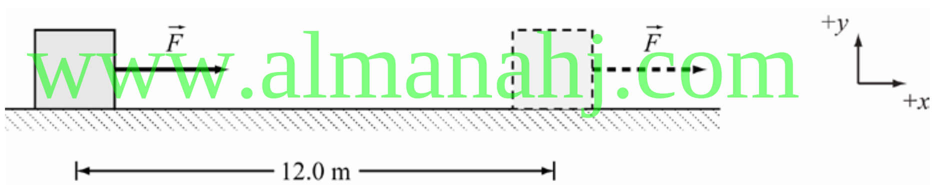
**CALCULATE:** $E = \frac{1}{2}(20.1 \text{ kg})(27.3 \text{ m/s})^2\cos^2(46.9°) = 3.497 \cdot 10^3 \text{ J}$

**ROUND:** 3 significant figures: $K = 3.50 \cdot 10^3$ J.

**DOUBLE-CHECK:** For such a heavy mass moving at a large speed, the result is reasonable.

**5.26.** **THINK:** A force $F = 5.00 \text{ N}$ moves an object a distance $d = 12.0 \text{ m}$. The object moves parallel to the force.

**SKETCH:**





**RESEARCH:** $W = Fd\cos\theta$

**SIMPLIFY:** $\theta = 0 \Rightarrow W = Fd$

**CALCULATE:** $W = (5.00 \text{ N})(12.0 \text{ m}) = 60.0 \text{ J}$

**ROUND:** Three significant figures: $W = 60.0 \text{ J}$.

**DOUBLE-CHECK:** This is a relatively small force over a moderate distance, so the work done is likewise moderate.

**5.27.** **THINK:** The initial speeds are the same for the two balls, so they have the same initial kinetic energy. Since the initial height is also the same for both balls, the gravitational force does the same work on them on their way down to the ground, adding the same amount of kinetic energy in the process. This automatically means that they hit the ground with the same value for their final kinetic energy. Since the balls have the same mass, they consequently have to have the same speed upon ground impact. This means that the difference in speeds that the problem asks for is 0. No further steps are needed in this solution.

**SKETCH:** Not necessary.

**RESEARCH:** Not necessary.

**SIMPLIFY:** Not necessary.

**CALCULATE:** Not necessary.

**ROUND:** Not necessary.

**DOUBLE-CHECK:** Even though our arguments based on kinetic energy show that the impact speed is identical for both balls, you may not find this entirely convincing. After all, most people expect the ball throw directly downward to have a higher impact speed. If you still want to perform a double-check, then you can return to the kinematic equations of chapter 3 and calculate the answer for both cases. Remember that the motion in horizontal direction is one with constant horizontal velocity component, and the motion in vertical direction is free-fall. In both cases we thus have:

$$v_x = v_{0x}$$
$$v_y = \sqrt{v_{0y}^2 + 2gh}$$

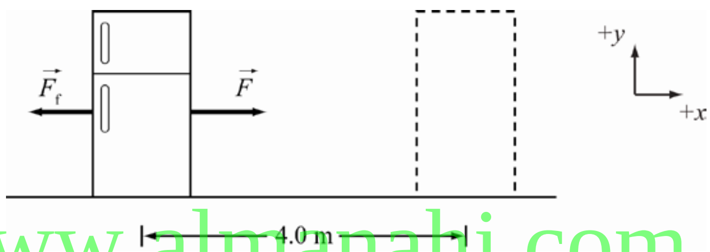
If you now square each equation and add them, you get:

$$v^2 = v_x^2 + v_y^2 = \left(v_{0x}^2\right) + \left(v_{0y}^2 + 2gh\right) = \left(v_{0x}^2 + v_{0y}^2\right) + 2gh = v_0^2 + 2gh$$

Then you see that indeed we have each time for the final speed $v = \sqrt{v_0^2 + 2gh}$, independent of the direction of the initial velocity vector. What we can learn from this double-check step is two-fold. First, our energy and work considerations yield the exact same results as our kinematic equations from Chapter 3 did. Second, and perhaps more important, the energy and work considerations required much less computational effort to arrive at the same result.

**5.28.** **THINK:** The object moves at constant speed so the net force is zero. The force applied is then equal to the force of friction. $F_\text{f} = 180$ N , $d = 4.0$ m , $m = 95$ kg.

**SKETCH:**






**RESEARCH:** $W = Fd\cos\theta$

**SIMPLIFY:** $\theta = 0 \Rightarrow W = -Fd = F_\text{f}d$

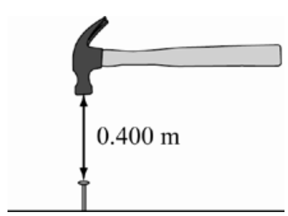
**CALCULATE:** $W = F_\text{f}d = (180 \text{ N})(4.0 \text{ m}) = 720 \text{ J}$

**ROUND:** 2 significant figures: $W = 720$ J

**DOUBLE-CHECK:** If we applied a force greater than 180 N, the object would accelerate. 720 J is reasonable for pushing a refrigerator 4.0 m.

**5.29.** **THINK:** The maximum amount of work that the hammerhead can do on the nail is equal to the work that gravity does on the hammerhead during the fall. $h = 0.400$ m and $m = 2.00$ kg.

**SKETCH:**




**RESEARCH:** The work done by gravity is $W_\text{g} = mgh$ and this is equal to the maximum work $W$ that the hammerhead can do on the nail.

**SIMPLIFY:** $W = mgh$

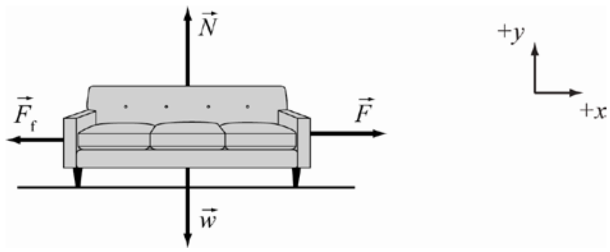
**CALCULATE:** $W = (2.00 \text{ kg})\left(9.81 \text{ m/s}^2\right)(0.400 \text{ m}) = 7.848 \text{ J}$

**ROUND:** Three significant figures: $W = 7.85$ J.

**DOUBLE-CHECK:** This result is reasonable. If the hammerhead had an initial velocity, more work could be done.

**5.30.** **THINK:** Only those forces that have a component along the couch's displacement contribute to the force. You push your couch with a force of $F = 200.0$ N a distance of $d = 4.00$ m. The frictional force opposes the motion, so the direction of $F_\text{f}$ is opposite to $F$.

**SKETCH:**




**RESEARCH:** $W = Fd\cos\theta$. Friction: $\theta = 180°$, $W_\text{f} = -F_\text{f}d$; You: $\theta = 0°$, $W_\text{you} = Fd$; Gravity: $\theta = 90°$, $W_g = Fd_\text{y}$; Net: $W_\text{net} = Fd - F_\text{f}d$.

**SIMPLIFY:** $W_\text{you} = Fd$

$W_\text{f} = -F_\text{f}d$

$W_\text{g} = Fd\cos 90.°$

$W_\text{net} = d(F - F_\text{f})$

**CALCULATE:** $W_\text{you} = (4.00 \text{ m})(200.0 \text{ N}) = 800.00 \text{ J}$

$W_\text{f} = -(4.00 \text{m})(150.0 \text{ N}) = -600.0 \text{ J}$

$W_\text{g} = 0$

$W_\text{net} = (4.00 \text{ m})(200.0 \text{ N} - 150.0 \text{ N}) = 200.0 \text{ J}$

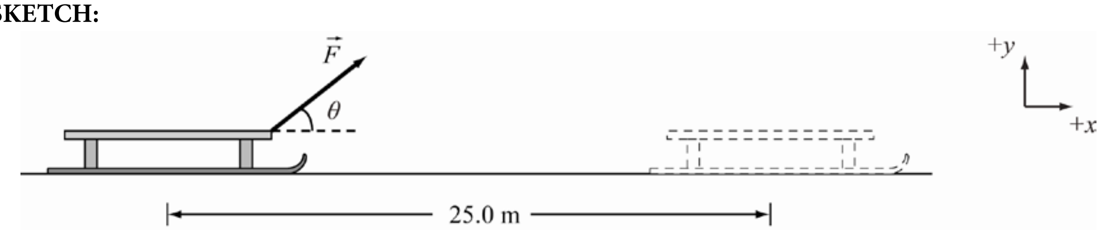
**ROUND:** Since the distance is given to three significant figures, $W_\text{you} = 8.00 \cdot 10^2$ J, $W_\text{f} = -6.00 \cdot 10^2$ J, $W_\text{g} = 0$, and $W_\text{net} = 2.00 \cdot 10^2$ J.



**DOUBLE-CHECK:** The work done by the person is greater than the work done by friction. If it was not, the couch would not move. The units of the work calculations are Joules, which are appropriate for work.

**5.31.** **THINK:** Only the component of the force parallel to the displacement does work.

**SKETCH:**




**RESEARCH:** $W = Fd\cos\theta$

**SIMPLIFY:** Not applicable.

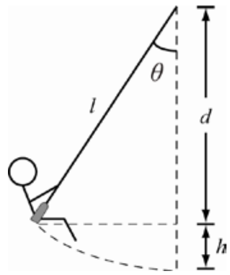
**CALCULATE:** $W = Fd\cos\theta = (25.0 \text{ N})(25.0 \text{ m})\cos 30.0° = 5.4127 \cdot 10^2 \text{ J}$

**ROUND:** Three significant figures: $W = 5.41 \cdot 10^2$ J.

**DOUBLE-CHECK:** The magnitude of the work done by the person is greater than the magnitude of the work done by friction. The units of the work calculations are joules, which are appropriate for work.

**5.32.** **THINK:** Neglect friction and use conservation of energy. Take the zero of gravitational potential energy to be the bottom of the swinging arc. Then, the speed at the bottom of the swinging motion can be determined from the fact that the initial potential energy is all converted to kinetic energy.

**SKETCH:**




**RESEARCH:** $\frac{1}{2}mv^2 + mgh = C$

**SIMPLIFY:** Initial: $v = 0,\ h = l - d = l(1-\cos\theta)$, $\frac{1}{2}mv^2 + mgh = 0 + mgl(1-\cos\theta) = E$

Final: $h = 0,\ \frac{1}{2}mv^2 + 0 = E$

$E_\text{i} = E_\text{f} \ \Rightarrow\ \frac{1}{2}mv^2 = mgl(1-\cos\theta) \ \Rightarrow\ v = \sqrt{2gl(1-\cos\theta)}$

**CALCULATE:** $v = \sqrt{2\left(9.81\text{ m/s}^2\right)(3.00\text{ m})(1-\cos 33.6°)} = 3.136\text{ m/s}$

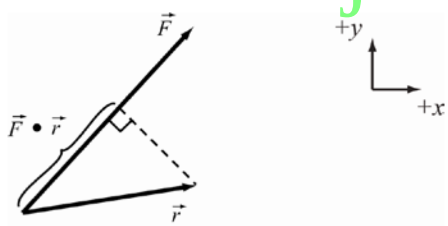
**ROUND:** Three significant figures: $v = 3.14$ m/s.

**DOUBLE-CHECK:** The result is independent of mass here because both potential and kinetic energy depend linearly on mass.

**5.33.** **THINK:** The scalar product can be used to determine the work done, since the vector components of the force $\vec{F} = (4.79, -3.79, 2.09)$ N and the displacement $\vec{r} = (4.25,\ 3.69,\ -2.45)$ m, are given.



**SKETCH:**




**RESEARCH:** $W = \vec{F} \bullet \vec{r}$

**SIMPLIFY:** $W = F_x r_x + F_y r_y + F_z r_z$

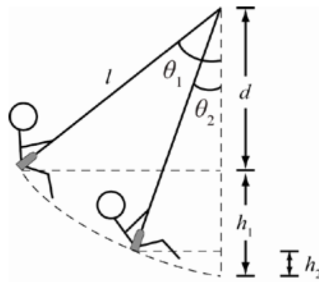
**CALCULATE:** $W = \left[(4.79)(4.25) + (-3.79)(3.69) + (2.09)(-2.45)\right] = 1.2519$ J

**ROUND:** 3 significant figures: $W = 1.25$ J .

**DOUBLE-CHECK:** The work done is much less than $\left|\vec{F}\right| \cdot \left|\vec{r}\right|$ since the force and the displacement are not parallel.

**5.34.** **THINK:** Take the zero of gravitational potential energy to be at the bottom of the swinging arc. The final speed can then be determined using conservation of energy.

**SKETCH:**




**RESEARCH:** $\frac{1}{2}mv^2 + mgh = \text{constant}$

**SIMPLIFY:** At $\theta_1$ : $v = 0,\ h = h_1 \ \Rightarrow\ \frac{1}{2}m(0)^2 + mgh_1 = \text{const.} = E \ \Rightarrow\ mgh_1 = E$

At $\theta_2$: $h = h_2 \ \Rightarrow\ \frac{1}{2}mv^2 + mgh_2 = E = mgh_1 \ \Rightarrow\ \frac{1}{2}v^2 = g\left(h_1 - h_2\right) \ \Rightarrow\ v = \sqrt{2g(h_1 - h_2)}$

$\Rightarrow\ v = \sqrt{2g\left[l(1-\cos\theta_1) - l(1-\cos\theta_2)\right]} = \sqrt{2gl(-\cos\theta_1 + \cos\theta_2)}$

**CALCULATE:** $v = \sqrt{2\left(9.81\ \text{m/s}^2\right)(3.50\ \text{m})(-\cos 35.0° + \cos 15.0°)} = 3.1747\ \text{m/s}$

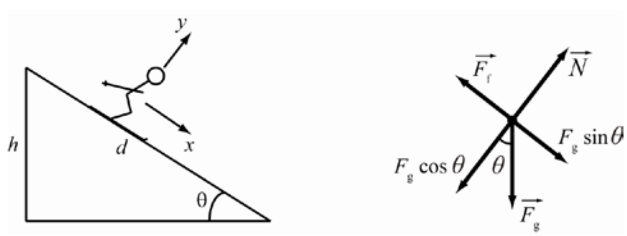
**ROUND:** 3 significant figures: $v = 3.17\ \text{m/s}$

**DOUBLE-CHECK:** The result is independent of mass and the final velocity seems reasonable.

**5.35.** **THINK:** The work done by gravity is $mgh$. In the absence of friction, the potential energy $mgh$ will be converted to kinetic energy. The actual kinetic energy, when friction is included is less than this. The "missing" energy is the work done by friction. If the work done by friction is known, the frictional force and the coefficient of friction can be determined.



**SKETCH:**




**RESEARCH:** $W = W_g + W_f = \frac{1}{2}mv^2,\ W_g = mgh,\ W_f = F_f d$

**SIMPLIFY:** $W_f = W - W_g \ \Rightarrow\ F_f d = \frac{1}{2}mv^2 - mgh \ \Rightarrow\ -\mu N d = m\left(\frac{1}{2}v^2 - gh\right)$

But $N = F_g \cos\theta = mg\cos\theta \ \Rightarrow\ -\mu mg\cos\theta d = m\left(\frac{1}{2}v^2 - gh\right) \ \Rightarrow\ \mu = \frac{1}{gd\cos\theta}\left(gh - \frac{1}{2}v^2\right)$

**CALCULATE:** $g = \left(9.81\ \text{m/s}^2\right)\left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right) = 32.185\ \text{ft/s}^2$,

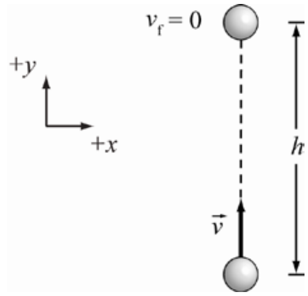
$$\mu = \frac{\left(32.185\ \text{ft/s}^2\right)(80.0\sin 30.0°\ \text{ft}) - (0.5)\left(45.0\ \text{ft/s}\right)^2}{\left(32.185\ \text{ft/s}^2\right)(80.0\ \text{ft})\cos 30.0°} = 0.123282$$

**ROUND:** Three significant figures: $\mu = 0.123$.

**DOUBLE-CHECK:** This is a reasonable result for the friction coefficient. If I had used SI units, the result would be the same because $\mu$ is dimensionless.

**5.36.** **THINK:** The molecule's initial speed can be determined from its mass and kinetic energy. At the highest point all the initial kinetic energy has been converted to potential energy.

**SKETCH:**




**RESEARCH:** $E = \frac{1}{2}mv^2 + mgh = \text{constant}$. When $h = 0$, $E_i = \frac{1}{2}mv^2$. When $v_f = 0$, $E_f = mgh$.

**SIMPLIFY:** $E$ is known: $E = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2E}{m}}$. At the maximum height $E = mgh \Rightarrow h = \frac{E}{mg}$.

% of Earth's radius: $p = \frac{h}{R_E} \times 100\%$

**CALCULATE:** $v = \sqrt{\frac{2(6.2 \cdot 10^{-21}\text{ J})}{4.7 \cdot 10^{-26}\text{ kg}}} = 513.64\text{ m/s}$

$$h = \frac{6.2 \cdot 10^{-21}\text{ J}}{(4.7 \cdot 10^{-26}\text{ kg})(9.81\text{ m/s}^2)} = 1.3447 \cdot 10^4\text{ m}$$

$$p = \frac{(1.3447 \cdot 10^4\text{ m})}{(6.37 \cdot 10^6\text{ m})} \times 100\% = 0.21110\%$$

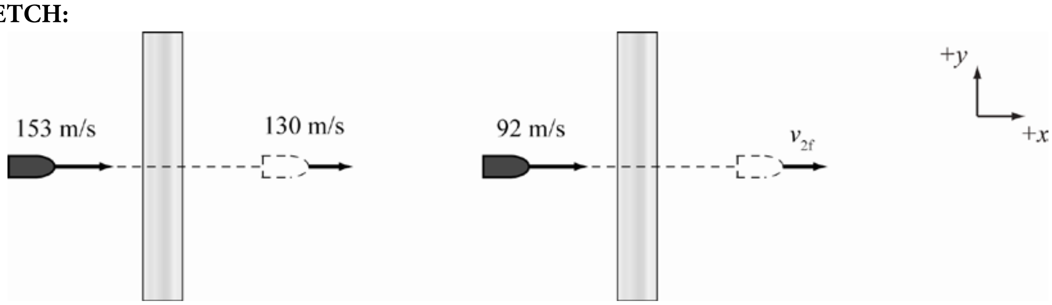


**ROUND:** 2 significant figures: $v = 510\text{ m/s}$, $h = 1.3 \cdot 10^4$ m, and $p = 0.21\%$.

**DOUBLE-CHECK:** The particle is not expected to escape the Earth's gravity, or to reach relativistic speeds. This lends support to the reasonableness of the answers.

**5.37.** **THINK:** If the resistance of the plank is independent of the bullet's speed, then both bullets should lose the same amount of energy while passing through the plank. From the first bullet, the energy loss can be determined. This can then be used to determine the second bullet's final speed.

**SKETCH:**




**RESEARCH:** $K = \frac{1}{2}mv^2$, $\Delta K = \frac{1}{2}m(v_{1f}^2 - v_{1i}^2) = \frac{1}{2}m(v_{2f}^2 - v_{2i}^2)$

**SIMPLIFY:** $(v_{1f}^2 - v_{1i}^2) = (v_{2f}^2 - v_{2i}^2)$, $v_{2f} = \sqrt{v_{1f}^2 - v_{1i}^2 + v_{2i}^2}$

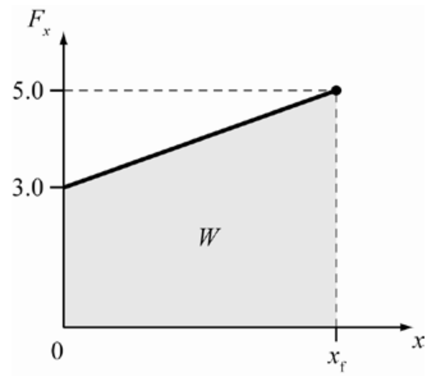
**CALCULATE:** $v_{2f} = \sqrt{(130.\text{ m/s})^2 - (153\text{ m/s})^2 + (92.0\text{ m/s})^2} = 44.215\text{ m/s}$

**ROUND:** By the rule for subtraction, the expression inside the square root has two significant figures. Rounding to two significant figures: $v_{2f} = 44$ m/s.

**DOUBLE-CHECK:** The final speed should be positive because the bullet is still moving to the right. The final speed should also be less than the initial speed. The answer is reasonable.

**5.38.** **THINK:** An expression $F_x$ as a function of $x$ is given, $F_x = (3.0+0.50x)$ N. The work done by the force must be determined.

**SKETCH:**




**RESEARCH:** Recall that the work done by a variable force is $W = \int_{x_i}^{x_f} F(x)dx$, or the area under the curve of $F$ versus $x$ plot.

**SIMPLIFY:** $W = \int_{x_i}^{x_f} F(x)dx = \int_0^4 (3.0+0.50x)dx$

**CALCULATE:** $W = \int_0^{4.0} (3.0+0.50x)dx = \left[3x+\frac{1}{4}x^2\right]_{x=0}^{x=4.0} = 3(4)+\frac{1}{4}(4)^2 - 0 = 12+4 = 16$ J

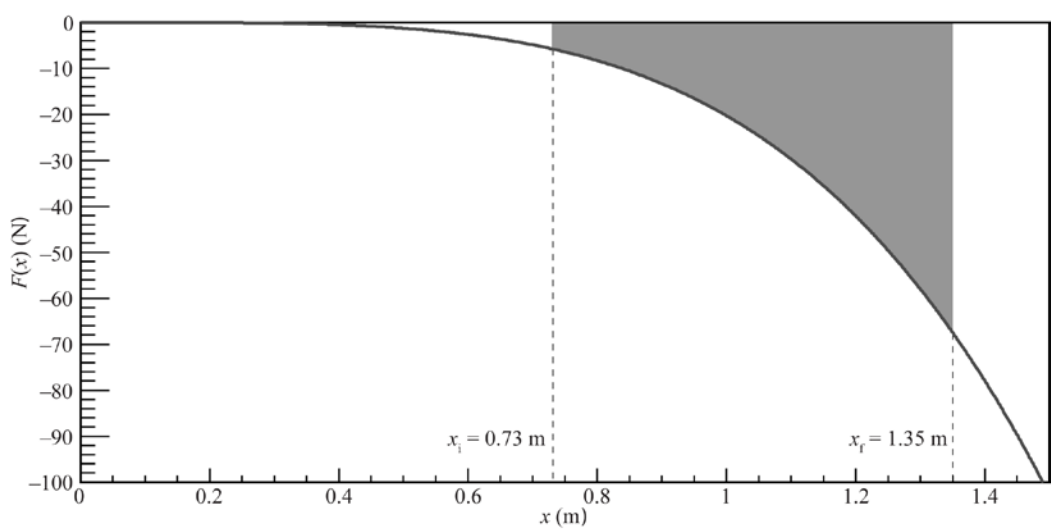


**ROUND:** $W = 16$ J

**DOUBLE-CHECK:** Since $F_x$ is a linear function of $x$, $W = \vec{F}(\Delta x)$. $\vec{F}$ is the average force. $\vec{F} = 4.0$ N and $\Delta x = 4.0$ m. So $W = 4.0(4.0) = 16$ J, as expected.

**5.39.** **THINK:** Determine the work necessary to change displacement from 0.730 m to 1.35 m for a force of $F(x) = -kx^4$ with a constant $k = 20.3$ N/m$^4$.

**SKETCH:**



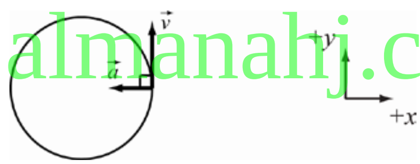
**RESEARCH:** The work done by the available force is $W = \int_{x_i}^{x_f} F(x)dx$.

**SIMPLIFY:** $W = \int_{x_i}^{x_f} -kx^4 dx = \left[-\frac{k}{5}x^5\right]_{x_i}^{x_f} = \frac{k}{5}x_i^{\,5} - \frac{k}{5}x_f^{\,5} = \frac{k}{5}\left(x_i^{\,5} - x_f^{\,5}\right)$

**CALCULATE:** $W = \frac{20.3 \text{ N/m}^4}{5}\left[(0.730 \text{ m})^5 - (1.35 \text{ m})^5\right] = -17.364 \text{ J}$

**ROUND:** Due to the difference, the answer has three significant figures. The work done *against* the spring force is the negative of the work done *by* the spring force: $W = 17.4$ J.
**DOUBLE-CHECK:** The negative work in this case is similar to the work done by a spring force.

**5.40.** **THINK:** Find a relationship between $\vec{a}(t)$ and $\vec{v}(t)$ for a body of mass $m$ moving along a trajectory $\vec{r}(t)$ at constant kinetic energy.
**SKETCH:** Not necessary.
**RESEARCH:** Kinetic energy $K(t) = \text{constant}$. Therefore, the work done by a force $\vec{F} = m\vec{a}$ is zero since $W = \Delta K = 0$ at all times. This means $P = dW/dt = 0$.
**SIMPLIFY:** $P = \vec{F} \bullet \vec{v} = m\vec{a} \bullet \vec{v} = 0 \Rightarrow \vec{a} \bullet \vec{v} = 0$. The acceleration vector is perpendicular to the velocity vector.
**CALCULATE:** Not necessary.
**ROUND:** Not necessary.
**DOUBLE-CHECK:** If a particle is moving in a circular motion at constant speed the kinetic energy is constant. The acceleration vector is always perpendicular to the velocity vector.





**5.41.** **THINK:** $\vec{F}(x) = 5x^3\hat{x} \text{ N/m}^3$ $F(x) = 5x^3\hat{x} \text{ N/m}^3$ is acting on a 1.00 kg mass. The work done from $x = 2.00$ m to $x = 6.00$ m must be determined.
**SKETCH:** Not applicable.
**RESEARCH:**

(a) Work done by a variable force is $W = \int_{x_i}^{x_f} F(x)dx$.

(b) Work-kinetic energy relation is $W = \Delta K$.
**SIMPLIFY:**

(a) $W = \int_{x_i}^{x_f} (5x^3)dx = \left[\frac{5}{4}x^4\right]_{x_i}^{x_f} = \frac{5}{4}\left(x_f^{\,4} - x_i^{\,4}\right)$

(b) $W = \Delta K = K_f - K_i = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2} = \frac{1}{2}m\left(v_f^{\,2} - v_i^{\,2}\right) \Rightarrow v_f^{\,2} = \frac{2W}{m} + v_i^{\,2} \Rightarrow v_f = \sqrt{\frac{2W}{m} + v_i^{\,2}}$

**CALCULATE:**

(a) $W = \left(\frac{5}{4}\frac{\text{N}}{\text{m}^3}\right)\left[(6.00 \text{ m})^4 - (2.00 \text{ m})^4\right] = 1600 \text{ J}$

(b) $v_f = \sqrt{\frac{2(1600 \text{ J})}{1.00 \text{ kg}} + (2.00 \text{ m/s})^2} = 56.6039 \text{ m/s}$

**ROUND:** Quantities in the problem are given to three significant figures.

(a) $W = 1.60 \cdot 10^3$ J

(b) $v_f = 56.6$ m/s

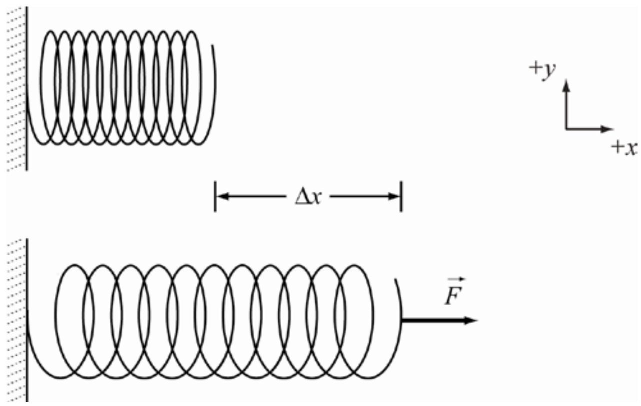
**DOUBLE-CHECK:** Since $v = dx/dt$, $a = dv/dt = (dv/dx)(dx/dt) = v(dv/dx)$. So, $vdv = adx$.

$$\int_{v_i}^{v_f} vdv = \int_{x_i}^{x_f} adx \Rightarrow \frac{1}{2}(v_f^2 - v_i^2) = \int_{x_i}^{x_f} \frac{F}{m}dx \Rightarrow \frac{1}{2}m(v_f^2 - v_i^2) = \int_{x_i}^{x_f} Fdx \Rightarrow \frac{1}{2}m(v_f^2 - v_i^2) = W$$

This is the same as above.

**5.42.** **THINK:** The spring has a constant $k$ = 440 N/m. The displacement from its equilibrium must be determined for $W$ = 25 J.

**SKETCH:**




**RESEARCH:** $W = \frac{1}{2}kx^2$

**SIMPLIFY:** $x = \sqrt{\frac{2W}{k}}$



**CALCULATE:** $x = \sqrt{\frac{2(25 \text{ J})}{440 \text{ N/m}}} = 0.3371$ m

**ROUND:** $x$ = 0.34 m

**DOUBLE-CHECK:** Because the value of $k$ is large, a small displacement is expected for a small amount of work.

**5.43.** **THINK:** The spring constant must be determined given that it requires 30.0 J to stretch the spring 5.00 cm $= 5.00 \cdot 10^{-2}$ m. Recall that the work done by the spring force is always negative for displacements from equilibrium.

**SKETCH:** Not necessary.

**RESEARCH:** $W_s = -\frac{1}{2}kx^2$

**SIMPLIFY:** $k = -\frac{2W_s}{x^2}$

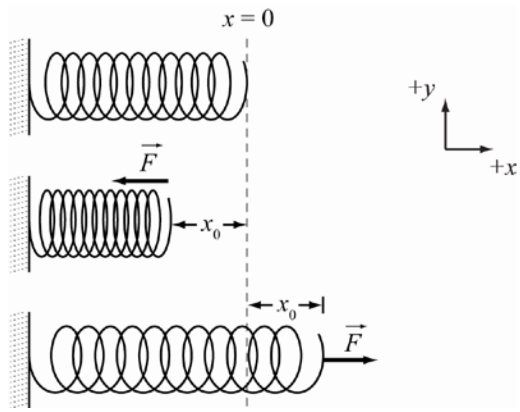
**CALCULATE:** $k = -\frac{2(-30.0 \text{ J})}{(5.00 \cdot 10^{-2} \text{ m})^2} = 2.40 \cdot 10^4$ N/m

**ROUND:** Variables in the question are given to three significant figures, so the answer remains $k = 2.40 \cdot 10^4$ N/m.

**DOUBLE-CHECK:** Because the displacement is in the order $10^{-2}$ m, the spring constant is expected to be in the order of $1/\left(10^{-2}\right)^2 \approx 10^4$.

**5.44.** **THINK:** The ratio of work done on a spring when it is stretched and compressed by a distance $x_0$ is to be determined.

**SKETCH:**




**RESEARCH:** $W=(1/2)kx^2$. When the spring is stretched, the work done is $W_s=(1/2)kx_0^{\ 2}$. When the spring is compressed, the work done is $W_c=(1/2)kx_0^{\ 2}$.

**SIMPLIFY:** $\text{Ratio}=\dfrac{W_s}{W_c}$

**CALCULATE:** $\text{Ratio}=\dfrac{W_s}{W_c}=\left(\dfrac{1}{2}kx_0^{\ 2}\right)/\left(\dfrac{1}{2}kx_0^{\ 2}\right)=1$

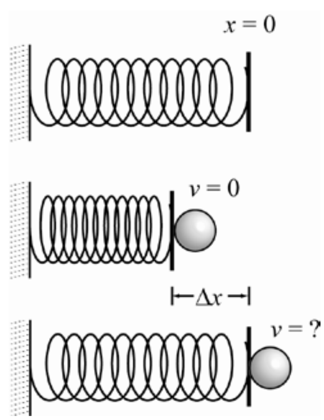
**ROUND:** Not necessary.

**DOUBLE-CHECK:** The work done on a spring is the same, regardless if it compressed or stretched; provided the displacement is the same.



**5.45.** **THINK:** The spring has a constant of 238.5 N/m and $\Delta x=0.231\text{ m}$. The steel ball has a mass of 0.0413 kg. The speed of the ball as it releases from the spring must be determined.

**SKETCH:**




**RESEARCH:** $W=\dfrac{1}{2}kx_\text{i}^{\ 2}-\dfrac{1}{2}kx_\text{f}^{\ 2}$, $W=\Delta K=K_\text{f}-K_\text{i}$

**SIMPLIFY:** $x_\text{f}=0$, $K_\text{i}=0$ and $K_\text{f}=\dfrac{1}{2}mv_\text{f}^{\ 2}$. It follows that:

$$W=\frac{1}{2}kx_\text{i}^{\ 2}-0=K_\text{f}-0\ \Rightarrow\ \frac{1}{2}kx_\text{i}^{\ 2}=\frac{1}{2}mv_\text{f}^{\ 2}\ \Rightarrow\ v_\text{f}=\sqrt{\frac{kx_\text{i}^{\ 2}}{m}}\ \Rightarrow\ v_\text{f}=|x_\text{i}|\sqrt{\frac{k}{m}}$$

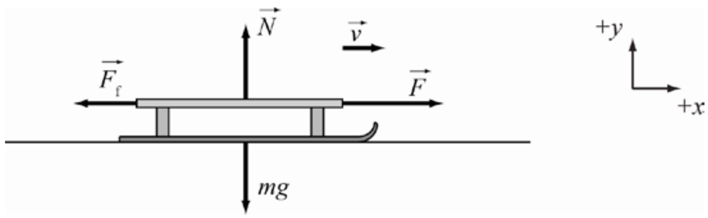
**CALCULATE:** $x_\text{i}=-\Delta x=-0.231\text{ m}$, $v_\text{f}=0.231\text{ m}\sqrt{\dfrac{238.5\text{ N/m}}{0.0413\text{ kg}}}=17.554\text{ m/s}$

**ROUND:** $v_f = 17.6$ m/s (three significant figures)

**DOUBLE-CHECK:** Energy stored in the spring is transferred to kinetic energy of the ball, $(1/2)kx^2 = (1/2)mv^2$.

**5.46.** **THINK:** Determine the power needed to move a sled and load with a combined mass of 202.3 kg at a speed of 1.785 m/s if the coefficient of friction between the sled and snow is 0.195.

**SKETCH:**




**RESEARCH:** Use Newton's second law, $F_f = \mu N$ and $P = \vec{F} \bullet \vec{v}$. Since $a_x = 0$, $\sum F_x = 0 \Rightarrow F - F_f = 0 \Rightarrow F = F_f = \mu N$. Also, $\sum F_y = ma_y = 0 \Rightarrow N - mg = 0 \Rightarrow N = mg$.

**SIMPLIFY:** $F = \mu mg$ and $P = \mu mgv$.

**CALCULATE:** $P = (0.195)(202.3 \text{ kg})(9.81 \text{ m/s}^2)(1.785 \text{ m/s}) = 690.78$ W

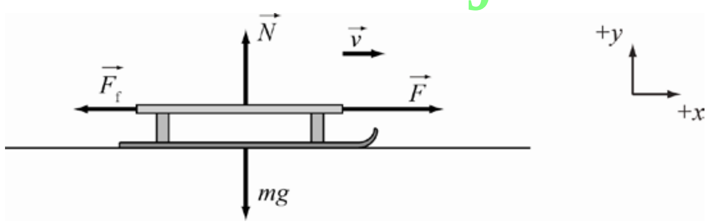
**ROUND:** $P = 691$ W

**DOUBLE-CHECK:** 1 horse power (hp) = 746 W. Our result is about 1 hp, which is reasonable since the sled is drawn by a horse.

**5.47.** **THINK:** Determine the constant speed of a sled drawn by a horse with a power of 1.060 hp. The coefficient of friction is 0.115 and the mass of the sled with a load is 204.7 kg. $P = 1.060 \text{ hp}(746 \text{ W/hp}) = 791 \text{ W}$.



**SKETCH:**




**RESEARCH:** Use Newton's second law, $F_f = \mu N$ and $P = \vec{F} \bullet \vec{v}$. $\sum F_x = 0$, since $a_x = 0$. So $F - F_f = 0$ $\Rightarrow F = F_f = \mu N$ and $\sum F_y = ma_y = 0 \Rightarrow N - mg = 0 \Rightarrow N = mg$.

**SIMPLIFY:** $P = Fv \Rightarrow v = \dfrac{P}{F} = \dfrac{P}{\mu N} = \dfrac{P}{\mu mg}$

**CALCULATE:** $v = \dfrac{791 \text{ W}}{(0.115)(204.7 \text{ kg})(9.81 \text{ m/s}^2)} = 3.42524$ m/s

**ROUND:** $v = 3.43$ m/s (three significant figures)

**DOUBLE-CHECK:** $v = 3.43$ m/s = 12.3 km/h, which is a reasonable speed.

**5.48.** **THINK:** Determine the power supplied by a towline with a tension of 6.00 kN which tows a boat at a constant speed of 12 m/s.

**SKETCH:** Not necessary.

**RESEARCH:** $P = Fv$

**SIMPLIFY:** Not required.

**CALCULATE:** $P = (6.00 \cdot 10^3 \text{ N})(12 \text{ m/s}) = 72.0 \cdot 10^3 \text{ W} = 72.0$ kW

**ROUND:** Not necessary.

**DOUBLE-CHECK:** $P = 72{,}000\text{ W}\left(\dfrac{1\text{ hp}}{746\text{ W}}\right) = 96\text{ hp}$, which is a reasonable value.

**5.49.** **THINK:** A car with a mass of 1214.5 kg is moving at 62.5 mph $\left(\dfrac{0.447\text{ m/s}}{1\text{ mph}}\right)$ = 27.94 m/s and comes to rest in 0.236 s. Determine the average power in watts.

**SKETCH:** Not required.

**RESEARCH:** Work, $W = \Delta K$ and the average power $\overline{P} = \dfrac{W}{\Delta t}$.

**SIMPLIFY:** $|P| = \left|\dfrac{W}{\Delta t}\right| = \left|\dfrac{\Delta K}{\Delta t}\right| = \left|\dfrac{\frac{1}{2}m(v_f^{\,2} - v_i^{\,2})}{\Delta t}\right|$. $v_f = 0$, so $|P| = \left|\dfrac{-\frac{1}{2}mv_i^{\,2}}{\Delta t}\right| = \dfrac{\frac{1}{2}mv_i^{\,2}}{\Delta t}$

**CALCULATE:** $|P| = \dfrac{\frac{1}{2}(1214.5\text{ kg})(27.94\text{ m/s})^2}{0.236\text{ s}} = 2.0087\cdot 10^6\text{ W}$

**ROUND:** Three significant figures: $|P| = 2.01\cdot 10^6\text{ W}$

**DOUBLE-CHECK:** Without the absolute values, the value would have been negative. The omitted negative sign on the power would indicate that the power is released by the car. It is expected that to make a car stop in a short time a large amount of power must be expended.

**5.50.** **THINK:** Determine the retarding force acting on a car travelling at 15.0 m/s with an engine expending $40.0\text{ hp}\left(\dfrac{746\text{W}}{1\text{ hp}}\right) = 29{,}840\text{W}$.



**SKETCH:** Not necessary.

**RESEARCH:** $P = Fv$

**SIMPLIFY:** $F = \dfrac{P}{v}$

**CALCULATE:** $F = \dfrac{29840\text{ W}}{15.0\text{ m/s}} = 1989.33\text{ N}$

**ROUND:** $F = 1990\text{ kN}$

**DOUBLE-CHECK:** Assume the mass of the car is 1000 kg and the coefficient of friction is about 0.1. The force of friction is about: $F = \mu N = \mu mg = 0.1(1000)(9.81) = 981\text{ N} \approx 1\text{ kN}$. So, the result is reasonable.

**5.51.** **THINK:** Determine the speed of a 942.4 kg car after 4.55 s, starting from rest with a power output of 140.5 hp. $140.5\text{ hp}(746\text{ W/hp}) = 104{,}813\text{ W}$.

**SKETCH:** Not necessary.

**RESEARCH:** Use the definition of average power, $\overline{P} = \dfrac{W}{\Delta t}$, and the work-kinetic energy relation, $W = \Delta K$.

**SIMPLIFY:** $\overline{P} = \dfrac{W}{\Delta t} = \dfrac{\Delta K}{\Delta t} = \dfrac{K_f - K_i}{\Delta t} = \dfrac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{\Delta t}$. $v_i = 0$, so $\overline{P} = \dfrac{mv_f^2}{2\Delta t} \Rightarrow v_f = \sqrt{\dfrac{2\overline{P}\Delta t}{m}}$.

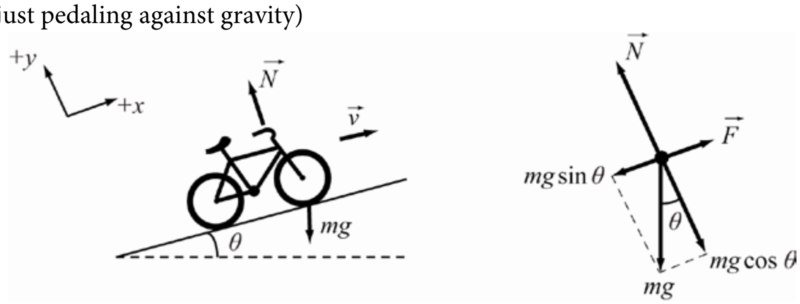
**CALCULATE:** $v_f = \sqrt{\dfrac{2(104{,}813\text{ W})(4.55\text{ s})}{942.4\text{ kg}}} = 31.81\text{ m/s}$

**ROUND:** $v_f = 31.8\text{ m/s}$

**DOUBLE-CHECK:** $v_f = 31.8\text{ m/s} = 114\text{ km/h}$. This represents a large acceleration, but the car is very light. This is consistent with a high performance sports car.

**5.52.** **THINK:** If you ride your bicycle on a horizontal surface and stop pedaling, you slow down to a stop. The force that causes this is the combination of friction in the mechanical components of the bicycle, air resistance, and rolling friction between the tires and the ground. In the first part of the problem statement we learn that the bicycle rolls down the hill at a constant speed. This automatically implies that the net force acting on it is zero. (Newton's First Law!) The force along the slope and downward is $mg\sin\theta$ (see sketch). For the net force to be zero this force has to be balanced by the force of friction and air resistance, which acts opposite to the direction of motion, in this case up the slope. So we learn from this first statement that the forces of friction and air resistance have exactly the same magnitude in this case as the component of the gravitational force along the slope. But if you go up the same slope, then gravity and the forces of air resistance and friction point in the same direction. Thus we can calculate the total work done against all forces in this case (and only in this case!) by just calculating the work done against gravity, and then simply multiplying by a factor of 2.

**SKETCH:** (for just pedaling against gravity)




**RESEARCH:** Again, let's just calculate the work done against gravity, and then in the end multiply by 2. The component of the gravitational force along the slope is $mg\sin\theta$. $F$ is the force exerted by the bicyclist. Power $= Fv$. Using Newton's second law:

$$\sum F_x = ma_x = 0 \Rightarrow F - mg\sin\theta = 0 \Rightarrow F = mg\sin\theta$$



**SIMPLIFY:** Power $= 2Fv = 2(mg\sin\theta)v$

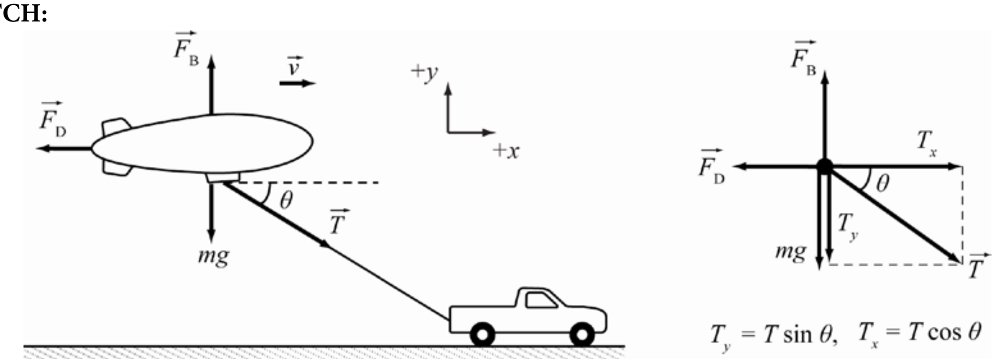
**CALCULATE:** $P = 2 \cdot 75\text{ kg}\left(9.81\text{ m/s}^2\right)\sin(7.0°)\left(5.0\text{ m/s}\right) = 896.654\text{ W}$

**ROUND:** $P = 0.90\text{ kW}$

**DOUBLE-CHECK:** $P = 900\text{ W}\left(\dfrac{1\text{ hp}}{746\text{ W}}\right) = 1.2\text{ hp}$. As this shows, going up a 7 degree slope at 5 m/s requires approximately 1.2 horsepower, which is what a good cyclist can expend for quite some time. (But it's hard!)

**5.53.** **THINK:** A blimp with a mass of 93.5 kg is pulled by a truck with a towrope at an angle 53.3° from the horizontal. The height of the blimp is $h = 19.5$ m and the truck moves for 840.5 m at a constant velocity $v = 8.90$ m/s. The drag coefficient of air is $k = 0.500$ kg/m. Determine the work done by the truck.

**SKETCH:**




**RESEARCH:** The tension in the towrope can be determined using Newton's second law.

$$\sum F_x = T_x - F_D = ma_x = 0$$
$$T_x = T\cos\theta = F_D = Kv^2$$

**SIMPLIFY:** The work done by the truck is: $W = \vec{F}_D \bullet \vec{d} = (T\cos\theta)(d) = Kv^2 d.$

**CALCULATE:** $W = (0.500 \text{ kg/m})(8.90 \text{ m/s})^2 (840.5 \text{ m}) = 33{,}288 \text{ J}$

**ROUND:** Rounding to three significant digits, $W = 3.33 \cdot 10^4$ J.

**DOUBLE-CHECK:** It is expected that the work is large for a long distance *d*.

**5.54.** **THINK:** A car of mass *m* accelerates from rest with a constant engine power *P*, along a track of length *x*.

(a) Find an expression for the vehicle's velocity as a function of time, $v(t)$.

(b) A second car has a constant acceleration $a$. I want to know which car initially takes the lead, and whether the other car overtakes it.

(c) Find the minimum power output, $P_0$, required to win a race against a car that accelerates at a constant rate of $a = 12 \text{ m/s}^2$. This minimum value occurs when the distance at which my car overtakes the other car is equal to the length of the track.

**SKETCH:** Not necessary.

**RESEARCH:**

(a) Use the relation between power and work, $P = W/\Delta t$ and $W = \Delta K$.

(b) $v_2 = at + v_0$ ,

(c) Use the result from part *b*, $x = \dfrac{512 P_o^2}{81 m_1^2 a^3} \Rightarrow P_0 = \sqrt{\dfrac{81 x m_1^2 a^3}{512}}$.

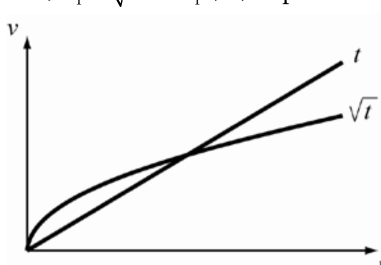
The typical track is a quarter mile long. $x = 0.250 \text{ mi}\left(\dfrac{1609 \text{ m}}{\text{mi}}\right) = 402.25 \text{ m} = 402 \text{ m}.$



**SIMPLIFY:**

(a) $P = \dfrac{W}{\Delta t} = \dfrac{\Delta E}{\Delta t} = \dfrac{\frac{1}{2} m \Delta v^2}{\Delta t}$, $v = \sqrt{\dfrac{2P\Delta t}{m}}$

(b) $v_0 = 0 \Rightarrow v_2 = at$. As a comparison, $v_1 = \sqrt{2P/m_1}\sqrt{t}$ , so plot $v = t$ and $v = \sqrt{t}$ .




By looking at the area under the curve for the distance traveled that the first car initially takes the lead but after a time *t*, the second car overtakes the first car. Assume this occurs at distances $x_1 = x_2$ .

$$x = \int_0^t v\,dt + x_0 \text{ , } x_0 = 0 \Rightarrow x = \int_0^t v\,dt$$

So, $x_1 = \sqrt{\dfrac{2P_0}{m_1}}\displaystyle\int_0^t t^{1/2}dt = \sqrt{\dfrac{2P_0}{m_1}}\left(\dfrac{2}{3}\right)t^{3/2}$, $x_2 = a\displaystyle\int_0^t t\,dt = \dfrac{1}{2}at^2$, $x_1 = x_2 = x_0$.

(c) $P_0 = m_1\sqrt{\dfrac{81}{512}x_0 a_0^3}$ (see below)

**CALCULATE:**
(a) Not necessary.

(b) $x_1 = x_2 \Rightarrow \sqrt{\frac{2P}{m_1}}\left(\frac{2}{3}\right)t^{3/2} = \frac{1}{2}at^2 \Rightarrow \sqrt{\frac{2P}{m_1}}\frac{4}{3a} = t^{1/2} \Rightarrow t = \left(\frac{2P}{m_1}\right)\left(\frac{4}{3a}\right)^2 = \frac{32P}{9m_1a^2}$

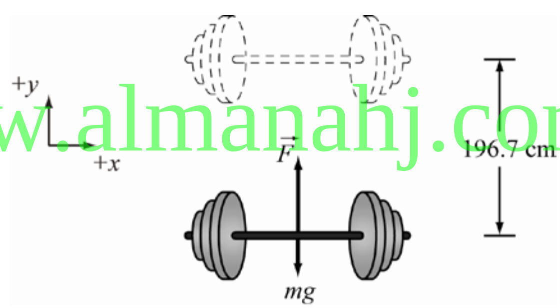
$$x_0 = \frac{1}{2}at^2 = \frac{1}{2}a\left(\frac{32P}{9m_1a^2}\right)^2 = \frac{512P^2}{81m_1^2a^3}$$

(c) $P_0 = (1000.\text{ kg})\sqrt{\frac{81}{512}(402\text{ m})(12.0\text{ m/s}^2)^3} = 331507\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = 444.38\text{ hp}$

**ROUND:**
(a) Not necessary.
(b) Not necessary.
(c) $P_o = 444$ hp

**DOUBLE-CHECK:**
(a) Not necessary.
(b) Not necessary.
(c) Even a typical sports car does not have an overall power band of 444 hp. This car must be some kind of professional drag racer.

**5.55.** **THINK:** Determine the work done by an athlete that lifted 472.5 kg to a height of 196.7 cm.
**SKETCH:**






**RESEARCH:** Use $W = Fd$. F is the combined force needed to the lift the weight, which is $F = mg$.

**SIMPLIFY:** $W = mgd$

**CALCULATE:** $W = (472.5\text{ kg})(9.81\text{ m/s}^2)(1.967\text{ m}) = 9117.49\text{ J}$

**ROUND:** Rounding to three significant figures, $W = 9.12$ kJ.
**DOUBLE-CHECK:** A large amount of work is expected for such a large weight.

**5.56.** **THINK:** Determine the amount of work done in lifting a 6 kg weight a distance of 20 cm.
**SKETCH:** Not necessary.
**RESEARCH:** Use $W = Fd$ and $F = mg$.

**SIMPLIFY:** $W = mgd$

**CALCULATE:** $W = (6.00\text{ kg})(9.81\text{ m/s}^2)(0.200\text{ m}) = 11.772\text{ J}$

**ROUND:** $W = 11.8$ J
**DOUBLE-CHECK:** For such a small distance, a small amount of work is expected.

**5.57.** **THINK:** Determine the power in kilowatts and horsepower developed by a tractor pulling with a force of 14.0 kN and a speed of 3.00 m/s.
**SKETCH:** Not necessary.
**RESEARCH:** $P = Fv$
**SIMPLIFY:** Not necessary.

**CALCULATE:** $P=(14.0\text{ kN})(3.00\text{ m/s})=42.0\text{ kW}=42000\text{ W}\left(\dfrac{1\text{ hp}}{746\text{ W}}\right)=56.3\text{ hp}$

**ROUND:** Variables in the question are given to three significant figures, so the answers remain $P=42.0\text{ kW}=56.3\text{ hp}$.

**DOUBLE-CHECK:** This is a reasonable value for a tractor.

**5.58.** **THINK:** It is given that a mass of $m$ = 7.3 kg with initial speed $v_\text{i}=0$ is accelerated to a final speed of $v_\text{f}=14\text{ m/s}$ in 2.0 s. Determine the average power of the motion.

**SKETCH:** Not necessary.

**RESEARCH:** $W=\Delta K=\frac{1}{2}m(v_\text{f}^2-v_\text{i}^2),\;\; P=\dfrac{W}{\Delta t}$

**SIMPLIFY:** $P=\dfrac{W}{\Delta t}=\dfrac{\Delta K}{\Delta t}=\dfrac{\frac{1}{2}m(v_\text{f}^2-v_\text{i}^2)}{\Delta t}.\;\; v_\text{i}=0,\text{ so }\; P=\dfrac{\frac{1}{2}mv_\text{f}^2}{\Delta t}.$

**CALCULATE:** $P=\dfrac{\frac{1}{2}(7.3\text{ kg})(14\text{ m/s})^2}{2.0\text{ s}}=357.7\text{ W}$

**ROUND:** $P=360\text{ W}$

**DOUBLE-CHECK:** 360 W is equivalent to about half a horsepower, so this is a reasonable result.

**5.59.** **THINK:** A car with mass $m=1200.$ kg can accelerate from rest to a speed of 25.0 m/s in 8.00 s. Determine the average power produced by the motor for this acceleration.

**SKETCH:** Not necessary.

**RESEARCH:** $W=\Delta K,\;\; P=\dfrac{W}{\Delta t}$



**SIMPLIFY:** $P=\dfrac{W}{\Delta t}=\dfrac{\Delta K}{\Delta t}=\dfrac{\frac{1}{2}m(v_\text{f}^2-v_\text{i}^2)}{\Delta t}.\;\; v_\text{i}=0,\text{ so }\; P=\dfrac{\frac{1}{2}mv_\text{f}^2}{\Delta t}.$

**CALCULATE:** $P=\dfrac{\frac{1}{2}(1200.\text{ kg})(25.0\text{ m/s})^2}{8.00\text{ s}}=46875\text{ W}\left(\dfrac{1\text{ hp}}{746\text{ W}}\right)=62.835\text{ hp}$

**ROUND:** Three significant figures: $P=62.8\text{ hp}$

**DOUBLE-CHECK:** An average car motor has a power between 100 and 500 hp. This result is reasonable for a small car.

**5.60.** **THINK:** Determine the work that must be done to stop a car of mass $m$ = 1250 kg traveling at a speed $v_0$ = 105 km/h (29.2 m/s).

**SKETCH:** Not necessary.

**RESEARCH:** $W=\Delta K,\;\; v_\text{i}=29.2\text{ m/s},\; v_\text{f}=0$

**SIMPLIFY:** $W=\frac{1}{2}m(v_\text{f}^2-v_\text{i}^2)=\frac{1}{2}m(0-v_\text{i}^2)=-\frac{1}{2}mv_\text{i}^2$

**CALCULATE:** $W=-\frac{1}{2}(1250\text{ kg})(29.2\text{ m/s})^2=-532900\text{ J}$

**ROUND:** $W=-533\text{ kJ}$

**DOUBLE-CHECK:** A negative amount of work means that the force to stop the car must be in the opposite direction to the velocity. This value is reasonable to stop a car moving at this speed.

**5.61.** **THINK:** A bowstring exerts an average force $F$ = 110. N on an arrow with a mass $m$ = 0.0880 kg over a distance $d$ = 0.780 m. Determine the speed of the arrow as it leaves the bow.

**SKETCH:** Not necessary.

**RESEARCH:** $W = Fd = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$

**SIMPLIFY:** $Fd = \frac{1}{2}m(v_f^2 - 0) \Rightarrow v_f = \sqrt{\frac{2Fd}{m}}$

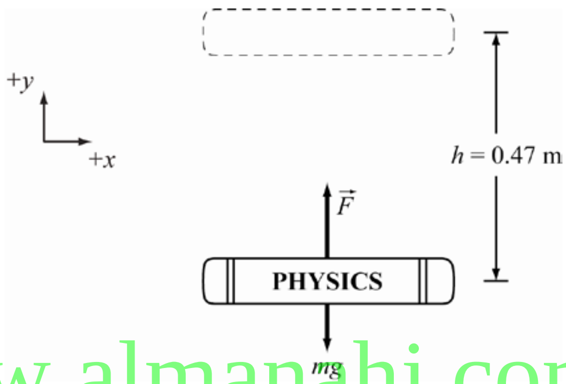
**CALCULATE:** $v_f = \sqrt{\frac{2(110.\text{ N})(0.780\text{ m})}{0.0880\text{ kg}}} = 44.159\text{ m/s}$

**ROUND:** $v_f = 44.2\text{ m/s}$

**DOUBLE-CHECK:** As a comparison, the speed of a rifle bullet is about 1000 m/s and the speed of sound is 343 m/s. This result is reasonable.

**5.62.** **THINK:** A textbook with a mass $m$ = 3.4 kg is lifted to a height $h$ = 0.47 m at a constant speed of $v$ = 0.27 m/s.

**SKETCH:**






**RESEARCH:**

(a) Work is given by $W = Fh\cos\theta$, where $\theta$ is the angle between $F$ and $h$. The force of gravity is given by $F_g = mg$.

(b) Power is given by $P = Fv$.

**SIMPLIFY:**

(a) $W_g = F_g h\cos\theta,\ \theta = 180° \Rightarrow W_g = -mgh$

(b) $P = F_g v$. From (a), $F_g = mg \Rightarrow P = mgv$.

**CALCULATE:**

(a) $W_g = -(3.4\text{ kg})(9.81\text{ m/s}^2)(0.47\text{ m}) = -15.676\text{ J}$

(b) $P = (3.4\text{ kg})(9.81\text{ m/s}^2)(0.27\text{ m/s}) = 9.006\text{ W}$

**ROUND:**

(a) $W_g = -16\text{ J}$

(b) $P = 9.0\text{ W}$

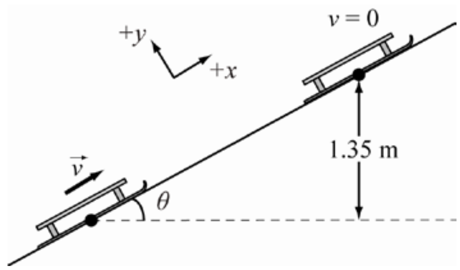
**DOUBLE-CHECK:**

(a) This is a reasonable result for a relatively light textbook moved a short distance.

(b) This result is much less than the output power of human muscle, which is of the order of $10^2$ W.

**5.63.** **THINK:** Determine the initial speed of a sled which is shoved up an incline that makes an angle of 28.0° with the horizontal and comes to a stop at a vertical height of $h$ = 1.35 m.

**SKETCH:**




**RESEARCH:** The work done by gravity must be equal to the change in kinetic energy: $W = \Delta K$ .

**SIMPLIFY:** $W_{\text{g}} = -mgh = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}m(v_{\text{f}}^2 - v_{\text{i}}^2)$. $v_{\text{f}} = 0$, so $-mgh = \frac{1}{2}m(0 - v_{\text{i}}^2) \Rightarrow v_{\text{i}} = \sqrt{2gh}$

**CALCULATE:** $v_{\text{i}} = \sqrt{2(9.81 \text{ m/s}^2)(1.35 \text{ m})} = 5.1466 \text{ m/s}$

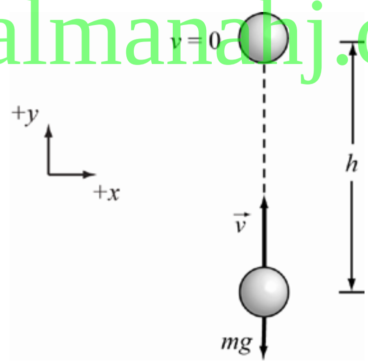
**ROUND:** The angle was given to three significant figures; so you may think that our result needs to be rounded to three digits. This is not correct, because the angle did not even enter into our calculations. The height was given to three digits, and so we round $v_{\text{i}}$ = 5.15 m/s.

**DOUBLE-CHECK:** $v_{\text{i}} = 5.15 \text{ m/s} = 18.5 \text{ km/h}$ is a reasonable value.

**5.64.** **THINK:** Determine the maximum height $h$ that a rock of mass $m$ = 0.325 kg reaches when thrown straight up and a net amount of work, $W_{\text{net}}$ = 115 J is done on the rock.

**SKETCH:**






**RESEARCH:** The amount of work done by the person's arm must equal the work done by gravity: $W_{\text{net}} = -W_{\text{g}}$ .

**SIMPLIFY:** $W_{\text{g}} = -mgh$, $W_{\text{net}} = mgh \Rightarrow h = \frac{W_{\text{net}}}{mg}$

**CALCULATE:** $h = \frac{115 \text{ J}}{0.325 \text{ kg}(9.81 \text{ m/s}^2)} = 36.0699 \text{ m}$

**ROUND:** $h = 36.1$ m

**DOUBLE-CHECK:** This is just under 120 ft—fairly high, but it is not unreasonable that an object with a small mass can be thrown this high.

**5.65.** **THINK:** Since we know the displacement, and we know that the car travels at constant velocity, the force must act in the same direction as the displacement. Then the work is simply the product of force times displacement.

**SKETCH:** Not necessary

**RESEARCH:** $W = Fx$

**SIMPLIFY:** $F = \dfrac{W}{x}$

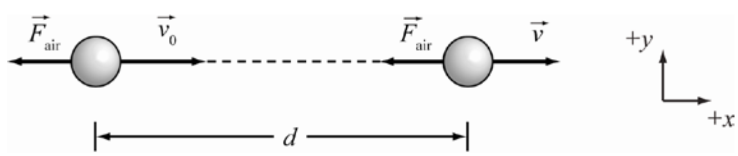
**CALCULATE:** $F = \dfrac{7.00 \cdot 10^4 \text{ J}}{2.8 \cdot 10^3 \text{ m}} = 25.0 \text{ N}$

**ROUND:** The variables in the question were given to three significant figures, so the answer remains $F = 25.0$ N.

**DOUBLE-CHECK:** It should take about 200 seconds to travel this distance. The average power is the net work done divided by the time interval, which under this assumption would compute to 350 W, which is realistic for a small car at relatively slow cruising speeds. The mass of the car should be around 1000 kg. A force of 25.0 N could accelerate it at 0.025 m/s$^2$, if it was not for friction and air resistance. These numbers are all of the right magnitude for a small passenger car, which gives us confidence in our solution.

**5.66.** **THINK:** A softball of mass $m$ = 0.250 kg is pitched at an initial speed of $v_0$ = 26.4 m/s. Air resistance causes the ball to slow down by 10.0% over a distance $d = 15.0$ m. I want to determine the average force of air resistance, $F_{\text{air}}$ , which causes the ball to slow down.

**SKETCH:**




**RESEARCH:** $W = Fd$ and $W = \Delta K$.

**SIMPLIFY:** Work done by air resistance: $W = -F_{\text{air}}d = \Delta K = \dfrac{1}{2}m(v_{\text{f}}^2 - v_{\text{i}}^2) \Rightarrow F_{\text{air}} = \dfrac{m}{2d}(v_{\text{i}}^2 - v_{\text{f}}^2)$.



**CALCULATE:** $v_{\text{f}} = 0.900(26.4 \text{ m/s}) = 23.76 \text{ m/s}, \ F_{\text{air}} = \dfrac{0.250 \text{ kg}}{2(15.0 \text{ m})}\left[(26.4 \text{ m/s})^2 - (23.76 \text{ m/s})^2\right] = 1.104 \text{ N}$

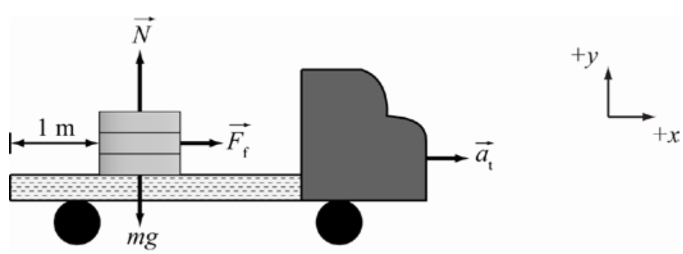
**ROUND:** Rounding to three significant figures, $F_{\text{air}} = 1.10$ N .

**DOUBLE-CHECK:** As a comparison, the force of gravity on the softball is

$$F_{\text{g}} = 0.25 \text{ kg}\left(9.81 \text{m/s}^2\right) = 2.45 \text{ N}$$

**5.67.** **THINK:** The stack of cement sacks has a combined mass $m$ = 1143.5 kg. The coefficients of static and kinetic friction between the sacks and the bed of the truck are 0.372 and 0.257, respectively. The truck accelerates from rest to $56.6 \text{ mph}\left(\dfrac{0.447 \text{ m/s}}{\text{mph}}\right) = 25.3 \text{ m/s}$ in $\Delta t = 22.9$ s. Determine if the sacks slide and the work done on the stack by the force of friction.

**SKETCH:**




**RESEARCH:** The acceleration of the truck $a_t$ and the acceleration of the stack $a_{\text{c}}$ must be determined: $a_{\text{t}} = v / \Delta t$. The maximum acceleration that will allow the cement sacks to stay on the truck is calculated by: $F_{\text{f,max}} = ma_{\text{c,max}} = \mu_{\text{s}}N$.

**SIMPLIFY:** $F_{\text{f,max}} = ma_{\text{c,max}} = \mu_{\text{s}}mg \Rightarrow a_{\text{c,max}} = \mu_{\text{s}}g$

**CALCULATE:** $a_{\text{t}} = \dfrac{25.3\text{ m/s}}{22.9\text{ s}} = 1.1048\text{ m/s}^2$, $a_{\text{c,max}} = (0.372)(9.81\text{ m/s}^2) = 3.649\text{ m/s}^2$

$a_{\text{c,max}}$ is larger than $a_{\text{t}}$. This means that the stack does not slide on the truck bed and $F_{\text{f}} < \mu_{\text{s}} N$. The acceleration of the stack must be the same as the acceleration of the truck $a_{\text{c}} = a_{\text{t}} = 1.10\text{ m/s}^2$. The work done on the stack by the force of friction is calculated using $W = \Delta K$:

$$W = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}m(v_{\text{f}}^2 - v_{\text{i}}^2).\ \text{Since } v_{\text{i}} = 0,\ W = \frac{1}{2}mv_{\text{f}}^2 = \frac{1}{2}(1143.5\text{ kg})(25.3\text{ m/s})^2 = 365971\text{ J}.$$

**ROUND:** $W = 366\text{ kJ}$

**DOUBLE-CHECK:** The work done by the force of friction can also be calculated by $W = F_{\text{f}}d$ ; where $F_{\text{f}} = ma_{\text{c}}$ and $d = \frac{1}{2}a_{\text{c}}t^2$ :

$$W = ma_{\text{c}}\left(\frac{1}{2}a_{\text{c}}t^2\right) = \frac{1}{2}ma_{\text{c}}^2t^2 = \frac{1}{2}m(a_{\text{c}}t)^2.\ \text{Using } v_{\text{f}} = a_{\text{c}}t,\ W = \frac{1}{2}m(v_{\text{f}})^2 \text{ as before.}$$

**5.68.** **THINK:** Determine the power needed to keep a car of mass $m$ = 1000. kg moving at a constant velocity $v$ = 22.2 m/s. When the car is in neutral, it loses power such that it decelerates from 25.0 m/s to 19.4 m/s in $t$ = 6.00 s. The average velocity over the period of deceleration is 22.2 m/s. Therefore, the power required to maintain this velocity is equal in magnitude to the power lost during the deceleration.

**SKETCH:** Not necessary.

**RESEARCH:** The power is given by the change in energy over time, $P = (K_{\text{f}} - K_{\text{i}})/t$. The energy is kinetic energy, $K = (1/2)mv^2$.

**SIMPLIFY:** $P = \dfrac{K_{\text{f}} - K_{\text{i}}}{t} = \dfrac{\frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2}{t} = \dfrac{m}{2t}(v_{\text{f}}^2 - v_{\text{i}}^2)$



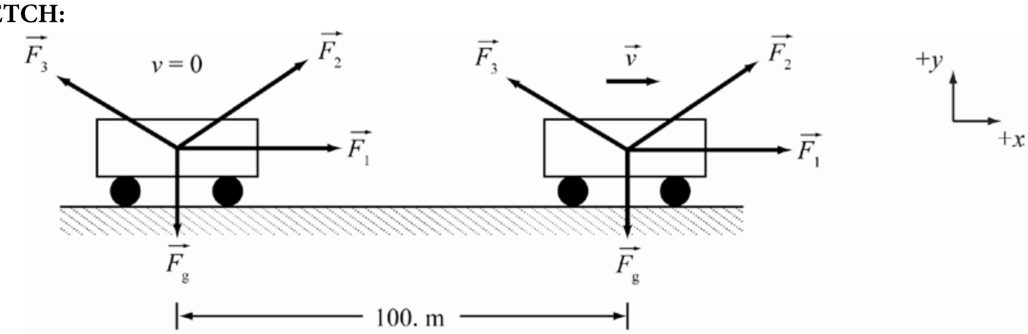
**CALCULATE:** $P = \dfrac{1000.\text{ kg}}{2(6.00\text{ s})}\left[(25.0\text{ m/s})^2 - (19.4\text{ m/s})^2\right] = 20720\text{ W}$

**ROUND:** Rounding to three significant figures, $P$ = 20.7 kW.

**DOUBLE-CHECK:** A 300 hp engine is equivalent to $300\text{ hp}(746\text{ W/hp}) = 223800\text{ W} = 223\text{ kW}$. Since the solution is smaller than 300 hp, the calculation is reasonable.

**5.69.** **THINK:** There are four forces acting on a 125 kg cart at rest. These forces are $\vec{F}_1 = 300.\text{ N at }0°$, $\vec{F}_2 = 300.\text{ N at }40.0°$, $\vec{F}_3 = 200.\text{ N at }150.°$ and $\vec{F}_{\text{g}} = mg$ downward. The cart does not move up or down, so the force of gravity, and the vertical components of the other forces, need not be considered. The horizontal components of the forces can be used to determine the net work done on the cart, and the Work-Kinetic Energy Theorem can be used to determine the velocity of the cart after 100. m.

**SKETCH:**



**RESEARCH:** $F_{1,x} = F_1 \cos\theta_1$, $F_{2,x} = F_2 \cos\theta_2$, $F_{3,x} = F_3 \cos\theta_3$, $W = \sum_{i=1}^{n} F_{i,x} \bullet \Delta x$, $W = \Delta K = K_f - K_i$,

$\sum_{i=1}^{3} F_{i,x} = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 = F_x$, $\Delta K = K_f - K_i = \frac{1}{2} m \left( v_f^2 - v_i^2 \right)$

**SIMPLIFY:** $W = \sum_{i=1}^{3} F_{i,x} \bullet \Delta x = K_f - K_i$, $v_f = \sqrt{\frac{2F_x \Delta x}{m}} = \sqrt{\frac{2(F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3)\Delta x}{m}}$.

**CALCULATE:** $v_f = \sqrt{\frac{2\left[(300.\text{ N})\cos 0° + (300.\text{ N})\cos 40.0° + (200.\text{ N})\cos 150.°\right](100.\text{ m})}{125\text{ kg}}}$

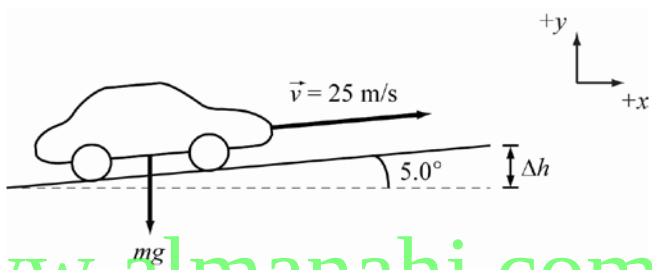
$= 23.89$ m/s in the direction of $F_1$.

**ROUND:** Rounding to three significant figures, $v_f = 23.9$ m/s in the direction of $F_1$.

**DOUBLE-CHECK:** If only $F_1$ was acting on the cart, the velocity would be 21.9 m/s. This is close to the answer above, so the answer is reasonable.

**5.70.** **THINK:** Determine the power required to propel a 1000.0 kg car up a slope of $5.0°$.

**SKETCH:**






**RESEARCH:** Since the speed is constant, the power is given by the change in potential energy over time, $P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t} = \frac{mg\Delta h}{\Delta t}$.

**SIMPLIFY:** $P = \frac{\Delta E}{\Delta t} = \frac{mg\Delta h}{\Delta t} = mgv\sin\theta$

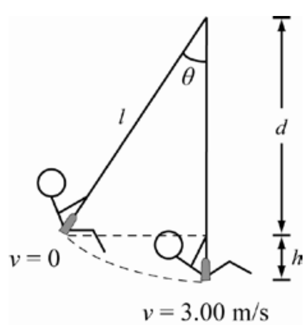
**CALCULATE:** $P = 1000.0\text{ kg}\left(9.81\text{ m/s}^2\right)\left(25.0\text{ m/s}\right)\sin\left(5.0°\right) = 21{,}374.95\text{ W}$

**ROUND:** Rounding to two significant figures, $P = 21$ kW.

**DOUBLE-CHECK:** This is a reasonable amount of power for a car.

**5.71.** **THINK:** Determine the angle $\theta$ that the granddaughter is released from to reach a speed of 3.00 m/s at the bottom of the swinging motion. The granddaughter has a mass of $m$ = 21.0 kg and the length of the swing is $l$ = 2.50 m.

**SKETCH:**



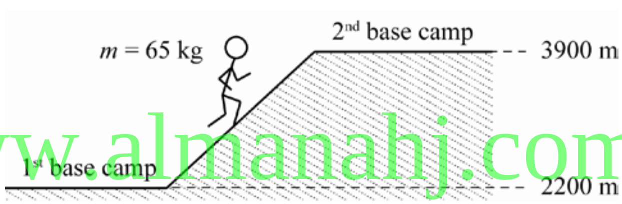
**RESEARCH:** The energy is given by the change in the height from the top of the swing, $mgh$. It can be seen from the geometry that $h = l - d = l - l\cos\theta = l(1-\cos\theta)$. At the bottom of the swinging motion, there is only kinetic energy, $K = (1/2)mv^2$.
**SIMPLIFY:** Equate the energy at the release point to the energy at the bottom of the swinging motion and solve for $\theta$:

$$mgh = \frac{1}{2}mv^2 \Rightarrow gl(1-\cos\theta) = \frac{1}{2}v^2 \Rightarrow \theta = \cos^{-1}\left(1 - \frac{v^2}{2gl}\right)$$

**CALCULATE:** $\theta = \cos^{-1}\left(1 - \frac{(3.00 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)(2.50 \text{ m})}\right) = 35.263°$

**ROUND:** Rounding to three significant figures, $\theta = 35.3°$.
**DOUBLE-CHECK:** This is a reasonable angle to attain such a speed on a swing.

**5.72** **THINK:**
(a) Determine the work done against gravity by a 65 kg hiker in climbing from height $h_1 = 2200 \text{ m}$ to a height $h_2 = 3900 \text{ m}$.
(b) The trip takes $t = 5.0 \text{ h}(3600 \text{ s/h}) = 18{,}000 \text{ s}$. Determine the average power output.
(c) Determine the energy input rate assuming the body is 15% efficient.
**SKETCH:**






**RESEARCH:**
(a) The work done against gravity is $W = mg(h_2 - h_1)$.
(b) $P = \frac{E_f - E_i}{t} = \frac{\Delta E}{t}$
(c) The energy output is given by $E_{in} \times \%\text{ conversion} = E_{out}$.
**SIMPLIFY:**
(a) Not necessary.
(b) Not necessary.
(c) $E_{in} = \frac{E_{out}}{\%\text{ conversion}}$
**CALCULATE:**
(a) $W = 65 \text{ kg}(9.81 \text{ m/s}^2)(3900 \text{ m} - 2200 \text{ m}) = 1{,}084{,}005 \text{ J}$
(b) $P = \frac{1{,}084{,}005 \text{ J}}{18{,}000 \text{ s}} = 60.22 \text{ W}$
(c) $E_{in} = \frac{1{,}084{,}005 \text{ J}}{0.15} = 7{,}226{,}700 \text{ J}$
**ROUND:**
(a) Rounding to two significant figures, $W = 1.1 \cdot 10^6 \text{ J}$.
(b) Rounding to two significant figures, $P = 60. \text{ W}$.
(c) Rounding to two significant figures, $E_{in} = 7.2 \cdot 10^6 \text{ J}$.
**DOUBLE-CHECK:**
(a) This is a reasonable value for such a long distance traveled.

(b) This value is reasonable for such a long period of time.

(c) The daily caloric requirements for a 65 kg man is 2432 calories, which is about $1.0 \cdot 10^7$ J. This is on the same order of magnitude as the result.

**5.73.** For work done by a force that varies with location, $W = \int_{x_1}^{x_2} F_x dx$. In order to oppose the force, equal work must be done opposite the direction of $F_x$.

$$W = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} (-cx^3) dx = -\frac{c}{4}\left[x^4\right]_{x_1}^{x_2} = \frac{c}{4}\left[x_1^4 - x_2^4\right]$$

This evaluates to:

$$W = \frac{19.1 \text{ N/m}^3}{4}\left[(0.810 \text{ m})^4 - (1.39 \text{ m})^4\right] = -15.77 \text{ J}$$

Therefore the work required to oppose $F_x$ is the opposite: $W = 15.77$ J or 15.8 J when rounded to three significant figures.

**5.74.** Apply Hooke's law to find the spring constant *k*:

$$F = -kx_0 \rightarrow |k| = \frac{F}{x_0}$$

The work done to compress the spring further is equal to the change in spring energy.

$$W = \Delta E = \frac{1}{2}k\left[x_f^2 - x_0^2\right] = \frac{1}{2}\frac{F}{x_0}\left[x_f^2 - x_0^2\right]$$

$$W = \frac{1}{2}\left(\frac{63.5 \text{ N}}{0.0435 \text{ m}}\right)\left[(0.0815 \text{ m})^2 - (0.0435 \text{ m})^2\right] = 3.47 \text{ J}$$



**5.75.** The amount of power required to overcome the force of air resistance is given by $P = F \cdot v$. And the force of air resistance is given by the Ch. 4 formula

$$F_d = \left(\frac{1}{2}c_d A\rho\right)v^2$$

$$\Rightarrow P = \left(\frac{1}{2}c_d A\rho v^2\right)\cdot v = \frac{1}{2}c_d A\rho v^3$$

This evaluates as:

$$P = \frac{1}{2}(0.333)(3.25 \text{ m}^2)(1.15 \text{ kg/m}^3)(26.8 \text{ m/s})^3 = 11{,}978.4 \text{ W} = (11{,}978.4 \text{ W})\left(\frac{1 \text{ hp}}{745.7 \text{ W}}\right) = 16.06 \text{ hp}$$

To three significant figures, the power is 16.1 hp.

## Multi-Version Exercises

**5.76.** **THINK:** This problem involves a variable force. Since we want to find the change in kinetic energy, we can find the work done as the object moves and then use the work-energy theorem to find the total work done.

**SKETCH:**

**RESEARCH:** Since the object started at rest, it had zero kinetic energy to start. Use the work-energy theorem $W = \Delta K$ to find the change in kinetic energy. Since the object started with zero kinetic energy, the total kinetic energy will equal the change in kinetic energy: $\Delta K = K$ . The work done by a variable force in the $x$-direction is given by $W = \int_{x_0}^{x} F_x(x')dx'$ and the equation for our force is $F_x(x') = A(x')^6$ . Since the object starts at rest at 1.093 m and moves to 4.429 m, we start at $x_0$ = 1.093 m and end at $x$ = 4.429 m.

**SIMPLIFY:** First, find the expression for work by substituting the correct expression for the force: $W = \int_{x_0}^{x} A(x')^6 dx'$ . Taking the definite integral gives $W = \frac{A}{7}(x')^7\Big|_{x_0}^{x} = \frac{A}{7}(x^7 - x_0^7)$. Combining this with the work-energy theorem gives $\frac{A}{7}(x^7 - x_0^7) = W = K$ .

**CALCULATE:** The problem states that $A$ = 11.45 N/m$^6$, that the object starts at $x_0$ = 1.093 m and that it ends at $x$ = 4.429 m. Plugging these into the equation and calculating gives:

$$
\begin{aligned}
K &= \frac{A}{7}(x^7 - x_0^7) \\
&= \frac{11.45 \text{ N/m}^6}{7}\left((4.429 \text{ m})^7 - (1.093 \text{ m})^7\right) \\
&= 5.467930659 \cdot 10^4 \text{ J}
\end{aligned}
$$

**ROUND:** The measured values in this problem are the constant $A$ in the equation for the force and the two distances on the $x$-axis. All three of these are given to four significant figures, so the final answer should have four significant figures: $5.468 \cdot 10^4$ J or 54.68 kJ.

**DOUBLE-CHECK:** Working backwards, if a variable force in the +$x$-direction changes the kinetic energy from zero to $5.468 \cdot 10^4$ J, then the object will have moved



$$
\begin{aligned}
x &= \sqrt[7]{\frac{7(5.468 \cdot 10^4 \text{ J})}{11.45 \text{ N/m}^6} + 1.093^7} \\
&= 4.429008023 \text{ m.}
\end{aligned}
$$

This is, within rounding error, the 4.429 m given in the problem, so it seems that the calculations were correct.

**5.77.** $K = \frac{A}{7}(x^7 - x_0^7)$

$$\frac{7K}{A} = x^7 - x_0^7$$

$$x = \sqrt[7]{\frac{7K}{A} + x_0^7} = \sqrt[7]{\frac{7(5.662 \cdot 10^3 \text{ J})}{13.75 \text{ N/m}^6} + (1.105 \text{ m})^7} = 3.121 \text{ m}$$

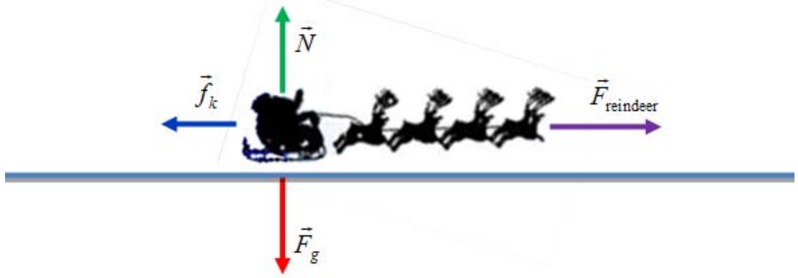
**5.78.** $K = \frac{A}{7}(x^7 - x_0^7)$

$$\frac{7K}{A} = x^7 - x_0^7$$

$$x_0 = \sqrt[7]{x^7 - \frac{7K}{A}} = \sqrt[7]{(3.313)^7 - \frac{7(1.00396 \cdot 10^4 \text{ J})}{16.05 \text{ N/m}^6}} = 1.114 \text{ m}$$

**5.79.** **THINK:** In this problem, the reindeer must pull the sleigh to overcome the friction between the runners of the sleigh and the snow. Express the friction force in terms of the speed and weight of the sleigh, and the coefficient of friction between the sleigh and the ground. It is then possible to find the power from the force and velocity.

**SKETCH:** Draw a free-body diagram for the sleigh:




**RESEARCH:** Since the sleigh is moving with a constant velocity, the net forces on the sleigh are zero. This means that the normal force and the gravitational force are equal and opposite ($\vec{N} = -\vec{F}_g$), as are the friction force and the force from the reindeer ($\vec{F}_{\text{reindeer}} = -\vec{f}_k$). From the data given in the problem, it is possible to calculate the friction force $f_k = \mu_k mg$. The power required to keep the sleigh moving at a constant speed is given by $P = F_{\text{reindeer}}v$. Eventually, it will be necessary to convert from SI units (Watts) to non-standard units (horsepower or hp). This can be cone using the conversion factor 1 hp = 746 W.

**SIMPLIFY:** To find the power required for the sleigh to move, it is necessary to express the force from the reindeer in terms of known quantities. Since the force of the reindeer is equal in magnitude with the friction force, use the equation for frictional force to find:

$$\begin{aligned}\left|\vec{F}_{\text{reindeer}}\right| &= \left|-\vec{f}_k\right| \\ &= f_k \\ &= \mu_k mg\end{aligned}$$

Use this and the speed of the sleigh to find that $P = F_{\text{reindeer}}v = \mu_k mgv$.



**CALCULATE:** With the exception of the gravitational acceleration, all of the needed values are given in the question. The coefficient of kinetic friction between the sleigh and the snow is 0.1337, the mass of the system (sleigh, Santa, and presents) is 537.3 kg, and the speed of the sleigh is 3.333 m/s. Using a gravitational acceleration of 9.81 m/s gives:

$$\begin{aligned}P &= \mu_k mgv \\ &= 0.1337 \cdot 537.3 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot 3.333 \text{ m/s} \\ &= 2348.83532 \text{ W}\end{aligned}$$

This can be converted to horsepower: $2348.83532 \text{ W} \cdot \dfrac{1 \text{ hp}}{746 \text{ W}} = 3.148572815 \text{ hp}$.

**ROUND:** The measured quantities in this problem are all given to four significant figures. Though the conversion from watts to horsepower and the gravitational acceleration have three significant figures, they do not count for the final answer. The power required to keep the sleigh moving is 3.149 hp.

**DOUBLE-CHECK:** Generally, it is thought that Santa has 8 or 9 reindeer (depending on how foggy it is on a given Christmas Eve). This gives an average of between 0.3499 and 0.3936 horsepower per reindeer, which seems reasonable. Work backwards to find that, if the reindeer are pulling the sled with 3.149 hp, then the speed they are moving must be (rounding to four significant figures):

$$v = \frac{3.149 \text{ hp}}{\mu_k mg}$$
$$= \frac{3.149 \text{ hp} \cdot 746 \text{ W/hp}}{0.1337 \cdot 537.3 \text{ kg} \cdot 9.81 \text{ m/s}^2}$$
$$= 3.333452207 \frac{\text{W}}{\text{kg} \cdot \text{m/s}^2}$$
$$= 3.333 \frac{\text{kg} \cdot \text{m}^2/\text{s}^3}{\text{kg} \cdot \text{m/s}^2} = 3.333 \text{ m/s}$$

This matches the constant velocity from the problem, so the calculations were correct.

**5.80.** $P = \mu_k mgv$

$$\mu_k = \frac{P}{mgv} = \frac{(2.666 \text{ hp})\frac{746 \text{ W}}{\text{hp}}}{(540.3 \text{ kg})(9.81 \text{ m/s}^2)(2.561 \text{ m/s})} = 0.1465$$

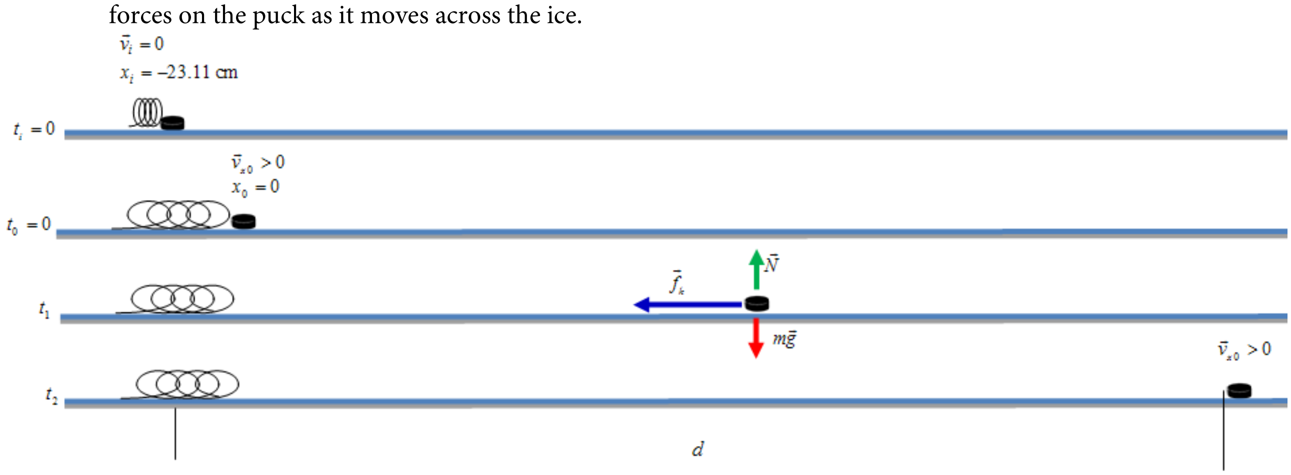
**5.81.** $P = \mu_k mgv$

$$m = \frac{P}{\mu_k gv} = \frac{(3.182 \text{ hp})\frac{746 \text{ W}}{\text{hp}}}{(0.1595)(9.81 \text{ m/s}^2)(2.791 \text{ m/s})} = 543.6 \text{ kg}$$

**5.82.** **THINK:** In this problem, the energy stored in the spring is converted to kinetic energy as the puck slides across the ice. The spring constant and compression of the spring can be used to calculate the energy stored in the spring. This is all converted to kinetic energy of the puck. The energy is dissipated as the puck slides across the ice. It is necessary to compute how far the puck must slide to dissipate all of the energy that was, originally, stored in the spring.



**SKETCH:** Sketch the puck when the spring is fully compressed, when it leaves contact with the spring, as it moves across the ice, and at the moment it comes to a stop. Include a free body diagram showing the forces on the puck as it moves across the ice.




**RESEARCH:** The potential energy stored in the spring is $U = \frac{1}{2}kx^2$, where $x$ is the compression of the spring. The energy dissipated by the force of friction is $\Delta U = Fd$. The force of friction on the puck is given by $F = \mu_k mg$. It is necessary to find the total distance traveled $d$.

**SIMPLIFY:** First, find the energy dissipated by the force of friction in terms of known quantities $\Delta U = \mu_k mgd$. This must equal the energy that was stored in the spring, $U = \frac{1}{2}kx^2$. Setting $\Delta U = U$, solve for the total distance traveled in terms of known quantities:

$$\Delta U = U$$
$$\mu_k mgd = \frac{1}{2}kx^2$$
$$d = \frac{kx^2}{2\mu_k mg}$$

It is important to note that $x$ represents the compression of the spring before the puck was released, and $d$ is the total distance traveled from the time that the puck was released (not from the time the puck left contact with the spring).

**CALCULATE:** Before plugging the values from the question into the equation above, it is important to make sure that all of the units are the same. In particular, note that it is easier to solve the equation directly if the compression is changed from 23.11 cm to 0.2311 m and the mass used is 0.1700 kg instead of 170.0 g. Then the distance is:

$$\begin{aligned} d &= \frac{kx^2}{2\mu_k mg} \\ &= \frac{15.19\text{ N/m}\cdot(-0.2311\text{ m})^2}{2\cdot 0.02221\cdot 0.1700\text{ kg}\cdot 9.81\text{ m/s}^2} \\ &= 10.95118667\text{ m} \end{aligned}$$

Of this distance, 0.2311 m is the distance the spring was compressed. So the distance traveled by the puck after leaving the spring is 10.95118667 m – 0.2311 m = 10.72008667 m.

**ROUND:** The measured values are all given to four significant figures, so the final answer is that the hockey puck traveled 10.72 m.

**DOUBLE-CHECK:** Working backwards, if the hockey puck weighs 0.1700 kg and traveled 10.95 m across the ice (including spring compression) with a coefficient of kinetic friction of 0.02221, then the energy dissipated was $\Delta U = \mu_k mgd = 0.0221\cdot 9.81\text{ m/s}^2\cdot 0.1700\text{ kg}\cdot 10.95\text{ m} = 0.4056\text{ J}$. Since the energy stored in this spring is $U = \frac{1}{2}kx^2 = \frac{15.19\text{ N/m}}{2}x^2$, it is necessary for the spring to have been compressed by $x = \sqrt{\frac{0.4056\text{ J}\cdot 2}{15.19\text{ N/m}}} = 0.231092\text{ m}$, within rounding of the value of 23.11 cm given in the problem.

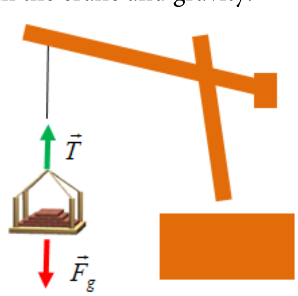


**5.83.** $d = \dfrac{kx^2}{2\mu_k mg}$

$$\mu_k = \frac{kx^2}{2mgd} = \frac{(17.49\text{ N/m})(0.2331\text{ m})^2}{2(0.1700\text{ kg})(9.81\text{ m/s}^2)(12.13\text{ m}+0.2331\text{ m})} = 0.02305$$

**5.84.** **THINK:** Since the bricks travel at a low, constant speed, use the information given in the problem to find the tension force that the crane exerts to raise the bricks. The power can be computed by finding the scalar product of the force vector and the velocity vector.

**SKETCH:** A free body diagram of the bricks as they are raised to the top of the platform is helpful. The only forces are tension from the crane and gravity.



**RESEARCH:** The average power is the scalar product of the force exerted by the crane on the bricks and the velocity of the bricks: $P = \vec{F} \bullet \vec{v}$, where the force is the tension from the crane. (The speed of the bricks is low, so air resistance is negligible in this case.) The bricks are moving at a constant velocity, so the sum of the forces is zero and $\vec{T} = -\vec{F}_g = -m\vec{g}$. The velocity is constant and can be computed as the distance divided by the time $\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$.

**SIMPLIFY:** Instead of using vector equations, note that the tension force and the velocity are in the same direction. The equation for the power then becomes $P = \vec{F} \bullet \vec{v} = Fv\cos\alpha$, where $\alpha$ is the angle between the velocity and force. Since $T = mg$ and $v = \frac{d}{t}$, the power is given by the equation $P = \frac{mgd}{t}\cos\alpha$.

**CALCULATE:** The mass, distance, and time are given in the problem. The velocity of the bricks is in the same direction as the tension force, so $\alpha = 0$.

$$\begin{aligned} P &= \frac{mgd}{t}\cos\alpha \\ &= \frac{75.0 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot 45.0 \text{ m}}{52.0 \text{ s}}\cos 0° \\ &= 636.7067308 \text{ W} \end{aligned}$$

**ROUND:** The mass of the bricks, height to which they are raised, and time are all given to three significant figures, and the answer should have four significant figures. The average power of the crane is 637 W.

**DOUBLE-CHECK:** To check, note that the average power is the work done divided by the elapsed time: $\bar{P} = \frac{W}{\Delta t}$. Combine this with the equation for the work done by the constant tension force $W = |\vec{F}||\Delta \vec{r}|\cos\alpha$ to find an equation for the average power: $\bar{P} = \frac{|\vec{F}||\Delta \vec{r}|\cos\alpha}{\Delta t}$. Plug in the values for the tension force $\vec{T} = -\vec{F}_g = -mg$ and distance $\Delta \vec{r} = 45.0$ m upward to find:

$$\begin{aligned} P &= \frac{mgd}{t}\cos\alpha \\ &= \frac{75.0 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot 45.0 \text{ m}}{52.0 \text{ s}}\cos 0° \\ &= 636.7067308 \text{ W} \end{aligned}$$

When this is rounded to three decimal places, it confirms the calculations.



**5.85.** $t = \frac{mgd}{P} = \frac{(75.0 \text{ kg})(9.81 \text{ m/s}^2)(45.0 \text{ m})}{725 \text{ W}} = 45.7 \text{ s}.$

**5.86.** $d = \frac{Pt}{mg} = \frac{(815 \text{ W})(52.0 \text{ s})}{(75.0 \text{ kg})(9.81 \text{ m/s}^2)} = 57.6 \text{ m}.$

# Chapter 6: Potential Energy and Energy Conservation

## Concept Checks

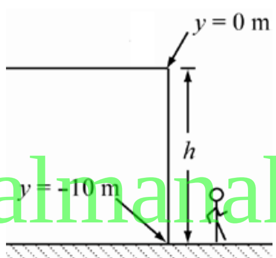
**6.1.** b **6.2.** a **6.3.** c **6.4.** e **6.5.** e **6.6.** d **6.7.** b

## Multiple-Choice Questions

**6.1.** a **6.2.** c **6.3.** e **6.4.** e **6.5.** d **6.6.** e **6.7.** d **6.8.** e **6.9.** a **6.10.** d **6.11.** c **6.12.** c **6.13.** b

## Conceptual Questions

**6.14.** The kinetic energy, *K*, of an object is proportional to the mass, *m*, of the object and the square of its speed, *v*. The formula is $K = mv^2/2$. The mass is always positive, and the square of the velocity is non-negative. Since the product of non-negative numbers is non-negative, the kinetic energy of an object cannot be negative. However, the potential energy of an object can be negative because it is a relative value. An example of negative potential energy is gravitational potential energy, given by the formula $U = mgh$, where *m* is the mass of the object, *g* is the acceleration due to gravity, and *h* is the vertical distance above the ground. Consider a person standing at the base of a bridge as in the figure below.





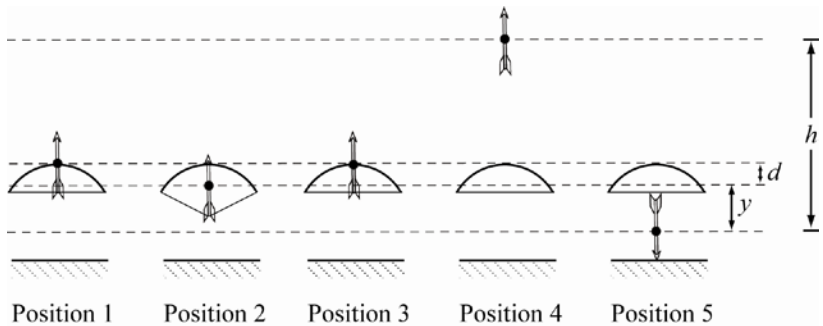

In this coordinate system, the person has potential energy due to gravity of $U = m\left(9.81 \text{ m/s}^2\right)\left(-10 \text{ m}\right)$ relative to the reference point of the bridge. Since a mass is always positive, the potential energy of the person standing on the ground relative to the bridge above has a negative value.

**6.15.** (a) If a person jumps off a table onto the floor, mechanical energy is not conserved. Mechanical energy is conserved while you are falling towards the floor (assuming energy lost to air resistance is ignored) because gravitational potential energy is being converted to kinetic energy. However, once you land on the floor all of the kinetic energy is absorbed by your body on impact. The energy is lost to non-conservative forces such as friction within your body and heat expelled by your muscles.

(b) The car's mechanical energy is not conserved. Assume a car is on a level plane so it has no gravitational potential energy. The car is in motion so its energy is in the form of kinetic energy. The energy is lost to non-conservative forces such as friction on the tires, thermal energy on the car's brakes and energy dissipated as the car's body is bent by the tree.

**6.16.** Work is defined as the dot product of force and displacement. This is indicated in the formula $W = \vec{F}\cdot\Delta\vec{r} = \left|\vec{F}\right|\left|\Delta\vec{r}\right|\cos\theta$, where $\vec{F}$ is the applied force, $\vec{r}$ is the displacement of the object, and $\theta$ is the angle between the vectors $\vec{F}$ and $\vec{r}$. When you are standing still, the bag of groceries does not travel any distance, i.e. $\left|\vec{r}\right| = 0$, so there is no work done. Assuming that you do not lift or lower the bag of groceries when you carry the bag a displacement $\vec{r}$ across the parking lot, then you do not do any work. This is because the applied force $\vec{F}$ is perpendicular to the displacement $\vec{r}$. Using $\theta = 90°$ in the formula gives $W = \left|\vec{F}\right|\left|\Delta\vec{r}\right|\cos 90° = \left|\vec{F}\right|\left|\Delta\vec{r}\right|\cdot 0 = 0 \text{ J}$.

**6.17.** The energy in the system, $E$, is the sum of the energy stored in the bow by flexing it, $E_b$, the kinetic energy of the arrow, $K$, and the gravitational potential energy of the arrow, $U$. Let the arrow have mass $m$ and the bow have spring constant $k$. Five separate positions of the arrow and bow system will be considered. Position 1 is where the arrow is put in the bow. Position 2 is where the arrow is pulled back in the bow. Position 3 is where the bow has returned to its relaxed position and the arrow is leaving the bowstring. Position 4 is where the arrow has reached its maximum height $h$. Position 5 is where the arrow has stuck in the ground.




At position 1 the arrow has gravitational potential energy $U = mg(y+d)$ (refer to diagram) relative to the ground. The total energy in the system at this position is $E_1 = mg(y+d)$. At position 2, the arrow now has gravitational potential energy $U = mgy$ and the elastic energy stored in the bow is $E_b = kd^2/2$ due to the downward displacement $d$. The total energy in the system at this position is $E_2 = mgy + \left(kd^2/2\right)$. The work done by the bowstring during this displacement is $E_{tot} = 2.0$ J At position 3, the bow's tension is released and the arrow is launched with a velocity, $v$. The total energy is given by $E_3 = \left(mv^2/2\right) + mg\left(y+d\right)$. The work done on the arrow by the bow is $W_3 = kd^2/2$. At position 4, the arrow has reached its maximum height, $h$. At this position, the velocity of the arrow is zero, so the kinetic energy is zero. The total energy is given by $E_4 = mgh$. The work done on the arrow by gravity is equal to the change in kinetic energy, $W_4 = \Delta K = 0 - mv^2/2$. At position 5, the arrow has hit the ground and stuck in. The total energy is $E_5 = 0$. When the arrow hits the ground the energy of the system is dissipated by friction between the arrow and the ground. The work done on the arrow by gravity during its fall is given by $W_5 = \Delta K = \left(mv^2/2\right) - 0$. This is equal to the kinetic energy of the arrow just before it strikes the ground.



**6.18.** (a) Assuming both billiard balls have the same mass, $m$, the initial energies, $E_{Ai}$ and $E_{Bi}$ are given by $E_{\text{Ai}} = mgh$ and $E_{\text{Bi}} = mgh$. The final energy is all due to kinetic energy, so the final energies are $E_{\text{Af}} = \left(mv_{\text{A}}^2\right)/2$ and $E_{\text{Bf}} = \left(mv_{\text{B}}^2\right)/2$. By conservation of energy (assuming no loss due to friction), $E_{\text{i}} = E_{\text{f}}$. For each ball the initial and final energies are equal. This means $mgh = \left(mv_{\text{A}}^2\right)/2 \Rightarrow v_{\text{A}} = \sqrt{2gh}$ and $mgh = \left(mv_{\text{B}}^2\right)/2 \Rightarrow v_{\text{B}} = \sqrt{2gh}$. Therefore, $v_{\text{A}} = v_{\text{B}}$. The billiard balls have the same speed at the end.

(b) Ball B undergoes an acceleration of $a$ and a deceleration of $-a$ due to the dip in the track. The effects of the acceleration and deceleration ultimately cancel. However, the ball rolling on track B will have a greater speed over of the lowest section of track. Therefore, ball B will win the race.

**6.19.** Because the girl/swing system swings out, then returns to the same point, the girl/swing system has moved over a closed path and the work done is zero. Therefore the forces acting on the girl/swing system are conservative. Assuming no friction, the only forces acting on the girl/swing system are the tension in the ropes holding up the girl/swing system and the force of gravity. Assume that the ropes cannot be stretched so that the tension in the ropes is conservative. Gravity is a conservative force, so it is expected that all forces are conservative for the girl/swing system.

**6.20.** No. Friction is a dissipative force (non-conservative). The work done by friction cannot be stored in a potential form.

**6.21.** No. The mathematical expression for the potential energy of a spring is $U=\left(kx^2\right)/2.$ The spring constant, $k$ is a positive constant. The square of the displacement of the spring, $x$, will always be non-negative. Hence, the potential energy of a spring will always be non-negative.

**6.22.** The elastic force is given by $\vec{F}=-k\vec{r}$ , where $\vec{r}$ is the displacement of the spring. The force is therefore a function of displacement, so denote that the force by $\vec{F}\left(\vec{r}\right).$ The sum of the inner product between $\vec{F}\left(\vec{r}\right)$ and the local displacements $\Delta r$ can be expressed as $\sum_{i=1}^{n}\vec{F}\left(\vec{r}_i\right)\cdot\Delta r.$ If the local displacements are chosen so they are infinitesimally small, the sum can be expressed as an integral:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\vec{F}\left(\vec{r}_i\right)\cdot\Delta r=\int_a^b\vec{F}\left(\vec{r}\right)\cdot d\vec{r}.$$

If the trajectory is a closed loop, then $a = b$ and the integral becomes $\int_a^b\vec{F}\left(\vec{r}\right)\cdot d\vec{r}=0$ because

$$\int_a^b\vec{F}\left(\vec{r}\right)\cdot d\vec{r}=f\left(b\right)-f\left(a\right)=f(a)-f(a)=0.$$

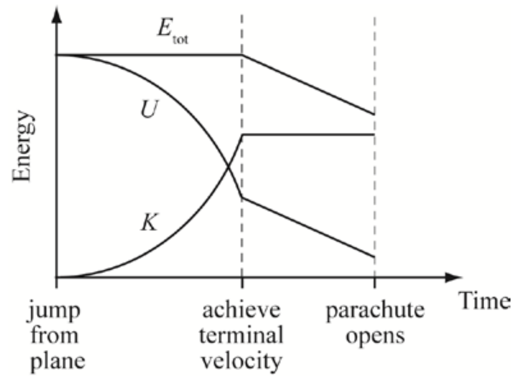
It should be noted that work, $W=\int\vec{F}\cdot d\vec{r}$ , is independent of a path because the force is conservative. If there was a dissipative force, such as friction present, the force would be non-conservative and therefore be path-dependent.



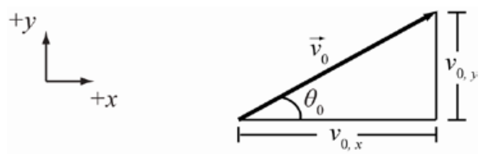
**6.23.** No. There is not a 1-1 correspondence between potential energy functions and conservative forces. The conservative force is the negative gradient of the potential energy. Therefore, two conservative forces will have the same potential energy function $U$ if they differ by a constant. For example, consider the force $F=0.$ The corresponding potential function is a constant, but it could be any constant depending on the situation. Therefore there is not necessarily a unique potential function corresponding to a conservative force.

**6.24.** When the person first steps out of the plane, all of the energy is potential energy and as they fall, the potential energy is converted to kinetic energy. In the first stage, before they reach the terminal velocity, they accelerate at a constant rate, so their velocity increases at a linear rate, and so $K=\frac{1}{2}mv^2$ increases at a quadratic rate. On the other hand, their height decreases at a quadratic rate, so $U=mgh$ decreases at a quadratic rate. Because there is no air resistance in the first stage of the model, the total energy, $E_{\text{tot}}=K+U$ , remains constant. In the second stage, their acceleration becomes zero, and their velocity becomes constant. This means that $K=\frac{1}{2}mv^2$ is constant, and $U=mgh$ decreases at a linear rate. The sum of the energies is no longer constant. The lost energy is due to the air resistance that counter-balances the acceleration due to gravity.

The rate of decrease of energy in the system is equal to the rate of decrease of potential energy.




**6.25.**




The lengths of the component vectors of $v_0$ are $v_{0,x} = v_0 \cos\theta_0$ and $v_{0,y} = v_0 \sin\theta_0$. Velocity is a vector quantity, so $\vec{v} = v_x \hat{x} + v_y \hat{y}$. Let $v = |\vec{v}|$. Then, $v^2 = {v_x}^2 + {v_y}^2$. The velocity vector $\vec{v}$ has component vectors $v_x = v_0 \cos\theta_0$ (horizontal component is constant) and $v_y = v_0 \sin\theta_0 - gt$ (which changes relative to time). To compute the kinetic energy, use the formula $K = mv^2/2$. First, compute

$$\begin{aligned} v^2 &= {v_0}^2 \cos{\theta_0}^2 + {v_0}^2 \sin^2\theta_0 - 2v_0 \sin\theta_0 gt + g^2t^2 \\ &= {v_0}^2\left(\cos^2\theta_0 + \sin^2\theta_0\right) - 2v_0 \sin\theta_0 gt + g^2t^2 \\ &= v_0^2 - 2v_0 \sin\theta_0 gt + g^2t^2. \end{aligned}$$



So, $K(t) = \left[m\left({v_0}^2 - 2v_0 \sin\theta_0 gt + g^2t^2\right)\right]/2$. The potential energy only changes with displacement in the vertical direction. The gravitational potential energy is given by $U = mgy$. From kinematics equations, $y = y_0 + v_{0,y}t - \left(gt^2\right)/2$. Because the projectile was launched from the ground, $y_0 = 0$. Substitute $v_{0,y} = v_0 \sin\theta_0$ into the equation to get $y = v_0 \sin\theta_0 t - \left(gt^2\right)/2$. Substituting this into the expression for $U$ yields $U(t) = mg\left(v_0 \sin\theta_0 t - \left(gt^2\right)/2\right)$. The total energy of the projectile is $E(t) = K(t) + U(t)$. This equation can be written as

$$E(t) = \frac{m\left({v_0}^2 - 2v_0 \sin\theta_0 gt + g^2t^2\right)}{2} + mg\left(v_0 \sin\theta_0 t - \frac{1}{2}gt^2\right).$$

Grouping like terms, the equation can be simplified:

$$E(t) = \frac{m}{2}\left(g^2t^2 - g^2t^2\right) - mgv_0 \sin\theta_0 t + mgv_0 \sin\theta_0 t + \frac{1}{2}m{v_0}^2 \Rightarrow E(t) = \frac{1}{2}m{v_0}^2.$$

Notice that $E$ is actually not time dependent. This is due to the conservation of energy.

**6.26.** (a) The total energy is given by the sum of the kinetic energy, $K = mv^2/2$, and potential energy, $U = mgh$. This gives the formula $E = \frac{1}{2}mv^2 + mgh$ for total energy. Therefore,

$$H(m,h,v) = \frac{\frac{1}{2}mv^2 + mgh}{mg} = \frac{\frac{1}{2}v^2 + gh}{g} = \frac{v^2}{2g} + h.$$

(b) The aircraft has a mass of $m = 3.5 \cdot 10^5$ kg, a velocity of $v$ = 250.0 m/s and a height of $h = 1.00 \cdot 10^4$ m Substituting these values gives

$$H = \frac{(250.0 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 1.00 \cdot 10^4 \text{ m} = 13{,}185.5 \text{ m} \approx 13{,}200 \text{ m}.$$

**6.27.** (a) The energy in the system is the sum of the kinetic energy and the gravitational potential energy, $E = K + U$. For motion in the $x$-direction, $U = 0$ and $K = mv^2/2$. So, $E = mv^2/2$. Newton's second law is $\vec{F} = m\vec{a}$, which can also be written as $\vec{F} = m\, d\vec{v}/dt$. By the work-kinetic energy theorem, $W = \Delta K$ and $W = \vec{F} \bullet \vec{x}$. If the work on the body as a function of position is to be determined,

$$W = \sum_{i=1}^{n} \vec{F}(x_i) \Delta \vec{x}.$$

If the motion is continuous, let the intervals become infinitesimal so that the sum becomes an integral, $W = \int_a^b \vec{F} \bullet d\vec{x}$. Since $\vec{F} = m\, d\vec{v}/dt$ and $\vec{v} = d\vec{x}/dt$, it must be that $d\vec{x} = \vec{v}\, dt$. Substituting these values into the equation:

$$W = \int m \frac{d\vec{v}}{dt} \bullet \vec{v}\, dt = \int m\vec{v} \bullet d\vec{v}.$$

Work is also equal to the change in kinetic energy, therefore,

$$\Delta K = \int_{v_i}^{v_f} m\vec{v} \bullet d\vec{v} = \frac{1}{2} mv^2 \Big|_{v_i}^{v_f} = \frac{1}{2} m\left(v_f^2 - v_i^2\right).$$

(b) Newton's second law, expressed as $\vec{F} = m\vec{a}$, does not hold for objects on the subatomic scale or for objects approaching the speed of light. The law of conservation of energy holds under all known circumstances.



**6.28.** (a) The force function is $F(x) = -\frac{dU(x)}{dx} = 4U_0\left[\frac{12x_0^{12}}{x^{13}} - \frac{6x_0^6}{x^7}\right]$.

(b) The two atoms experience zero force from each other when $F$ = 0, which is when $\left[12\frac{x_0^{12}}{x^{13}} - \frac{6x_0^6}{x^7}\right] = 0$.

Solving for $x$ yields $\frac{6x_0^6}{x^7} = \frac{12x_0^{12}}{x^{13}} \Rightarrow x^6 = 2x_0^6$ or $x = \pm\sqrt[6]{2}x_0$. Since $x$ is the separation, $x = \sqrt[6]{2}x_0$.

(c) For separations larger than $x = \sqrt[6]{2}x_0$, let $x = 3x_0$:

$$U(3x_0) = 4U_0\left[\left(\frac{x_0}{3x_0}\right)^{12} - \left(\frac{x_0}{3x_0}\right)^6\right] = 4U_0\left[\left(\frac{1}{3}\right)^{12} - \left(\frac{1}{3}\right)^6\right].$$

The factor $\left[\left(1/3^{12}\right) - \left(1/3^6\right)\right]$ is negative and the potential is negative. Therefore, for $x > \sqrt[6]{2}x_0$, the nuclei attract. For separations smaller than $x = \sqrt[6]{2}x_0$, let $x = x_0/2$:

$$U(x_0/2) = 4U_0\left[\left(\frac{2x_0}{x_0}\right)^{12} - \left(\frac{2x_0}{x_0}\right)^6\right] = 4U_0\left[2^{12} - 2^6\right].$$

The term $\left[2^{12} - 2^6\right]$ is positive and the potential is positive. So, when $x < \sqrt[6]{2}x_0$, the potential is positive and the nuclei repel.

**6.29.** (a) In two-dimensional situations, the force components can be obtained from the potential energy using the equations $F_x = -\frac{\partial U(x,y)}{\partial x}$ and $W_a = (10.0 \text{ N/cm})\left((5.00 \text{ cm})^2 - (-5.00 \text{ cm})^2\right)/2 = 0 \text{ J}$ The net force is given by:

$$\vec{F} = F_x\hat{x} + F_y\hat{y} = -\frac{\partial U(x,y)}{\partial x}\hat{x} - \frac{\partial U(x,y)}{\partial y}\hat{y} = -\frac{1}{2}k\left(\frac{\partial}{\partial x}(x^2+y^2)\hat{x} + \frac{\partial}{\partial y}(x^2+y^2)\hat{y}\right)$$

$$= -\frac{1}{2}k(2x\hat{x} + 2y\hat{y}) = -k(x\hat{x} + y\hat{y}).$$

(b) The equilibrium point will be where $\vec{F} = 0.$ This occurs if and only if $x$ and $y$ are both zero.

(c) These forces will accelerate the mass in the $-\hat{x}$ and $-\hat{y}$ directions for positive values of $x$ and $y$ and vice versa for negative values of $x$ and $y$.

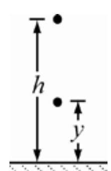
(d) $|\vec{F}| = \left[(F_x)^2 + (F_y)^2\right]^{\frac{1}{2}}.$ For $x$ = 3.00 cm, $y$ = 4.00 cm and $k$ = 10.0 N/cm:

$$|\vec{F}| = \left[\left(-(10.0 \text{ N/cm})(3.00 \text{ cm})\right)^2 + \left(-(10.0 \text{ N/cm})(4.00 \text{ cm})\right)^2\right]^{\frac{1}{2}} = 50.0 \text{ N}.$$

(e) A turning point is a place where the kinetic energy, $K$ is zero. Since $K = E - U$, the turning point will occur when $U = E$, so the turning points occurs when $U$ = 10 J. Solve $U(x,y) = 10 \text{ J} = \frac{1}{2}k(x^2+y^2)$. This gives $20.0 \text{ J} = \frac{10.0 \text{ N}}{\text{cm}} \cdot \frac{100 \text{ cm}}{\text{m}}(x^2+y^2)$, or $x^2 + y^2 = 0.0200 \text{ m}^2$. The turning points are the points on the circle centered at the origin of radius 0.141 m.

**6.30.** Setting the kinetic energy equal to the potential energy will normally not yield useful information. To use the example in the problem, if the rock is dropped from a height, $h$ above the ground, then solving for the speed at two different locations:



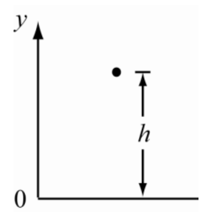



Point 1: $U_1 = mgh$ and $K_1 = (mv_1^2)/2$. If $mgh = mv^2/2$, then solving for $v_1$: $v_1 = \sqrt{2gh}$. But the rock has not been dropped yet so in fact $v_1$ is really zero. Point 2: just before the rock hits the ground. In this case, the rock's height above the ground, $y$, is almost zero. If $U_2 = K_2$, then $mgy = mv_2^2/2$ or $v_2 = \sqrt{2gy}$. But if $y$ is about 0 m, then $v_2 \approx 0$ m/s. At point 2, the rock's velocity is reaching its maximum value, so by setting the potential and kinetic energy equal to one another at this point, the wrong value is calculated for the rock's speed.

## Exercises

**6.31.** **THINK:** The mass of the book is $m$ = 2.00 kg and its height above the floor is $h$ = 1.50 m. Determine the gravitational potential energy, $U_g$.

**SKETCH:**




**RESEARCH:** Taking the floor's height as $U_g = 0$, $U_g$ for the book can be determined from the formula $U_g = mgh$.

**SIMPLIFY:** It is not necessary to simplify.

**CALCULATE:** $U_g = (2.00 \text{ kg})(9.81 \text{ m/s}^2)(1.50 \text{ m}) = 29.43 \text{ J}$

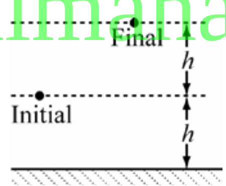
**ROUND:** The given initial values have three significant figures, so the result should be rounded to $U_g = 29.4 \text{ J}$.

**DOUBLE-CHECK:** This is a reasonable value for a small mass held a small distance above the floor.

**6.32.** **THINK:** The rock's mass is $m$ = 40.0 kg and the gravitational potential energy is $U_g = 500. \text{ J}$. Determine:
(a) the height of the rock, $h$, and
(b) the change, $\Delta U_g$ if the rock is raised to twice its original height, $2h$.

**SKETCH:**






**RESEARCH:** Use the equation $U_g = mgh$. Note: $\Delta U_g = U_g - U_{g,0}$.

**SIMPLIFY:**

(a) $U_g = mgh \Rightarrow h = \dfrac{U_g}{mg}$

(b) 
$$\begin{aligned} \Delta U_g &= U_g - U_{g,0} \\ &= mg(2h) - mgh \\ &= mgh \\ &= U_g \end{aligned}$$

**CALCULATE:**

(a) $h = \dfrac{500. \text{ J}}{40.0 \text{ kg}(9.81 \text{ m/s}^2)} = 1.274 \text{ m}$

(b) $\Delta U_g = 500. \text{ J}$

**ROUND:**

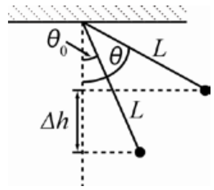
(a) $h = 1.27 \text{ m}$

(b) $\Delta U_g = 500. \text{ J}$ does not need to be rounded.

**DOUBLE-CHECK:** The initial height is reasonable for such a large mass, despite the large $U_g$. Since the potential energy is proportional to height, it should double when the height is doubled.

**6.33.** **THINK:** The rock's mass is $m$ = 0.773 kg. The length of the string is $L$ = 2.45 m. The gravitational acceleration on the Moon is $g_M = g/6$. The initial and final angles are $\theta_0 = 3.31°$ and $\theta = 14.01°$, respectively. Determine the rock's change in gravitational potential energy, $\Delta U$.

**SKETCH:**




**RESEARCH:** To determine $\Delta U$, the change in height of the rock, $\Delta h$, is needed. This can be determined using trigonometry. Then $\Delta U = mg_M \Delta h$.

**SIMPLIFY:** To determine $\Delta h$: $\Delta h = L\cos\theta_0 - L\cos\theta = L(\cos\theta_0 - \cos\theta)$. Then

$$\Delta U = mg_M \Delta h = \frac{1}{6}mgL(\cos\theta_0 - \cos\theta).$$

**CALCULATE:** $\Delta U = \frac{1}{6}(0.773 \text{ kg})(9.81 \text{ m/s}^2)(2.45 \text{ m})(\cos(3.31°) - \cos(14.01°)) = 0.08694 \text{ J}$

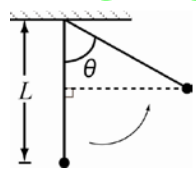
**ROUND:** With three significant figures in the values, the result should be rounded to $\Delta U = 0.0869$ J.

**DOUBLE-CHECK:** $\Delta U$ is small, as it should be considering the smaller gravitational acceleration and the small change in height.

**6.34.** **THINK:** The child's mass is $m$ = 20.0 kg. Each rope has a length of $L$ = 1.50 m. Determine (a) $U_g$ at the lowest point of the swing's trajectory, (b) $U_g$ when the ropes are $\theta = 45.0°$ from the vertical and (c) the position with the higher potential energy.



**SKETCH:**




**RESEARCH:** Use $U_g = mgh$.

**SIMPLIFY:**

(a) Relative to the point where $U_g = 0$, the height of the swing is $-L$. Then $U_g = -mgL$.

(b) Now, the height of the swing is $-L\cos\theta$. Then $U_g = -mgL\cos\theta$.

**CALCULATE:**

(a) $U_g = -(20.0 \text{ kg})(9.81 \text{ m/s}^2)(1.50 \text{ m}) = -294.3 \text{ J}$

(b) $U_g = -(20.0 \text{ kg})(9.81 \text{ m/s}^2)(1.50 \text{ m})\cos 45.0° = -208.1 \text{ J}$

(c) Relative to the point $U_g = 0$, the position in part (b) has greater potential energy.

**ROUND:** With three significant figures in $m$ and $L$:

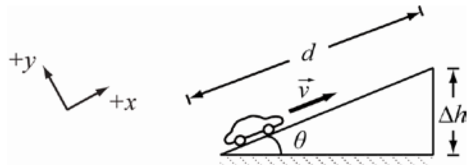
(a) $U_g = -294$ J

(b) $U_g = -208$ J

**DOUBLE-CHECK:** Had $U_g = 0$ been set at the lowest point of the swing's trajectory, the potential energy in part (b) would still be greater than the potential energy in part (a), as it should be.

**6.35.** **THINK:** The mass of the car is $m = 1.50 \cdot 10^3$ kg. The distance traveled is $d$ = 2.50 km = $2.50 \cdot 10^3$ m. The angle of inclination is $\theta = 3.00°$. The car travels at a constant velocity. Determine the change in the car's potential energy, $\Delta U$ and the net work done on the car, $W_{net}$.

**SKETCH:**




**RESEARCH:** To determine $\Delta U$ the change of height of the car $\Delta h$ must be known. From trigonometry, the change in height is $\Delta h = d\sin\theta$. Then, $\Delta U = mg\Delta h$. To determine $W_{\text{net}}$ use the work-kinetic energy theorem. Despite the fact that non-conservative forces are at work (friction force on the vehicle), it is true that $W_{\text{net}} = \Delta K$.

**SIMPLIFY:** $\Delta U = mg\Delta h = mgd\sin\theta$

$$W_{\text{net}} = \Delta K = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(v_{\text{f}}^2 - v_0^2\right)$$

**CALCULATE:** $\Delta U = \left(1.50\cdot 10^3 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)\left(2.50\cdot 10^3 \text{ m}\right)\sin\left(3.00°\right) = 1925309 \text{ J}$

$$W_{\text{net}} = \frac{1}{2}m\left(v_{\text{f}}^2 - v_0^2\right) = \frac{1}{2}m(0) = 0$$

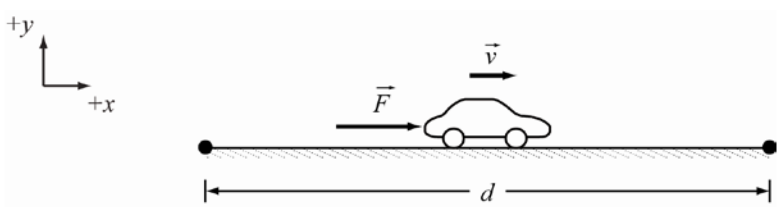
**ROUND:** Since $\theta$ has two significant figures, $\Delta U = 1.93\cdot 10^6$ J, and there is no net work done on the car.

**DOUBLE-CHECK:** The change in potential energy is large, as the car has a large mass and a large change in height, $\Delta h = \left(2.50\cdot 10^3 \text{ m}\right)\sin\left(3.00°\right) = 131$ m. The fact that the net work done is zero while there is a change in potential energy means that non-conservative forces did work on the car (friction, in this case).

**6.36.** **THINK:** The constant force is $F$ = 40.0 N. The distance traveled is $d = 5.0\cdot 10^3$ m. Assume the force is parallel to the distance traveled. Determine how much work is done, and if it is done on or by the car. The car's speed is constant.



**SKETCH:**




**RESEARCH:** In general,

$$W = \int_{x_0}^{x} F(r)\,dr \quad \text{(in one dimension).}$$

Here the force is constant, so $F(r) = F$. Bearing in mind that $W_{\text{net}} = \Delta K = 0$, due to the constant speed, the work done by the constant force, $F$ can still be calculated.

**SIMPLIFY:** $W = \int_{x_0}^{x_1} F\,dr = F\int_{x_0}^{x_1} dr = F\Delta x = Fd$

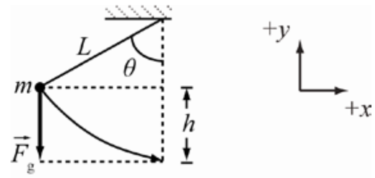
**CALCULATE:** $W = (40.0 \text{ N})\left(5.0\cdot 10^3 \text{ m}\right) = 200{,}000 \text{ J}$. This is the work done on the car by the constant force, as it is a positive value.

**ROUND:** With two significant figures in $d$, $W = 2.0\cdot 10^5$ J.

**DOUBLE-CHECK:** This is a reasonable amount of work done by $F$, given the large distance the force acts over.

**6.37.** **THINK:** The piñata's mass is $m$ = 3.27 kg. The string length is $L$ = 0.810 m. Let $h$ be the height of the piñata at its initial position, at an initial angle of $\theta = 56.5°$ to the vertical. Determine the work done by gravity, $W_{\text{g}}$, by the time the string reaches a vertical position for the first time.

**SKETCH:**




**RESEARCH:** Since the force of gravity is constant, the work is given by $W_g = \vec{F}_g h = mgh$.

**SIMPLIFY:** $W_g = mgh = mg(L - L\cos\theta) = mgL(1-\cos\theta)$

**CALCULATE:** $W_g = (3.27 \text{ kg})(9.81 \text{ m/s}^2)(0.810 \text{ m})(1-\cos(56.5°)) = 11.642 \text{ J}$

**ROUND:** With three significant figures in $L$, $W_g = 11.6$ J.

**DOUBLE-CHECK:** The work done by gravity should be positive because $F_g$ pulls the piñata downward.

**6.38.** **THINK:** $U(x) = \frac{1}{x} + x^2 + x - 1$. Determine (a) a function which describes the force on the particle, and (b) a plot of the force and the potential functions and (c) the force on the particle when $x =$ 2.00 m.

**SKETCH:** A sketch will be provided when part (b) is completed.

**RESEARCH:** The relationship between $F$ and $U$, in one dimension, is $F(x) = -\frac{d}{dx}U(x)$.

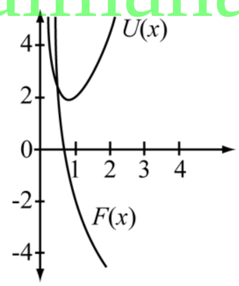
**SIMPLIFY:** (a) $F(x) = -\frac{d}{dx}\left(\frac{1}{x} + x^2 + x - 1\right) = -\left(-x^{-2} + 2x + 1\right) = \frac{1}{x^2} - 2x - 1$

**CALCULATE:**

(a) Not necessary.

(b) Plotting yields:






(c) At $x = 2.00$ m, $F(2.00) = \frac{1}{(2.00)^2} - 2(2.00) - 1 = -4.75$ N (SI units are assumed).

**ROUND:** $F(2.00 \text{ m}) = -4.75$ N

**DOUBLE-CHECK:** $F(x)$ is the negative of the slope of $U(x)$. $F(x)$ crosses the $x$-axis where $U(x)$ has a local minimum, as would be expected.

**6.39.** **THINK:** The potential energy functions are (a) $U(y) = ay^3 - by^2$ and (b) $U(y) = U_0 \sin(cy)$. Determine $F(y)$ from $U(y)$.

**SKETCH:** A sketch is not necessary.

**RESEARCH:** $F(y) = -\frac{\partial U(y)}{\partial y}$

**SIMPLIFY:**

(a) $F(y) = -\frac{\partial(ay^3 - by^2)}{\partial y} = 2by - 3ay^2$

(b) $F(y) = -\dfrac{\partial\left(U_0 \sin(cy)\right)}{\partial y} = -cU_0\cos(cy)$

**CALCULATE:** There are no numerical calculations to perform.
**ROUND:** It is not necessary to round.
**DOUBLE-CHECK:** The derivative of a cubic polynomial should be a quadratic, so the answer obtained for (a) makes sense. The derivative of a sine function is a cosine function, so it makes sense that the answer obtained for (b) involves a cosine function.

**6.40.** **THINK:** The potential energy function is of the form $U(x,z) = ax^2 + bz^3$. Determine the force vector, $\vec{F}$, associated with $U$.
**SKETCH:** Not applicable.

**RESEARCH:** $\vec{F}(x,y,z) = -\vec{\nabla}U(x,y,z) = -\left(\dfrac{\partial}{\partial x}U\hat{x} + \dfrac{\partial}{\partial y}U\hat{y} + \dfrac{\partial}{\partial z}U\hat{z}\right)$

**SIMPLIFY:** The expression cannot be further simplified.

**CALCULATE:** $\vec{F} = -\dfrac{\partial\left(ax^2+bz^3\right)\hat{x}}{\partial x} - \dfrac{\partial\left(ax^2+bz^3\right)\hat{y}}{\partial y} - \dfrac{\partial\left(ax^2+bz^3\right)\hat{z}}{\partial z}$

$$= -(2ax)\hat{x} - 0\hat{y} - \left(3bz^2\right)\hat{z}$$

$$= -(2ax)\hat{x} - \left(3bz^2\right)\hat{z}$$

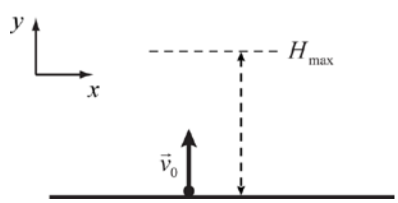
**ROUND:** Not applicable.
**DOUBLE-CHECK:** Notice that $U$ is the sum of a function of $x$ and a function of $z$, namely,

if $G(x) = ax^2$ and $H(z) = bz^3$ then $U(x,z) = G(x) + H(z)$.

Since $G(x)$ has a critical point at $x = 0$ and $H(z)$ has a critical point at $z = 0$, we may expect that $\vec{F} = 0$. And in fact, $\vec{F} = -\left(2a(0)\right)\hat{x} - \left(3b(0)^2\right)\hat{z} = 0$. Therefore, the answer is reasonable.



**6.41.** **THINK:** The maximum height achieved is $H_{max} = 5.00$ m, while the initial height $h_0$ is zero. The speed of the ball when it reaches its maximum height is $v = 0$. Determine the initial speed.
**SKETCH:**




**RESEARCH:** In an isolated system with only conservative forces, $\Delta E_{mec} = 0$. Then, $\Delta K = -\Delta U$. Use $U = mgH_{max}$ and $K = mv^2/2$.

**SIMPLIFY:** $K_f - K_i = -\left(U_f - U_i\right) = U_i - U_f$, so $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\;2} = mgh_0 - mgH_{max}$.

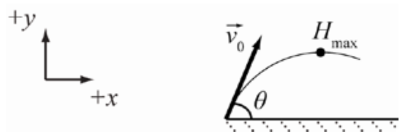
Substituting $v = 0$ and $h_0 = 0$ gives the equation $-\frac{1}{2}mv_0^{\;2} = -mgH_{max}$. Therefore, $v_0 = \sqrt{2gH_{max}}$.

**CALCULATE:** $v_0 = \sqrt{2\left(9.81 \text{ m/s}^2\right)\left(5.00 \text{ m}\right)} = 9.9045$ m/s

**ROUND:** With three significant figures in $H_{max}$, $v_0 = 9.90$ m/s.
**DOUBLE-CHECK:** This is a reasonable speed to throw a ball that reaches a maximum height of 5 m.

**6.42.** **THINK:** The cannonball's mass is $m$ = 5.99 kg. The launch angle is $\theta = 50.21°$ above the horizontal. The initial speed is $v_0 = 52.61$ m/s and the final vertical speed is $v_y = 0$. The initial height is zero. Determine the gain in potential energy, $\Delta U$.

**SKETCH:**




**RESEARCH:** Neglecting air resistance, there are only conservative forces at work. Then, $\Delta K = -\Delta U$ or $\Delta U = -\Delta K$. Determine $\Delta K$ from $K = mv^2/2$. From trigonometry, $v_x = v_0 \cos\theta$.

**SIMPLIFY:** $\Delta U = -\Delta K = -(K_f - K_i) = K_i - K_f$. Note that initially the ball has a horizontal speed $v_x$ (which is constant throughout the cannonball's motion) and a vertical speed $v_{y0}$. At its maximum height, $v_y = 0$. Then, $\Delta U = \frac{1}{2}mv_0^2 - \frac{1}{2}m(v_0\cos\theta)^2 = \frac{1}{2}mv_0^2(1-\cos^2\theta)$.

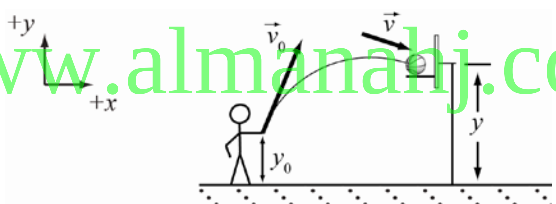
**CALCULATE:** $\Delta U = \frac{1}{2}(5.99 \text{ kg})(52.61 \text{ m/s})^2(1-\cos^2(50.21°)) = 4894.4 \text{ J}$

**ROUND:** With three significant figures in *m*, $\Delta U = 4890$ J.

**DOUBLE-CHECK:** The change in potential energy is positive, implying that the ball gained potential energy, which it would if raised any height above its initial point. Since the horizontal velocity of the cannonball is constant, it makes sense that the initial vertical velocity is converted entirely into potential energy when the cannonball reaches the highest point.

**6.43.** **THINK:** The initial height of the basketball is $y_0 = 1.20$ m. The initial speed of the basketball is $v_0 = 20.0$ m/s. The final height is $y = 3.05$ m. Determine the speed of the ball at this point.

**SKETCH:**






**RESEARCH:** Neglecting air resistance, there are only conservative forces, so $\Delta K = -\Delta U$. The kinetic energy *K* can be determined from $K = mv^2/2$ and *U* from $U = mgh$.

**SIMPLIFY:** $K_f - K_i = U_i - U_f$, so $(1/2)mv^2 - (1/2)mv_0^2 = mgy_0 - mgy$. Dividing through by the mass *m* yields the equation $(1/2)v^2 - (1/2)v_0^2 = gy_0 - gy$. Then solving for *v* gives

$$v = \sqrt{2\left(g(y_0 - y) + \frac{1}{2}v_0^2\right)}.$$

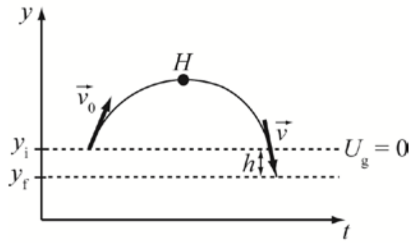
**CALCULATE:**
$$v = \sqrt{2\left((9.81 \text{ m/s}^2)(1.20 \text{ m} - 3.05 \text{ m}) + \frac{1}{2}(20.0 \text{ m/s})^2\right)}$$
$$= \sqrt{2(-18.1485 \text{ m}^2/\text{s}^2 + 200.0 \text{ m}^2/\text{s}^2)}$$
$$= 19.071 \text{ m/s}$$

**ROUND:** The initial height is given with the fewest number of significant figures. Since it has three significant figures the value of *v* needs to be rounded to three significant figures: *v* = 19.1 m/s.

**DOUBLE-CHECK:** The final speed should be less than the initial speed since the final height is greater than the initial one.

**6.44.** **THINK:** The book's mass is *m* = 1.0 kg. The initial height is $y_0 = 1.0$ m, where $U_g = 0$, the maximum height is *H* = 3.0 m, and the final height is $y_f = 0$ m. Determine (a) the potential energy of the book when it hits the ground, $U_g$, and (b) the velocity of the book as it hits the ground, $v_f$. The book is thrown straight up into the air, so the launch angle is vertical. The sketch is not a plot of the trajectory of the book, but a plot of height versus time.

**SKETCH:**




**RESEARCH:**

(a) Gravitational potential energy is given by $U_g = mgh$. To compute the final energy, consider the height relative to the height of zero potential, $y_i = 1.0$ m.

(b) To determine $v_f$, consider the initial point to be at $y = H$ (where $v = 0$), and the final point to be at the point of impact $y = y_f = 0$. Assume there are only conservative forces, so that $\Delta K = -\Delta U$. $\Delta U$ between $H$ and $y_f$ is unaffected by the choice of reference point.

**SIMPLIFY:**

(a) Relative to $U_g = 0$ at $y_i$, the potential energy of the book when it hits the ground is given by $U_g = mgh = mg(y_f - y_i)$.

(b) $\Delta K = -\Delta U \Rightarrow K_f - K_i = -(U_f - U_i)$. With $v = 0$ at the initial point, $K_f = U_i - U_f$ and $(1/2)mv^2 = mgH - mgy_f = mgH$. Solving for $v_f$ gives the equation: $v_f = -\sqrt{2gH}$. The negative root is chosen because the book is falling.



**CALCULATE:**

(a) $U_g = (1.0 \text{ kg})(9.81 \text{ m/s}^2)(0 - 1.0 \text{ m}) = -9.81$ J

(b) $v_f = -\sqrt{2(9.81 \text{ m/s}^2)(3.0 \text{ m})} = -7.6720$ m/s

**ROUND:** With two significant figures in $m$, $y_i$ and $H$:

(a) $U_g = -9.8$ J

(b) $v_f = -7.7$ m/s, or 7.7 m/s downward.

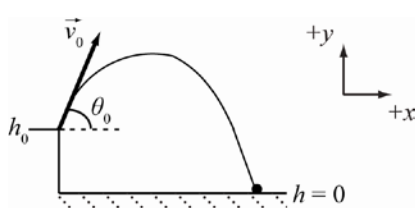
**DOUBLE-CHECK:** $U_g$ should be negative at $y_f$, relative to $U_g = 0$ at $y_0$, because there should be a loss of potential energy. Also, it is sensible for the final velocity of the book to be directed downward.

**6.45.** **THINK:** The ball's mass is $m$ = 0.0520 kg. The initial speed is $v_0 = 10.0$ m/s. The launch angle is $\theta_0 = 30.0°$. The initial height is $h_0 = 12.0$ m. Determine:

(a) kinetic energy of the ball when it hits the ground, $K_f$ and

(b) the ball's speed when it hits the ground, $v$.

**SKETCH:**




**RESEARCH:** Assuming only conservative forces act on the ball (and neglecting air resistance), $\Delta K = -\Delta U$. $K_f$ can be determined using the equations $\Delta K = -\Delta U$, $K = mv^2/2$ and $U = mgh$. Note that $U_f = 0$, as $h = 0$. With $K_f$ known, $v$ can be determined.

**SIMPLIFY:**

(a) $\Delta K = -\Delta U \Rightarrow K_f - K_i = U_i - U_f = U_i \Rightarrow K_f = U_i + K_i = mgh_0 + \frac{1}{2}mv_0^2$

(b) $K_f = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2K_f / m}$

**CALCULATE:**

(a) $K_f = (0.0520 \text{ kg})(9.81 \text{ m/s}^2)(12.0 \text{ m}) + \frac{1}{2}(0.0520 \text{ kg})(10.0 \text{ m/s})^2 = 6.121 \text{ J} + 2.60 \text{ J} = 8.721 \text{ J}$

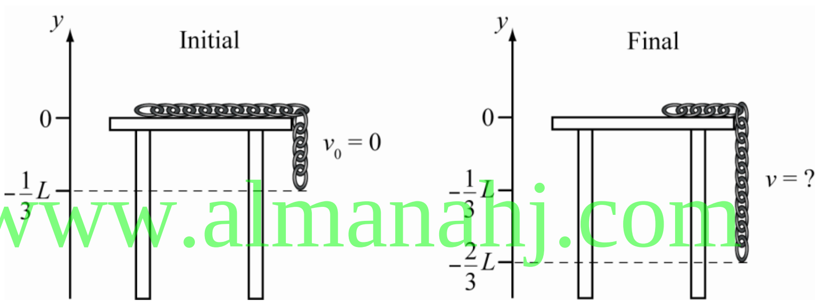
(b) $v = \sqrt{2(8.721 \text{ J})/(0.0520 \text{ kg})} = 18.32 \text{ m/s}$

**ROUND:** With $m$ having three significant figures, $K_f = 8.72$ J and $v$ = 18.3 m/s.

**DOUBLE-CHECK:** The amount of kinetic energy computed is a reasonable amount for a ball. The final speed should be greater than the initial speed because the mechanical energy has been completely transformed to kinetic energy. It is, so the calculated value is reasonable.

**6.46.** **THINK:** The chain's mass is $m$ and has a length of $L$ = 1.00 m. A third of the chain hangs over the edge of the table and held stationary. After the chain is released, determine its speed, $v$, when two thirds of the chain hangs over the edge.

**SKETCH:**






**RESEARCH:** Consider the center of mass (com) location for the part of the chain that hangs over the edge. Since the chain is a rigid body, and it is laid out straight (no slack in the chain), $v_{com} = v$. $\Delta K = -\Delta U$, $K = (mv^2)/2$ and $U = mgh$.

**SIMPLIFY:** Initially, 1/3 of the chain is hanging over the edge and then $m_{com,0} = m/3$, and $h_{com,i} = -L/6$. When 2/3 of the chain is hanging over the edge, the hanging mass is $m_{com} = 2m/3$. Then, $\Delta K = -\Delta U \Rightarrow K_f - K_i = U_i - U_f$ and $K_i = 0$, so $K_f = U_i - U_f$. Substituting gives $(1/2)mv_{com}^2 = m_{com,i}gh_{com,i} - m_{com}gh_{com}$, so $\frac{1}{2}mv^2 = \left(\frac{m}{3}\right)g\left(-\frac{L}{6}\right) - \left(\frac{2m}{3}\right)g\left(-\frac{L}{3}\right)$, and dividing through by $m$ gives the equation $\frac{1}{2}v^2 = -\frac{1}{18}gL + \frac{2}{9}gL$. Solving for $v$ yields $v = \sqrt{gL/3}$.

**CALCULATE:** $v = \sqrt{(9.81 \text{ m/s}^2)(1.00 \text{ m})/3} = 1.808 \text{ m/s}$

**ROUND:** With three significant figures in $L$, $v$ = 1.81 m/s.

**DOUBLE-CHECK:** This is a reasonable speed for the chain to achieve while sliding off the table.

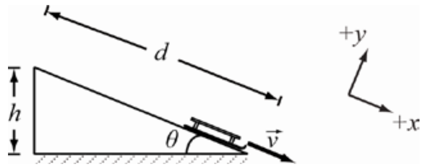
**6.47.** **THINK:** The initial height is $h$ = 40.0 m. Determine:

(a) the speed $v_f$ at the bottom, neglecting friction,

(b) if the steepness affects the final speed; and

(c) if the steepness affects the final speed when friction is considered.

**SKETCH:**




**RESEARCH:**

(a) With conservative forces, $\Delta K = -\Delta U$. $v$ can be determined from $K = \left(mv_{\text{f}}^{2}\right)/2$ and $U = mgh$.

(b and c) Note that the change in the angle $\theta$ affects the distance, *d*, traveled by the toboggan: as $\theta$ gets larger (the incline steeper), $d$ gets smaller.

(c) The change in thermal energy due to friction is proportional to the distance traveled: $\Delta E_{\text{th}} = \mu_{\text{k}} Nd$. The total change in energy of an isolated system is $\Delta E_{\text{tot}} = 0$, where $\Delta E_{\text{tot}} = \Delta K + \Delta U + \Delta E_{\text{th}}$, and $\Delta E_{\text{th}}$ denotes the non-conservative energy of the toboggan-hill system (in this case, friction).

**SIMPLIFY:**

(a) With $K_i = 0$ (assuming $v_0 = 0$) and $U_{\text{f}} = 0$ (taking the bottom to be $h = 0$):

$$K_{\text{f}} = U_{\text{i}} \Rightarrow \frac{1}{2}mv_{\text{f}}^{2} = mgh \Rightarrow v_{\text{f}} = \sqrt{2gh}$$

(b) The steepness does not affect the final speed, in a system with only conservative forces, the distance traveled is not used when conservation of mechanical energy is considered.

(c) With friction considered, then for the toboggan-hill system,

$$\Delta E = \Delta K + \Delta U + \Delta E_{\text{th}} = 0 \Rightarrow \Delta K = -\Delta U - \Delta E_{\text{th}} \Rightarrow K_{\text{f}} = U_{\text{i}} - \Delta E_{\text{th}} = mgh - \mu_{\text{k}} Nd$$

The normal force *N* is given by $N = mg\cos\theta$, while on the hill. With $d = h/\sin\theta$,



$$K_{\text{f}} = mgh - \mu_{\text{k}}\left(mg\cos\theta\right)\left(\frac{h}{\sin\theta}\right) = mgh\left(1 - \mu_{\text{k}}\cot\theta\right).$$

The steepness of the hill does affect $K_{\text{f}}$ and therefore $v$ at the bottom of the hill.

**CALCULATE:**

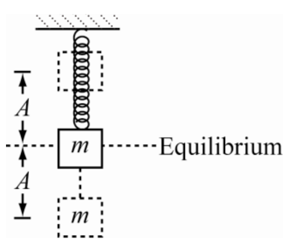
(a) $v_{\text{f}} = \sqrt{2\left(9.81 \text{ m/s}^2\right)\left(40.0 \text{ m}\right)} = 28.01 \text{ m/s}$

**ROUND:** Since *h* has three significant figures, *v* = 28.0 m/s.

**DOUBLE-CHECK:** This is a very fast, but not unrealistic speed for the toboggan to achieve.

**6.48.** **THINK:** The block's mass is *m* = 0.773 kg, the spring constant is *k* = 239.5 N/m and the amplitude is *A* = 0.551 m. The block oscillates vertically. Determine the speed *v* of the block when it is at *x* = 0.331 m from equilibrium.

**SKETCH:**




**RESEARCH:** The force of gravity in this system displaces the equilibrium position of the hanging block by *mg*/*k*. Since the distance from equilibrium is given, the following equation can be used to determine *v*: $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$.

**SIMPLIFY:** $v=\sqrt{\frac{k}{m}\left(A^2-x^2\right)}$

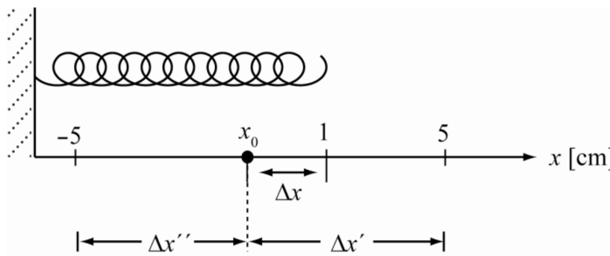
**CALCULATE:** $v=\sqrt{\frac{239.5\text{ N/m}}{0.773\text{ kg}}\left(\left(0.551\text{ m}\right)^2-\left(0.331\text{ m}\right)^2\right)}=7.7537\text{ m/s}$

**ROUND:** The least precise value has three significant figures, so round the answer to three significant figures: $v$ = 7.75 m/s.

**DOUBLE-CHECK:** This is a reasonable speed for the block on the spring.

**6.49.** **THINK:** It is known that $k=$ 10.0 N/cm and $\Delta x=1.00$ cm. Determine (a) the energy needed to further stretch the spring to $\Delta x'=5.00$ cm and (b) the energy needed to compress the spring from $\Delta x'=5.00$ cm to $\Delta x''=-5.00$ cm.

**SKETCH:**




**RESEARCH:** Assume the spring is stationary at all positions given above. The energy required to stretch the spring is the work applied to the spring, $W_a$, and $W_a=-W_s$ for $\Delta k=0$. It is known that $W_s=\left[\left(kx_i^2\right)/2\right]-\left[\left(kx_f^2\right)/2\right]$.



**SIMPLIFY:** $W_a=-W_s=-\left[\left(kx_i^2\right)/2\right]+\left[\left(kx_f^2\right)/2\right]=k\left(x_f^2-x_i^2\right)/2$

**CALCULATE:**

(a) $W_a=\left(10.0\text{ N/cm}\right)\left(\left(5.00\text{ cm}\right)^2-\left(1.00\text{ cm}\right)^2\right)/2=120.\text{ N cm}=1.20\text{ J}$

(b) $W_a=\left(10.0\text{ N/cm}\right)\left(\left(5.00\text{ cm}\right)^2-\left(-5.00\text{ cm}\right)^2\right)/2=0\text{J}$

**ROUND:** With three significant figures in each given value, (a) $W_a=1.20$ J and (b) $W_a=0$. Take this zero to be precise.

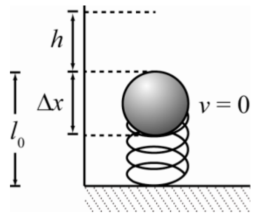
**DOUBLE-CHECK:**

(a) $W_a$ should be positive because energy must be transferred to the spring to stretch it further from equilibrium.

(b) The spring is the same distance from the equilibrium point, so the net energy transferred to the spring must be zero.

**6.50.** **THINK:** The mass of the ball is $m$ = 5.00 kg. The initial height is $h$ = 3.00 m. The initial speed is $v_0=5.00$ m/s. The spring constant is $k$ = 1600. N/m. The final speed of the ball is zero. Determine (a) the maximum compression $\Delta x$ of the spring and (b) the total work done on the ball while the spring compresses. The spring is initially at equilibrium, so the height given is the height above the equilibrium point of the spring.

**SKETCH:**




**RESEARCH:**

(a) There are no non-conservative forces, so $\Delta K = -\Delta U$, $U_{\text{s}} = \left(kx^2\right)/2$, $U_{\text{g}} = mgh$ and $K = \left(mv^2\right)/2$.

(b) Use the work-kinetic energy theorem to find the net work done on the ball while the spring compresses $\Delta x$ by $W_{\text{net}} = \Delta K$.

**SIMPLIFY:**

(a) $\Delta K = -\Delta U$ so $K_{\text{f}} - K_{\text{i}} = U_{\text{si}} - U_{\text{sf}} + U_{\text{gi}} - U_{\text{gf}}$. Note that the equilibrium position of the spring is $l_0$. Since $K_{\text{f}}$ and $U_{\text{si}}$ are zero, $0 - K_{\text{i}} = 0 - U_{\text{sf}} + U_{\text{gi}} - U_{\text{gf}}$, and $-\frac{1}{2}mv_0^2 = -\frac{1}{2}k(\Delta x)^2 + mg(l_0 + h) - mg(l_0 - \Delta x)$, which simplifies to $-\frac{1}{2}mv_0^2 = -\frac{1}{2}k(\Delta x)^2 + mgh + mg\Delta x$ and subsequently $\frac{1}{2}k(\Delta x)^2 - mg\Delta x - \frac{1}{2}mv_0^2 - mgh = 0$. Solving the quadratic equation gives

$$\Delta x = \frac{mg \pm \sqrt{(-mg)^2 + 2k\left(\frac{1}{2}mv_0^2 + mgh\right)}}{k} = \frac{mg \pm \sqrt{(mg)^2 + mk\left(v_0^2 + 2gh\right)}}{k}.$$



(b) $W_{\text{net}} = \Delta K = -\Delta U = -\Delta U_{\text{s}} - \Delta U_{\text{g}}$

$$\begin{aligned} W_{\text{net}} &= U_{\text{si}} - U_{\text{sf}} + U_{\text{gi}} - U_{\text{gf}} \\ &= 0 - \frac{1}{2}k(\Delta x)^2 + mgl_0 - mg(l_0 - \Delta x) \\ &= -\frac{1}{2}k(\Delta x)^2 + mg\Delta x \end{aligned}$$

**CALCULATE:**

(a) $$\Delta x = \frac{(5.00\text{ kg})(9.81\text{ m/s}^2)}{1600.\text{ N/m}} \pm \frac{\sqrt{(5.00\text{ kg})^2(9.81\text{ m/s}^2)^2 + (1600.\text{ N/m})(5.00\text{ kg})\left((5.00\text{ m/s})^2 + 2(9.81\text{ m/s}^2)(3.00\text{ m})\right)}}{1600.\text{ N/m}}$$

$$= 0.54349\text{ m},\ -0.48218\text{ m}$$

Since $\Delta x$ is defined as a positive distance (not a displacement), the solution must be positive. Take $\Delta x = 0.54349$ m.

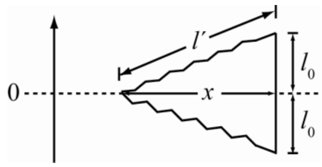
(b) $W_{\text{net}} = -\frac{1}{2}(1600.\text{ N/m})(0.54349\text{ m})^2 + (5.00\text{ kg})(9.81\text{ m/s}^2)(0.54349\text{ m}) = -209.6\text{ J}$

**ROUND:** Since the least precise value given in the question has three significant figures, both answers will have three significant figures: $\Delta x = 0.543$ m and $W_{\text{net}} = -210.$ J.

**DOUBLE-CHECK:** $\Delta x$ should be positive. Relative to the height, $h$, the value of $\Delta x$ is reasonable. Because the net work is negative, and since $\left|\Delta U_{\text{s}}\right| > \left|\Delta U_{\text{g}}\right|$ for the distance $\Delta x$, the clay ball does positive work on the spring and the spring does negative work on the clay ball. This makes sense for spring compression.

**6.51.** **THINK:** The spring constant for each spring is $k$ = 30.0 N/m. The stone's mass is $m$ = 1.00 kg. The equilibrium length of the springs is $l_0 = 0.500$ m. The displacement to the left is $x$ = 0.700 m. Determine the system's total mechanical energy, $E_{\text{mec}}$ and (b) the stone's speed, $v$, at $x$ = 0.

**SKETCH:**




Note: The sketch is a side view. The word "vertical" means that the springs are oriented vertically above the ground. The path the stone takes while in the slingshot is completely horizontal so that gravity is neglected.

**RESEARCH:**

(a) In order to determine $E_{\text{mec}}$, consider all kinetic and potential energies in the system. Since the system is at rest, the only form of mechanical energy is spring potential energy, $U_s = \left(kx^2\right)/2.$

(b) By energy conservation, $\Delta E_{\text{mec}}$ (no non-conservative forces). $v$ can be determined by considering $\Delta E_{\text{mec}} = 0.$

**SIMPLIFY:**

(a) $E_{\text{mec}} = K + U = U_s = U_{s1} + U_{s2} = \frac{1}{2}k_1\left(l_0 - l'\right)^2 + \frac{1}{2}k_2\left(l_0 - l'\right)^2 = k\left(l_0 - l'\right)^2$. To determine $l'$, use the Pythagorean theorem, $l' = \sqrt{l_0^2 + x^2}$. Then, $E_{\text{mec}} = k\left(l_0 - \sqrt{l_0^2 + x^2}\right)^2$.

(b) As the mechanical energy is conserved, $E_{\text{mec f}} = E_{\text{mec i}}$ so $K_{\text{f}} + U_{s\text{f}} = E_{\text{mec}}$ (with $U_{\text{f}} = 0$), and therefore $K_{\text{f}} = E_{\text{mec}}$. Solving the equation for kinetic energy, $\frac{1}{2}mv^2 = E_{\text{mec}} \Rightarrow v = \sqrt{2E_{\text{mec}}/m}.$



**CALCULATE:**

(a) $E_{\text{mec}} = 30.0 \text{ N/m}\left(0.500 \text{ m} - \sqrt{\left(0.500 \text{ m}\right)^2 + \left(0.700 \text{ m}\right)^2}\right)^2 = 3.893 \text{ J}$

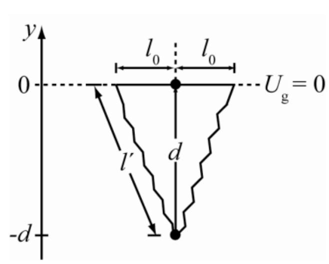
(b) $v = \sqrt{2\left(3.893 \text{ J}\right)/1.00 \text{ kg}} = 2.790 \text{ m/s}$

**ROUND:** Since all of the given values have three significant figures, the results should be rounded to $E_{\text{mec}} = 3.89$ J and $v$ = 2.79 m/s.

**DOUBLE-CHECK:** The values are reasonable considering the small spring constant.

**6.52.** **THINK:** The spring constant for each spring is $k$ = 30.0 N/m. The stone's mass is $m$ = 0.100 kg. The equilibrium length of each spring is $l_0 = 0.500$ m. The initial vertical displacement is $d$ = 0.700 m. Determine (a) the total mechanical energy, $E_{\text{mec}}$ and (b) the stone's speed, $v$, when it passes the equilibrium point.

**SKETCH:**



**RESEARCH:**
(a) To determine $E_{\text{mec}}$, all forms of kinetic and potential energy must be calculated for the system. Note that initially $K = 0$. Use the equations $U_s = \left(kx^2\right)/2$ and $U_g = mgh$.
(b) As there are no non-conservative forces, $E_{\text{mec}}$ is conserved. The speed, $v$, can be determined from $E_{\text{mec f}} = E_{\text{mec i}}$, using $K = \left(mv^2\right)/2$.
**SIMPLIFY:**
(a) $E_{\text{mec}} = K + U = U_g + U_{s1} + U_{s2} = mg(-d) + 2\left(\frac{1}{2}k\left(l' - l_0\right)^2\right)$. Note $l' = \sqrt{l_0^2 + d^2}$ from Pythagorean's theorem. Then, $E_{\text{mec}} = k\left(\sqrt{l_0^2 + d^2} - l_0^2\right)^2 - mgd$.
(b) $E_{\text{mec i}} = E_{\text{mec f}} = E_{\text{mec}}$ so $K_f = E_{\text{mec}}$ (as $U_{gf} = U_{sf} = 0$) and therefore $\frac{1}{2}mv^2 = E_{\text{mec}} \Rightarrow v = \sqrt{2E_{\text{mec}}/m}$.

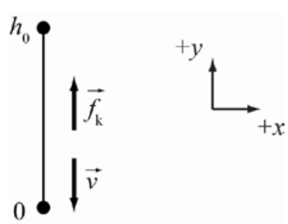
**CALCULATE:**

(a) $$E_{\text{mec}} = (30.0\text{ N/m})\left(\sqrt{(0.500\text{ m})^2 + (0.700\text{ m})^2} - 0.500\text{ m}\right)^2 - (0.100\text{ kg})(9.81\text{ m/s}^2)(0.700\text{ m})$$
$$= 3.893\text{ J} - 0.6867\text{ J}$$
$$= 3.206\text{ J}$$
(b) $v = \sqrt{2(3.206\text{ J})/(0.100\text{ kg})} = 8.0075\text{ m/s}$

**ROUND:** As each given value has three significant figures, the results should be rounded to $E_{\text{mec}} = 3.21\text{ J}$ and $v = 8.01$ m/s.
**DOUBLE-CHECK:** $E_{\text{mec}}$ is decreased by the gravitational potential energy. The stone's speed is reasonable considering its small mass.



**6.53.** **THINK:** The mass of the man is $m = 80.0$ kg. His initial height is $h_0 = 3.00$ m. The applied frictional force is $f_k = 400.$ N. His initial speed is $v_0 = 0$. What is his final speed, $v$?
**SKETCH:**



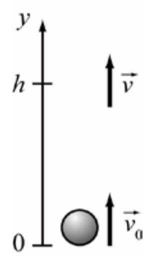

**RESEARCH:** In an isolated system, the total energy is conserved. $\Delta E_{\text{tot}} = \Delta K + \Delta U + \Delta E_{\text{th}} = 0$. Using $K = \left(mv^2\right)/2$, $U_g = mgh_0$ and $\Delta E_{\text{th}} = f_k d$, $v$ can be determined.
**SIMPLIFY:** Note $K_i = U_{gf} = 0$. Then, $\Delta K + \Delta U + \Delta E_{\text{th}} = 0 \Rightarrow K_f - U_{gi} + \Delta E_{\text{th}} = 0$. Note that the force of friction acts over the length of the pole, $h_0$. Then, $\frac{1}{2}mv^2 - mgh_0 + f_k h_0 = 0 \Rightarrow v = \sqrt{2\left(gh_0 - f_k h_0/m\right)} = \sqrt{2h_0\left(g - f_k/m\right)}$.

**CALCULATE:** $$v = \sqrt{2\left((9.81\text{ m/s}^2)(3.00\text{ m}) - (400.\text{ N})(3.00\text{ m})/(80.0\text{ kg})\right)}$$
$$= \sqrt{2\left(29.43\text{ m}^2/\text{s}^2 - 15.0\text{ m}^2/\text{s}^2\right)}$$
$$= 5.372\text{ m/s}$$
**ROUND:** With three significant figures in each given value, the result should be rounded to $v = 5.37$ m/s.

**DOUBLE-CHECK:** This velocity is less than it would be if the man had slid without friction, in which case $v$ would be $\sqrt{2gh_0} \approx 8$ m/s.

**6.54.** **THINK:** The ball's mass is $m = 0.100$ kg. The initial speed is $v_0 = 10.0$ m/s. The final height is $h = 3.00$ m and the final speed is $v = 3.00$ m/s. Determine the fraction of the original energy lost to air friction. Note that the initial height is taken to be zero.

**SKETCH:**




**RESEARCH:** For an isolated system, $\Delta E_{\text{tot}} = \Delta K + \Delta U + \Delta E_{\text{other}} = 0$. The fraction that must be determined is as follows: $\dfrac{\Delta E_{\text{friction}}}{E_{\text{initial}}} = \dfrac{\Delta E_{\text{f}}}{E_{\text{i}}}$.

**SIMPLIFY:** $\Delta K + \Delta U + \Delta E_{\text{f}} = 0$. Note $U_i = 0$. This means that

$$\Delta E_{\text{f}} = -\Delta K - \Delta U = -\left(\frac{1}{2}m\left(v^2 - v_0^{\,2}\right) + mgh\right) \text{ and } E_i = K_i + U_i = K_i = \frac{1}{2}mv_0^{\,2}.$$

Then, $\dfrac{\Delta E_{\text{f}}}{E_{\text{i}}} = \dfrac{\frac{1}{2}m\left(v_0^{\,2} - v^2 - 2gh\right)}{\frac{1}{2}mv_0^{\,2}} = \dfrac{\left(v_0^{\,2} - v^2 - 2gh\right)}{v_0^{\,2}}$.



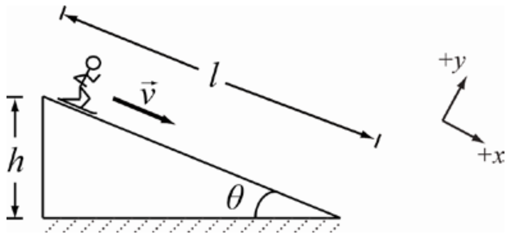
**CALCULATE:** $\dfrac{\Delta E_{\text{f}}}{E_{\text{i}}} = \dfrac{(10.0 \text{ m/s})^2 - (3.00 \text{ m/s})^2 - 2\left(9.81 \text{ m/s}^2\right)(3.00 \text{ m})}{(10.0 \text{ m/s})^2} = 0.3214$

**ROUND:** Each given value has three significant figures, so the result should be rounded as $\Delta E_{\text{f}} = 0.321 E_{\text{i}}$. The final answer is 32.1% of $E_i$ is lost to air friction.

**DOUBLE-CHECK:** If there were no friction and the ball started upward with an initial speed of $v_0 = 10$ m/s, its speed at a height of 3 m would be using kinematics $v = \sqrt{v_0^2 - 2gh} = \sqrt{(10 \text{ m/s})^2 - 2\left(9.81 \text{ m/s}^2\right)(3 \text{ m})} = 6.41$ m/s. This corresponds to a mechanical energy of $E = \frac{1}{2}(0.1 \text{ kg})(6.41 \text{ m/s})^2 + (0.1 \text{ kg})\left(9.81 \text{ m/s}^2\right)(3 \text{ m}) = 5.00$ J. The ball actually had a mechanical energy of $E = \frac{1}{2}(0.1 \text{ kg})(3 \text{ m/s})^2 + (0.1 \text{ kg})\left(9.81 \text{ m/s}^2\right)(3 \text{ m}) = 3.93$ J, which corresponds to a 32.1% loss, which agrees with the result using energy concepts.

**6.55.** **THINK:** The skier's mass is $m$ = 55.0 kg. The constant speed is $v$ = 14.4 m/s. The slope length is $l = 123.5$ m and the angle of the incline is $\theta = 14.7°$. Determine the mechanical energy lost to friction, $\Delta E_{\text{th}}$.

**SKETCH:**




**RESEARCH:** The skier and the ski slope form an isolated system. This implies that $\Delta E_{tot} = \Delta K + \Delta U + \Delta E_{th} = 0$. Note that $\Delta K = 0$ since $v$ is constant. Use the equation $U = mgh$, where the height of the ski slope can be found using trigonometry: $h = l\sin\theta$.

**SIMPLIFY:** At the bottom of the slope, $U_f = 0$. Then,

$$\Delta E_{th} = -\Delta U = -(U_f - U_i) = U_i = mgh = mgl\sin\theta.$$

**CALCULATE:** $\Delta E_{th} = (55.0 \text{ kg})(9.81 \text{ m/s}^2)(123.5 \text{ m})\sin 14.7° = 16909 \text{ J}$

**ROUND:** With three significant figures in $m$, $g$ and $\theta$, the result should be rounded to $\Delta E_{th} = 16.9 \text{ kJ}$.

**DOUBLE-CHECK:** If this energy had been transformed completely to kinetic energy (no friction), and if the skier had started from rest, their final velocity would have been 24.8 m/s at the bottom of the slope. This is a reasonable amount of energy transferred to thermal energy generated by friction.

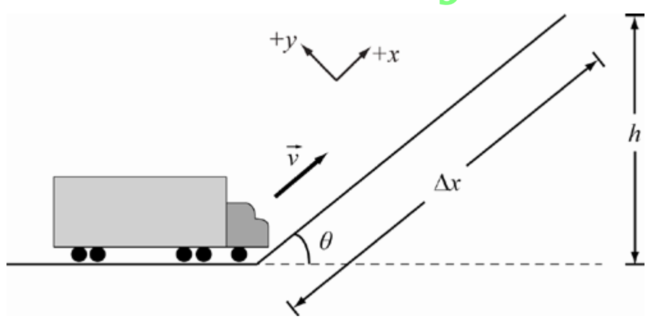
**6.56.** **THINK:** The truck's mass is $m$ = 10,212 kg. The initial speed is

$$v_0 = 61.2 \text{ mph}\left(\frac{1609.3 \text{ m}}{1 \text{ mile}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 27.3581 \text{ m/s}.$$

The incline angle is $\theta = 40.15°$ and the coefficient of friction is $\mu_k = 0.634$. Determine the distance traveled along the incline, $\Delta x$, before the truck stops (when $v = 0$).



**SKETCH:**




**RESEARCH:** The truck and the gravel incline form an isolated system. Use energy conservation to determine $\Delta x$. The initial energy is purely kinetic, $K = (mv^2)/2$. The final energies are thermal, $\Delta E_{th} = f_k d$ and gravitational potential, $U = mgh$.

**SIMPLIFY:**

$$\Delta E_{tot} = 0$$

$$\Delta K + \Delta U + \Delta E_{th} = 0$$

$$-K_i + U_f + \Delta E_{th} = 0$$

$$-\frac{1}{2}mv_0^2 + mgh + f_k\Delta x = 0$$

$$-\frac{1}{2}mv_0^2 + mg\Delta x\sin\theta + \mu_k N\Delta x = 0$$

Note that $N = mg\cos\theta$ on the incline. This gives:

$$-\frac{1}{2}mv_0^{\ 2} + mg\Delta x\sin\theta + \mu_k mg\cos\theta\Delta x = 0$$

$$\Delta x\left(g\sin\theta + \mu_k g\cos\theta\right) = \frac{1}{2}v_0^{\ 2}$$

$$\Delta x = \frac{v_0^{\ 2}}{2g\left(\sin\theta + \mu_k\cos\theta\right)}$$

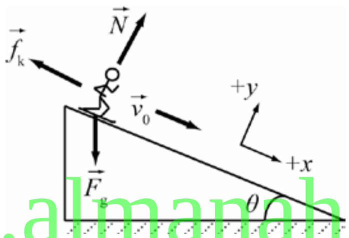
**CALCULATE:** $\Delta x = \dfrac{\left(27.3581 \text{ m/s}\right)^2}{2\left(9.81 \text{ m/s}^2\right)\left(\sin\left(40.15°\right) + 0.634\cos\left(40.15°\right)\right)} = 33.777 \text{ m}$

**ROUND:** With three significant figures in $v_0$, the result should be rounded to $\Delta x = 33.8$ m.

**DOUBLE-CHECK:** This is a reasonable stopping distance given the incline angle and high coefficient of friction.

**6.57.** **THINK:** The snowboarder's mass is $m$ = 70.1 kg. The initial speed is $v_0 = 5.10$ m/s. The slope angle is $\theta = 37.1°$. The coefficient of kinetic friction is $\mu_k = 0.116$. Determine the net work, $W_{net}$ done on the snowboarder in the first $t$ = 5.72 s.

**SKETCH:**






**RESEARCH:** It is known that $W_{net} = \Delta K$. By considering the forces acting on the skier, and assuming constant acceleration, $v_f$ can be determined at $t$ = 5.72 s. Use $f_k = \mu_k N$, $F_{x\text{ net}} = \sum F_x$ and $v = v_0 + at$.

**SIMPLIFY:** In the $x$-direction (along the slope), $F_{x\text{ net}} = F_{gx} - f_k$. Since $N = mg\cos\theta$, the force equation is expanded to

$$ma_{net} = mg\sin\theta - \mu_k mg\cos\theta \Rightarrow a_{net} = g\left(\sin\theta - \mu_k\cos\theta\right).$$

Then, the velocity is given by the formula $v = v_0 + at = v_0 + g\left(\sin\theta - \mu_k\cos\theta\right)t$, and $W_{net} = K_f - K_i = \frac{1}{2}m\left(\left(v_0 + g\left(\sin\theta - \mu_k\cos\theta\right)t\right)^2 - v_0^{\ 2}\right)$.

**CALCULATE:**

$$W_{net} = \frac{1}{2}(70.1 \text{ kg})\left(\left(5.10 \text{ m/s} + \left(9.81 \text{ m/s}^2\right)\left(\sin\left(37.1°\right) - 0.116\cos\left(37.1°\right)\right)\left(5.72 \text{ s}\right)\right)^2 - \left(5.10 \text{ m/s}\right)^2\right)$$

$$= \frac{1}{2}(70.1 \text{ kg})\left(\left(5.10 \text{ m/s} + 28.66 \text{ m/s}\right)^2 - \left(5.10 \text{ m/s}\right)^2\right)$$

$$= \frac{1}{2}(70.1 \text{ kg})\left(1139.5 \text{ m}^2/\text{s}^2 - 26.01 \text{ m}^2/\text{s}^2\right)$$

$$= 39027.5 \text{ J}$$

**ROUND:** Because the $m$ and $v_0$ have three significant figures, the result should be rounded to $W_{net} = 39.0$ kJ.

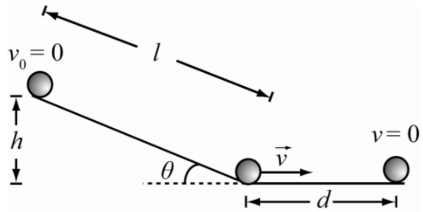
**DOUBLE-CHECK:** This is a reasonable energy required to change the snowboarder's speed.

**6.58.** **THINK:** The ball's mass is $m$ = 0.0459 kg. The length of the bar is $l = 30.0 \text{ in}(0.0254 \text{ m/in}) = 0.762$ m. The incline angle is $\theta = 20.0°$. The distance traveled on the green is

$$d = 11.1 \text{ ft}\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right) = 3.38328 \text{ m}.$$

Determine the coefficient of friction between the green and the ball. Assume the bar is frictionless.

**SKETCH:**




**RESEARCH:** The ball-bar-green system is isolated, so $\Delta E_{\text{tot}} = 0$. Take the initial point to be when the ball starts to roll down the bar, and the final point where the ball has stopped rolling on the green after traveling a distance, $d$, on the green. $K_{\text{i}} = K_{\text{f}} = U_{\text{f}} = 0$. Then, $\Delta K + \Delta U + \Delta E_{\text{th}} = 0$, with $U = mgh$ and $\Delta E_{\text{th}} = f_{\text{k}}d$ can be used to determine $\mu_{\text{k}}$.

**SIMPLIFY:** $\Delta K + \Delta U + \Delta E_{\text{th}} = 0 \Rightarrow -U_{\text{i}} + \Delta E_{\text{th}} = 0 \Rightarrow \Delta E_{\text{th}} = U_{\text{i}} \Rightarrow f_{\text{k}}d = mgh \Rightarrow \mu_{\text{k}}mgd = mgl\sin\theta$

$\Rightarrow \mu_{\text{k}} = \dfrac{l\sin\theta}{d}$

**CALCULATE:** $\mu_{\text{k}} = \dfrac{(0.762 \text{ m})\sin(20.0°)}{3.38328 \text{ m}} = 0.0770316$

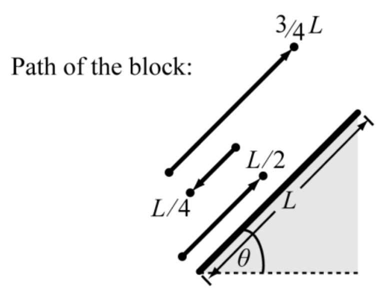
**ROUND:** With three significant figures in each given value, the result should be rounded to $\mu_{\text{k}} = 0.0770$.



**DOUBLE-CHECK:** $\mu_{\text{k}}$ has no units and has a small value, which is reasonable for golf greens.

**6.59.** **THINK:** The block's mass is $m$ = 1.00 kg. The length of the plank is $L$ = 2.00 m. The incline angle is $\theta = 30.0°$. The coefficient of kinetic friction is $\mu_{\text{k}} = 0.300$. The path taken by the block is $L/2$ upward, $L/4$ downward, then up to the top of the plank. Determine the work , $W_b$, done by the block against friction.

**SKETCH:**




**RESEARCH:** Friction is a non-conservative force. The work done by friction, $W_{\text{f}}$, is therefore dependent on the path. It is known that $W_{\text{f}} = -f_{\text{k}}d$, and with $W_{\text{b}} = -W_{\text{f}}$, the equation is $W_{\text{b}} = f_{\text{k}}d$. The total path of the block is $d = L/2 + L/4 + 3L/4 = 1.5L$.

**SIMPLIFY:** $W_{\text{b}} = f_{\text{k}}d = \mu_{\text{k}}Nd = \mu_{\text{k}}mg(\cos\theta)d$ ($N = mg\cos\theta$ on the incline)

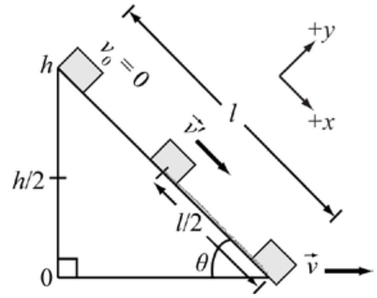
**CALCULATE:** $W_b = (0.300)(1.00 \text{ kg})(9.81 \text{ m/s}^2)\cos(30.0°)(1.50(2.00 \text{ m})) = 7.646$ J

**ROUND:** Each given value has three significant figures, so the result should be rounded to $W_{\text{b}} = 7.65$ J.

**DOUBLE-CHECK:** This is a reasonable amount of work done against friction considering the short distance traveled.

**6.60.** **THINK:** The block's mass is $m$ = 1.00 kg. The initial velocity is $v_0 = 0$ m/s. The incline's length is $l = 4.00$ m. The angle of the incline is $\theta = 45.0°$. The coefficient of friction is $\mu_k = 0.300$ for the lower half of the incline. Determine (a) the block's speed just before the rough section, $v'$, and (b) the block's speed at the bottom, $v$.

**SKETCH:**




**RESEARCH:** Energy is conserved in the block/incline system. Recall $K = (mv^2)/2$, $U$ = $mgh$ and $\Delta E_{th} = f_k d = \mu_k N d$.

(a) With no friction, $\Delta K + \Delta U = 0$.

(b) With friction, $\Delta K + \Delta U + \Delta E_{th} = 0$.

**SIMPLIFY:**

(a) With $v_0 = 0$ m/s and $K_i = 0$, $K_f - K_i + U_f - U_i = 0$ becomes $K_f = U_i - U_f$.

$$\frac{1}{2}mv'^2 = mgh - mg\left(\frac{h}{2}\right) = \frac{mgh}{2} \Rightarrow \frac{1}{2}v'^2 = \frac{1}{2}gl\sin\theta \Rightarrow v' = \sqrt{gl\sin\theta}$$

(b) Consider the initial point to be halfway down $l$ (when the velocity is $v'$), and the final point where $U_f = 0$: $\Delta K + \Delta U + \Delta E_{th} = 0 \Rightarrow K_f - K_i + U_f - U_i + \Delta E_{th} = 0 \Rightarrow K_f = K_i + U_i - \Delta E_{th}$

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + mg\left(\frac{h}{2}\right) - f_k d \quad \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}mgl\sin\theta - \mu_k mg\cos\theta\left(\frac{l}{2}\right)$ since $N = mg\cos\theta$ and $f_k = \mu_k N$. So $v^2 = v'^2 + gl\sin\theta - \mu_k lg\cos\theta$. Since $v'^2 = gl\sin\theta$, $v = \sqrt{gl(2\sin\theta - \mu_k\cos\theta)}$.



**CALCULATE:**

(a) $v' = \sqrt{(9.81 \text{ m/s}^2)(4.00 \text{ m})\sin(45.0°)} = 5.2675$ m/s

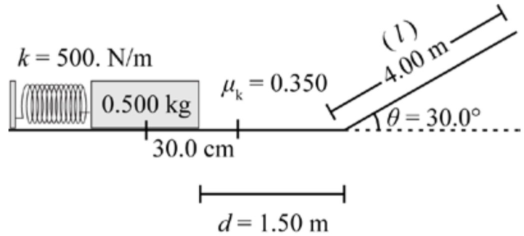
(b) $v = \sqrt{(9.81 \text{ m/s}^2)(4.00 \text{ m})(2\sin(45.0°) - 0.300\cos(45.0°))} = 6.868$ m/s

**ROUND:** With $l$ having three significant figures, the results should be rounded to $v' = 5.27$ m/s and $v = 6.87$ m/s.

**DOUBLE-CHECK:** In the complete absence of friction, the speed at the bottom would be $v = \sqrt{2gh} = 7.45$ m/s. The velocity calculated in part (b) is less than this due to the thermal energy dissipated by friction.

**6.61.** **THINK:** The spring constant is $k$ = 500. N/m. The mass is $m$ = 0.500 kg. The spring compression is $x = 30.0$ cm. The length of the plane is $l = 4.00$ m. The incline angle is $\theta = 30.0°$. The coefficient of kinetic friction is $\mu_k = 0.350$. With the spring compressed, the mass is 1.50 m from the bottom of the inclined plane. Determine (a) the speed of the mass at the bottom of the inclined plane, (b) the speed of the mass at the top of the inclined plane, and (c) the total work done by friction from beginning to end.

**SKETCH:**




**RESEARCH:**

(a) The elastic potential energy is $U_{\text{spring}} = \left(kx^2\right)/2$. The mass loses energy $W_{\text{f}} = -F_{\text{f}}d = -\mu_k mgd$ due to friction. Therefore, the kinetic energy at the bottom is given by $K_{\text{b}} = \frac{1}{2}mv_{\text{b}}^2 = \frac{1}{2}kx^2 - \mu_{\text{k}}mgd.$

(b) To reach the top of the incline, the gravitational potential energy must also be considered: $\Delta U_{\text{gravity}} = U_{\text{top}} - U_{\text{bottom}}$. Since the plane has length, $l$, and incline angle, $\theta$, $\Delta U_{\text{gravity}} = mgl\sin\theta.$ The kinetic energy at the top (and thus the speed) can then be calculated by subtracting the gravitational potential energy and work due to friction from the kinetic energy at the bottom: $K_{\text{top}} = K_{\text{b}} - \mu_{\text{k}}mgl\cos\theta - mgl\sin\theta.$

(c) The total work due to friction is given by $W_{\text{f}} = -F_{\text{f}}\left(d + l\right).$

**SIMPLIFY:**

(a) $K_{\text{b}} = \frac{1}{2}mv_{\text{b}}^2 = \frac{1}{2}kx^2 - \mu_{\text{k}}mgd \Rightarrow v_b = \sqrt{\frac{kx^2}{m} - 2\mu_{\text{k}}gd}$

(b) $K_{\text{top}} = \frac{1}{2}mv_{\text{top}}^2 = K_{\text{b}} - \mu_{\text{k}}mgl\cos\theta - mgl\sin\theta \Rightarrow v_{\text{top}} = \sqrt{\frac{2}{m}\left[K_{\text{b}} - mgl\left(\mu_{\text{k}}\cos\theta + \sin\theta\right)\right]}$



(c) $W_{\text{f}} = -\mu_{\text{k}}mgd - \mu_{\text{k}}mg\left(\cos\theta\right)l$

**CALCULATE:**

(a) $v_{\text{b}} = \sqrt{\frac{\left(500.\text{ N/m}\right)\left(0.300\text{ m}\right)^2}{0.500\text{ kg}} - 2\left(0.350\right)\left(9.81\text{ m/s}^2\right)\left(1.50\text{ m}\right)} = 8.927\text{ m/s}$

$K_{\text{b}} = \frac{1}{2}\left(0.500\text{ kg}\right)\left(8.927\text{ m/s}\right)^2 = 19.92\text{ J}$

(b) $v_{\text{top}} = \sqrt{\frac{2}{0.500\text{ kg}}\left[\left(19.92\text{ J}\right) - \left(0.500\text{ kg}\right)\left(9.81\text{ m/s}^2\right)\left(4.00\text{ m}\right)\left(0.350\cos 30.0^\circ + \sin 30.0^\circ\right)\right]} = 4.08\text{ m/s}$

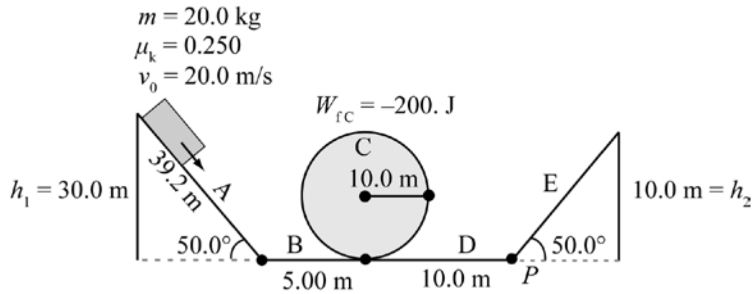
(c) $W_{\text{f}} = -\left(0.350\right)\left(0.500\text{ kg}\right)\left(9.81\text{ m/s}^2\right)\left(1.50\text{ m} + 4.00\cos\left(30.0^\circ\right)\text{ m}\right) = -8.52\text{ J}$

**ROUND:** Rounding to three significant figures, $v_{\text{b}} = 8.93\text{ m/s}$, $v_{\text{top}} = 4.08\text{ m/s}$ and $W_{\text{f}} = -8.52\text{ J}.$

**DOUBLE-CHECK:** The results are reasonable for the given values.

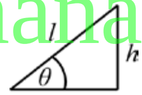
**6.62.** **THINK:** Determine the speed of the sled at the end of the track or the maximum height it reaches if it stops before reaching the end. The initial velocity is $v_0 = 20.0$ m/s.

**SKETCH:**




**RESEARCH:** The total initial energy is given by $E_0 = K_0 - U_0$. When the sled reaches point $p$ at the bottom of the second incline, it has lost energy due to friction given by $W_p = W_A + W_B + W_C + W_D$, where $W_A = A\mu_k mg\cos\theta$, $W_B = B\mu_k mg$, $W_C = 200.$ J, and $W_D = D\mu_k mg$. As the sled reaches point $p$, it has kinetic energy $K_p = E_0 - W_p$. In order for the sled to reach the end of the incline, it needs to have enough energy to cover the work due to friction as well as the gravitational potential energy at the top. Therefore, if $K_p > U_E + W_E$, then it does reach the top and the speed can be determined from the kinetic energy at the top: $K_{top} = K_p - P_E - W_E$. If $K_p < U_E + W_{fE}$, then it stops before reaching the top and the height the sled reaches can be determined by considering the gravitational potential energy equation: $U = mgh = K_p - W_{fE}$, where $W_{fE}$ is the work due to friction for the section of the incline up to $h$. The height can be related to the distance covered on the incline by recalling that $h = l\sin\theta \Rightarrow l = h/\sin\theta$.






Therefore,

$$W_{fE} = \mu_k mg(\cos\theta)l = \mu_k mg\frac{(\cos\theta)h}{\sin\theta} = \mu_k mg(\cot\theta)h.$$

**SIMPLIFY:** It is convenient to evaluate the following terms separately:
$E_0$, $W_A$, $W_B, W_C, W_D, U_E, W_E$ and $W_{fE}$.

$$E_0 = \frac{1}{2}mv_0^2 + mgh_1,\ U_E = mgh_2,\ W_E = E\mu_k mg\cos\theta = h_2\mu_k mg\cot\theta.$$

**CALCULATE:** $E_0 = \frac{1}{2}(20.0 \text{ kg})(20.0 \text{ m/s})^2 + (20.0 \text{ kg})(9.81 \text{ m/s}^2)(30.0 \text{ m}) = 9886 \text{ J}$

$W_A = (39.2 \text{ m})(0.250)(20.0 \text{ kg})(9.81 \text{ m/s}^2)\cos(50.0°) = 1236 \text{ J}$

$W_B = (5.00 \text{ m})(0.250)(20.0 \text{ kg})(9.81 \text{ m/s}^2) = 245 \text{ J},\qquad W_C = 200. \text{ J}$

$W_D = (10.0 \text{ m})(0.250)(20.0 \text{ kg})(9.81 \text{ m/s}^2) = 491 \text{ J},\quad U_E = (20.0 \text{ kg})(9.81 \text{ m/s}^2)(10.0 \text{ m}) = 1962 \text{ J}$

$W_E = (10.0 \text{ m})(0.250)(20.0 \text{ kg})(9.81 \text{ m/s}^2)\cot(50.0°) = 412 \text{ J}$

Therefore, $K_p = (9886 \text{ J}) - (1236 \text{ J}) - (245 \text{ J}) - 200. \text{ J} - (491 \text{ J}) = 7714 \text{ J}$, and
$U_E + W_E = (1962 \text{ J}) + (412 \text{ J}) = 2374 \text{ J}$. Therefore, since $K_p > U_E + W_E$, the sled will reach the top and have speed: $K_{top} = K_p - U_E - W_E \Rightarrow \frac{1}{2}mv_{top}^2 = K_p - U_E - W_E$

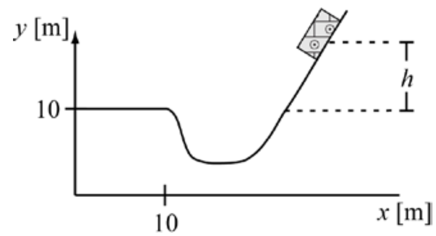
$$\Rightarrow v_{top} = \sqrt{\frac{2}{m}(K_p - U_E - W_E)} = \sqrt{\left(\frac{2}{20.0 \text{ kg}}\right)((7714 \text{ J}) - (2374 \text{ J}))} = 23.11 \text{ m/s}.$$

**ROUND:** Rounding to three significant figures, $v_{top} = 23.1$ m/s.

**DOUBLE-CHECK:** The fact that the sled reaches the top of the second ramp is reasonable given how much higher the second ramp is than the first. The value of the velocity is of the same order of magnitude as the initial velocity so it is reasonable.

**6.63.** **THINK:** The mass of the cart is 237.5 kg. The initial velocity is $v_0 = 16.5$ m/s. The surface is frictionless. Determine the turning point shown on the graph in the question, sketched below.

**SKETCH:**




**RESEARCH:** Since the system is conservative, $E_{tot} = \text{constant} = U_{max} = K_{max}$. Therefore, the kinetic energy at $x = 0$, $y = 10.$ m is the same as the kinetic energy whenever the track is at $y = 10.$ m again. Set $y = 10.$ m as the origin for gravitational potential energy. Therefore,

$$E_{tot} = K_{max} = \frac{mv_0^2}{2}.$$

This is the available energy to climb the track from $y = 10.$ m. The turning point is when $v = 0$ and

$$U_{max} = K_{max} \Rightarrow mgh = \frac{mv_0^2}{2}.$$



**SIMPLIFY:** $h = \dfrac{v_0^2}{2g}$, $y = 10.\text{ m} + h$

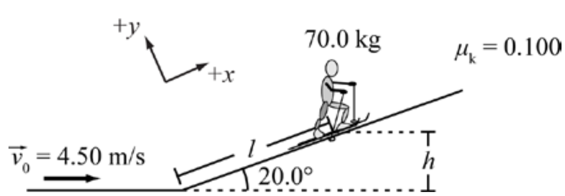
**CALCULATE:** $h = \dfrac{(16.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 13.9\text{ m}$, $y = 10.\text{ m} + 13.9\text{ m} = 23.9\text{ m}$

**ROUND:** Reading off the graph is accurate to the nearest integer, so round the value of $y$ to 24 m. Reading off the graph, the value of $x$ at $y = 24$ m is $x = 42$ m.

**DOUBLE-CHECK:** It is reasonable that the cart will climb about 18 m with an initial velocity of $v_0 = 16.5$ m/s.

**6.64.** **THINK:** A 70.0 kg skier's initial velocity is $v_0 = 4.50$ m/s towards a $20.0°$ incline. Determine (a) the range up the incline if there is no friction and (b) the range up the incline if $\mu_k = 0.100$.

**SKETCH:**




**RESEARCH:**

(a) Since the system is conservative, $E_{tot} = K_{max} = U_{max} \Rightarrow \left(mv_0^2\right)/2 = mgh_1 = mgl_1 \sin\theta$.

(b) The work due to friction is determined by $W_f = F_f l_2 = \mu_k mgl_2 \cos\theta$. Therefore,

$$K_{bottom} = U_{top} - W_f.$$

**SIMPLIFY:**

(a) $\frac{1}{2}mv_0^2 = mgl_1\sin\theta \Rightarrow l_1 = \frac{v_0^2}{2g\sin\theta}$

(b) $\frac{1}{2}mv_0^2 = mgl_2\sin\theta + \mu_k mgl_2\cos\theta \Rightarrow \frac{v_0^2}{2} = l_2 g(\sin\theta + \mu_k\cos\theta) \Rightarrow l_2 = \frac{v_0^2}{2g(\sin\theta + \mu_k\cos\theta)}$

**CALCULATE:**

(a) $l = \frac{(4.50 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)\sin(20.0°)} = 3.0177 \text{ m}$

(b) $l_2 = \frac{(4.50 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)(\sin(20.0°) + 0.100\cos(20.0°))} = 2.3672 \text{ m}$

**ROUND:** The final rounded answer should contain 3 significant figures:

(a) $l = 3.02$ m

(b) $l = 2.37$ m

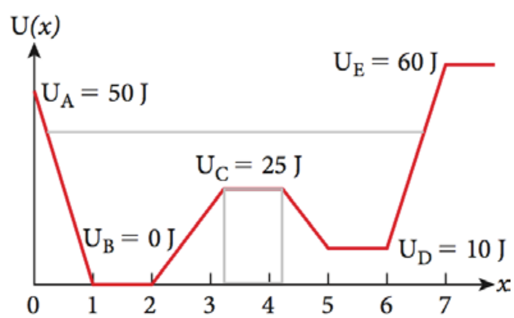
**DOUBLE-CHECK:** As expected, introducing friction into the system will decrease the available mechanical energy.

**6.65.** **THINK:** The particle has a total energy of $E_{tot} = 40.0$ J at its initial position and retains it everywhere. Thus we can draw a horizontal line (gray) for its total energy, approximately at 4/5 of the value of the potential energy at point A (= 50.0 J) for this value of the total energy. The locations of the turning points are here this horizontal line intersects the potential energy curve (red).

Further, we can determine the shape of the potential curve in a more analytical form. From the drawing we can clearly see that it is piecewise linear, falling from 50.0 J at $x = 0$ to 0 J at $x = 1$, rising from 0 J at $x = 2$ to 25.0 J at $x = 3.25$, falling again from 25.0 J at $x = 4.25$ to 10.0 J at $x = 5$, and finally rising from 10.0 J at $x = 6$ to 60.0 J at $x = 7$. (We have drawn in a gray rectangle; this way it is easier to see at what $x$-values the slopes change.)

The turning points are where $v = 0$, which is where the total energy is equal to the potential energy.

**SKETCH:**






**RESEARCH:** Assume a conservative system and $E_{tot} = K + U$.

(a) Consider the potential energy at the point $x = 3$ m and call it $U_3$:

$$E_{tot} = K + U \Rightarrow K_3 = E_{tot} - U_3, \text{ and } K_3 = \frac{mv_3^2}{2}.$$

(b) Similarly, $K_{4.5} = E_{tot} - U_{4.5}$, and $K_{4.5} = (mv_{4.5}^2)/2$.

(c) Since $E_{tot} = 40.0$ J at $x = 4.00$ and $U_C = 25.0$ J, then $E_{tot} - U_C = K_C$. This kinetic energy will become potential energy to reach the turning point.

**SIMPLIFY:**

(a) $\frac{1}{2}mv_3^2 = E_{tot} - U_3 \Rightarrow v_3 = \sqrt{\frac{2}{m}(E_{tot} - U_3)}$. $U_3$ is obtained from the graph.

(b) $v_{4.5} = \sqrt{\frac{2}{m}\left(E_{\text{tot}} - U_{4.5}\right)}$. $U_{4.5}$ is obtained from the graph.

(c) $E_{\text{tot}} - U_{\text{C}} = K_{\text{C}} = U_4$. Therefore, $U_{\text{turning}} = U_{\text{C}} + U_{\text{t}} = E_{\text{tot}}$.

**CALCULATE:**

(a) Interpolation between $x = 2$ and $x = 3.25$ yields

$$U(x) = U_{\text{C}}(x-2)/(3.25-2) \Rightarrow U_3 \equiv U(x{=}3) = (25.0 \text{ J})(3-2)/(3.25-2) = 20.0 \text{ J}$$

$$v_3 = \sqrt{\left(\frac{2}{0.200 \text{ kg}}\right)\left(40.0 \text{ J} - 20.0 \text{ J}\right)} = 14.14 \text{ m/s}$$

(b) Interpolation between $x = 4.25$ and $x = 5$ yields

$U(x) = U_{\text{C}} - (U_{\text{C}} - U_{\text{D}})(x-4.25)/(5-4.25) \Rightarrow U_{4.5} \equiv U(x{=}4.5) = (25.0 \text{ J}) - (15.0 \text{ J})(4.5-4.25)/(0.75) = 20.0 \text{ J}$

$v_{4.5} = \sqrt{\left(\frac{2}{0.200 \text{ kg}}\right)\left(40.0 \text{ J} - 20.0 \text{ J}\right)} = 14.14 \text{ m/s}$

(c) Graphical interpolation between 0 and 1 and between 6 and 7 then results in turning points results in $x_L = 1 - E/U_A = 1 - (40.0 \text{ J})/(50.0 \text{ J}) = 0.2$ for the left turning point, and $x_R = 6 + (E - U_D)/(U_E - U_D) = 6 + (30.0 \text{ J})/(50.0 \text{ J}) = 6.6$ for the right one.

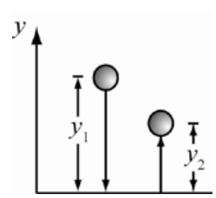
**ROUND:** Since we are reading data of a graph, we should probably round to two figures and state our results as $v_3 = v_{4.5} = 14$ m/s and $x_L = 0.2$ m and $x_R = 6.6$ m.

**DOUBLE-CHECK:** Our numerical findings for the turning points agree with our graphical estimation, within the uncertainties stated here.

**6.66.** **THINK:** The mass of the ball is $m = 1.84$ kg. The initial height is $y_1 = 1.49$ m and the second height is $y_2 = 0.87$ m. Determine the energy lost in the bounce.



**SKETCH:**




**RESEARCH:** Consider the changes in the potential energy from $y_1$ to $y_2$. The energy lost in the bounce is given by $U_1 - U_2$.

**SIMPLIFY:** $E_{\text{lost}} = mgy_1 - mgy_2 = mg\left(y_1 - y_2\right)$

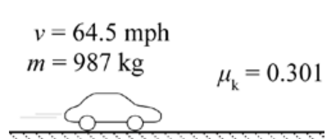
**CALCULATE:** $E_{\text{lost}} = \left(1.84 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)\left(1.49 \text{ m} - 0.87 \text{ m}\right) = 11.2 \text{ J}$

**ROUND:** Since the least precise value is given to two significant figures, the result is $E_{\text{lost}} = 11$ J.

**DOUBLE-CHECK:** The ball lost roughly half of its height, so it makes sense that it lost roughly half of its potential energy (which was about 27 J).

**6.67.** **THINK:** The mass of the car is $m = 987$ kg. The speed is $v = 64.5$ mph. The coefficient of kinetic friction is $\mu_{\text{k}} = 0.301$. Determine the mechanical energy lost.

**SKETCH:**



**RESEARCH:** Since all of the mechanical energy is considered in the form of kinetic energy, the energy lost is equal to the kinetic energy before applying the brakes. Using the conversion 1 mph is equal to 0.447 m/s, the speed can be converted to SI units. Convert the speed: $v=(64.5 \text{ mph})\left(\frac{0.447 \text{ m/s}}{1 \text{ mph}}\right)=28.8 \text{ m/s}.$

**SIMPLIFY:** $E_{\text{lost}}=\frac{1}{2}mv^2$

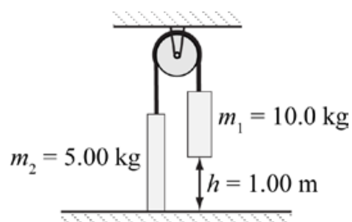
**CALCULATE:** $E_{\text{lost}}=\frac{1}{2}(987 \text{ kg})(28.8 \text{ m/s})^2=4.10\cdot 10^5 \text{ J}$

**ROUND:** Rounding to three significant figures, $E_{\text{lost}}=4.10\cdot 10^5 \text{ J}.$

**DOUBLE-CHECK:** For an object this massive, it is reasonable that it requires such a large amount of energy to stop it.

**6.68.** **THINK:** Two masses, $m_1=10.0 \text{ kg}$ and $m_2=5.00 \text{ kg}$ are attached to a frictionless pulley. The first mass drops $h$ = 1.00 m. Determine (a) the speed of the 5.00 kg mass before the 10.0 kg mass hits the ground and (b) the maximum height of the 5.00 kg mass.

**SKETCH:**




**RESEARCH:**



(a) Since energy is conserved, $\Delta K=-\Delta U$. Since the masses are attached to each other, their speeds are the same before one touches the ground.

(b) When $m_1$ hits the ground, $m_2$ is at $h$ = 1.00 m with a speed $v$. The kinetic energy for $m_2$ is then $(m_2v^2)/2$ and this is given to potential energy for a height above $h$ = 1.00 m. Let $h_t$ be the height where the potential and kinetic energies are equal. When the kinetic energies are equal, $U=K \Rightarrow m_2gh_t=(m_2v^2)/2 \Rightarrow h_t=v^2/2g$. Therefore, the maximum height is $h_{\text{max}}=h+h_t$.

**SIMPLIFY:**

(a)

$$K_{\text{f}}-K_{\text{i}}=U_{\text{i}}-U_{\text{f}}$$

$$\left(\frac{1}{2}m_1v^2+\frac{1}{2}m_2v^2\right)-0=(m_1gh+m_2gh)-0$$

$$(m_1+m_2)v^2=2gh(m_1-m_2)$$

$$v^2=2gh\left(\frac{m_1-m_2}{m_1+m_2}\right)$$

$$v=\pm\sqrt{2gh\left(\frac{m_1-m_2}{m_1+m_2}\right)}$$

(b) $h_{\text{max}}=h+\frac{v^2}{2g}$

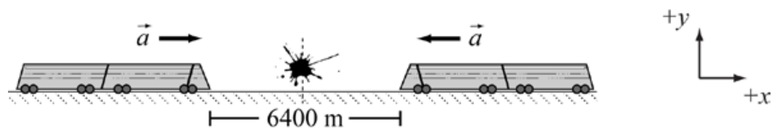
**CALCULATE:**

(a) $v=\sqrt{2(9.81 \text{ m/s}^2)(1.00 \text{ m})\left(\frac{10.0 \text{ kg}-5.00 \text{ kg}}{10.0 \text{ kg}+5.00 \text{ kg}}\right)}=2.557 \text{ m/s}$

(b) $h_{\text{max}} = 1.00 \text{ m} + \dfrac{(2.557 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 1.333 \text{ m}$

**ROUND:** Rounding to three significant figures, $v$ = 2.56 m/s and $h_{\text{max}} = 1.33$ m.
**DOUBLE-CHECK:** The calculated values have appropriate units and are of reasonable orders of magnitude for a system of this size.

**6.69.** **THINK:** The distance that each train covered is $\Delta x = 3200$ m. The weight of each train is $w = 1.2 \cdot 10^6$ N. Their accelerations have a magnitude of $a = 0.26 \text{ m/s}^2$, but are in opposite directions. Determine the total kinetic energy of the two trains just before the collision. The trains start from rest.
**SKETCH:**




**RESEARCH:** The total kinetic energy will be twice the kinetic energy for one train. With $K = (mv^2)/2$, $m$ can be determined from $w = mg$ and $v$ from $v^2 = v_0^2 + 2a\Delta x$.

**SIMPLIFY:** $m = w/g$. Then, $K_{\text{tot}} = 2K = 2\left((mv^2)/2\right) = w(2a\Delta x)/g$.

**CALCULATE:** $K_{\text{tot}} = \dfrac{2(1.2 \cdot 10^6 \text{ N})(0.26 \text{ m/s}^2)(3200 \text{ m})}{(9.81 \text{ m/s}^2)} = 2.035 \cdot 10^8 \text{ J}$

**ROUND:** With two significant figures in each given value, $K_{\text{tot}} = 2.0 \cdot 10^8$ J.
**DOUBLE-CHECK:** For such a horrific explosion, a very large kinetic energy is expected before impact.

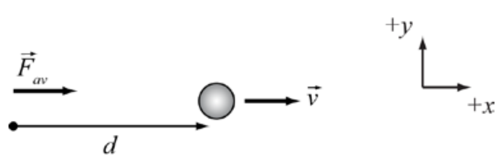


**6.70.** **THINK:** The ball's mass is $m = 5.00 \text{ oz}(0.02835 \text{ kg/oz}) = 0.14175$ kg. The final speed is

$$v = 90.0 \frac{\text{miles}}{\text{h}}\left(\frac{1609.3 \text{ m}}{1 \text{ mile}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 40.2325 \text{ m/s}.$$

The distance traveled is $d = 2(28.0 \text{ in})\left(\dfrac{0.0254 \text{ m}}{1 \text{ in}}\right) = 1.4224$ m. Determine the average force, $F_{\text{av}}$.

**SKETCH:**




**RESEARCH:** There are no non-conservative forces in the system. So, $\Delta K = -\Delta U$. With $F_{\text{av}}$ as a conservative force, the work it does is given by $W_{\text{c}} = -\Delta U$ and $W_{\text{c}} = \vec{F} \bullet \vec{d}$. From this, $F_{\text{av}}$ can be determined.

**SIMPLIFY:** Note $\vec{F}_{\text{av}}$ and $\vec{d}$ are in the same direction, so $W_{\text{c}} = F_{\text{av}} d$ and
$\Delta K = -\Delta U = W_{\text{c}} \Rightarrow K_{\text{f}} - K_{\text{i}} = W_{\text{c}}$. Since $K_{\text{i}} = 0$, $K_{\text{f}} = W_{\text{c}}$.

$$\frac{1}{2}mv^2 = F_{\text{av}} d \Rightarrow F_{\text{av}} = \frac{mv^2}{2d}$$

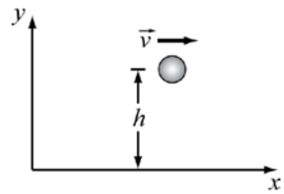
**CALCULATE:** $F_{\text{av}} = \dfrac{(0.14175 \text{ kg})(40.2325 \text{ m/s})^2}{2(1.4224 \text{ m})} = 80.654 \text{ N}$

**ROUND:** Since the values are given to three significant figures, $F_{\text{av}} = 80.7$ N.

**DOUBLE-CHECK:** This average force is equal to holding an object that has a mass of 14.8 kg ($m = F/g = (145\text{ N})/\left(9.81\text{ m/s}^2\right)$), so it is reasonable.

**6.71.** **THINK:** The mass of the ball is $m$ = 1.50 kg. Its speed is $v$ = 20.0 m/s and its height is $h$ = 15.0 m. Determine the ball's total energy, $E_{\text{tot}}$.

**SKETCH:**




**RESEARCH:** Total energy is the sum of the mechanical energy and other forms of energy. As there are no non-conservative forces (neglecting air resistance), the total energy is the total mechanical energy. $E_{\text{tot}} = K + U.$ Use $K = \left(mv^2\right)/2$ and $U = mgh.$

**SIMPLIFY:** $E_{\text{tot}} = m\left(\frac{1}{2}v^2 + gh\right)$

**CALCULATE:** $E_{\text{tot}} = 1.50\text{ kg}\left(\frac{1}{2}(20.0\text{ m/s})^2 + \left(9.81\text{ m/s}^2\right)(15.0\text{ m})\right)$

$$= 1.50\text{ kg}\left(200\text{ m}^2/\text{s}^2 + 147.15\text{ m}^2/\text{s}^2\right)$$

$$= 520.725\text{ J}$$

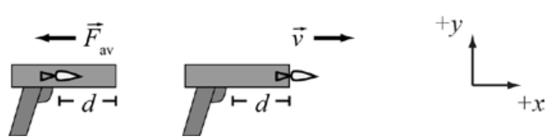
**ROUND:** As the speed has three significant figures, the result should be rounded to $E_{\text{tot}} = 521$ J.

**DOUBLE-CHECK:** The energy is positive and has the correct unit of measurement. It is also on the right order of magnitude for the given values. This is a reasonable energy for a ball.



**6.72.** **THINK:** The average force used to load the dart gun is $F_{\text{av}} = 5.5$ N. The dart's mass is $m = 4.5\cdot 10^{-3}$ kg and the distance the dart is inserted into the gun is $d$ = 0.060 m. Determine the speed of the dart, $v$, as it exits the gun.

**SKETCH:**




**RESEARCH:** Assuming the barrel is frictionless, and neglecting air resistance, the conservation of mechanical energy can be used to determine $v$. Use $\Delta K = -\Delta U$, $K = \left(mv^2\right)/2$ and $W_{\text{c}} = -\Delta U = \vec{F}\bullet\vec{d}$ ($W_c$ is work done by a conservative force).

**SIMPLIFY:** Note $\vec{F}$ and $\vec{d}$ are in the same direction so the equation can be reduced to $-\Delta U = W_{\text{c}} = F_{\text{av}}d.$

$$\Delta K = -\Delta U \Rightarrow K_f - K_i = F_{\text{av}}d \;\Rightarrow\; \frac{1}{2}mv^2 = F_{\text{av}}d \;\left(\text{as } v_0 = 0\right) \;\Rightarrow\; v = \sqrt{2F_{\text{av}}d/m}$$

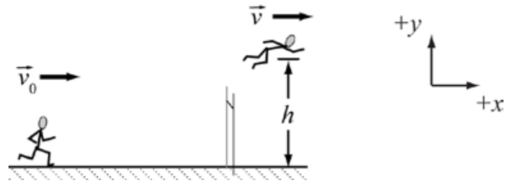
**CALCULATE:** $v = \sqrt{2(5.5\text{ N})(0.060\text{ m})/\left(4.5\cdot 10^{-3}\text{ kg}\right)} = 12.111\text{ m/s}$

**ROUND:** All given values have two significant figures, so the result should be rounded to $v$ = 12 m/s.

**DOUBLE-CHECK:** This is a reasonable velocity for a dart to exit a dart gun.

**6.73.** **THINK:** The jumper's initial speed is $v_0 = 9.00$ m/s and his final speed as he goes over the bar is $v = 7.00$ m/s. Determine his highest altitude, $h$.

**SKETCH:**




**RESEARCH:** As there are no non-conservative forces in the system, the conservation of mechanical energy can be used to solve for $h$ as follows, $\Delta K = -\Delta U$.

**SIMPLIFY:** $K_f - K_i = U_i - U_f \Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = -mgh$

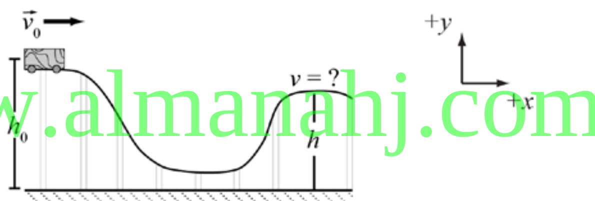
**CALCULATE:** $h = \frac{(9.00 \text{ m/s})^2 - (7.00 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 1.63099$ m

**ROUND:** There are three significant figures in $v_0$ and $v$, so the result should be rounded to $h = 1.63$ m.

**DOUBLE-CHECK:** Since $v < v_0$, it is necessary that $h > h_0$ to conserve mechanical energy.

**6.74.** **THINK:** The initial speed of the roller coaster is $v_0 = 2.00$ m/s and its initial height is $h_0 = 40.0$ m. Determine the speed, $v$ at the top of the second peak at a height of $h = 15.0$ m.

**SKETCH:**





**RESEARCH:** As there are no non-conservative forces in this system, to solve for $v$, the conservation of mechanical energy can be used: $\Delta K = -\Delta U$, where $K = (mv^2)/2$ and $U = mgh$.

**SIMPLIFY:**

$$\Delta K = -\Delta U$$

$$K_f - K_i = U_i - U_f$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mgh_0 - mgh$$

$$v = \sqrt{2g(h_0 - h) + v_0^2}$$

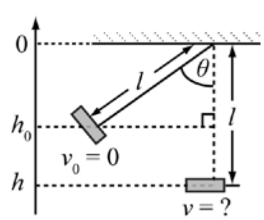
**CALCULATE:** $v = \sqrt{2(9.81 \text{ m/s}^2)(40.0 \text{ m} - 15.0 \text{ m}) + (2.00 \text{ m/s})^2} = 22.24$ m/s

**ROUND:** As $h_0$ has three significant figures, the result should be rounded to $v = 22.2$ m/s.

**DOUBLE-CHECK:** The speed on the lower hill must be greater than the speed on the higher hill.

**6.75.** **THINK:** The length of the chain is $l = 4.00$ m and the maximum displacement angle is $\theta = 35°$. Determine the speed of the swing, $v$, at the bottom of the arc.

**SKETCH:**



**RESEARCH:** At the maximum displacement angle, the speed of the swing is zero. Assuming there are no non-conservative forces, to determine the speed, $v$, the conservation of mechanical energy can be used: $\Delta K = -\Delta U$. Use $K = \left(mv^2\right)/2$ and $U = mgh$. The initial height can be determined using trigonometry. Take the top of the swing to be $h = 0$.

**SIMPLIFY:** $v_0 = 0$ and $K_i = 0$. From the sketch, $h_0 = -l\cos\theta$ and $h = -l$. Then,

$$K_f = U_i - U_f \Rightarrow \frac{1}{2}mv^2 = mg\left(-l\cos\theta\right) - mg\left(-l\right) \Rightarrow \frac{1}{2}v^2 = g\left(l - l\cos\theta\right) \Rightarrow v = \sqrt{2g\left(l - l\cos\theta\right)}$$

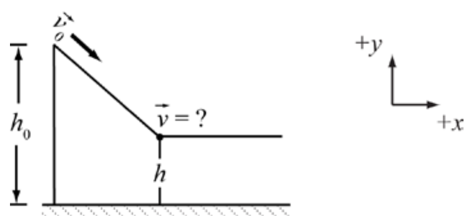
**CALCULATE:** $v = \sqrt{2\left(9.81\text{ m/s}^2\right)\left(4.00\text{ m} - \left(4.00\text{ m}\right)\cos 35.0°\right)} = 3.767\text{ m/s}$

**ROUND:** $l$ and $\theta$ have two significant figures, so the result should be rounded to $v = 3.77$ m/s.

**DOUBLE-CHECK:** This is a reasonable speed for a swing to achieve when initially displaced from the vertical by $35°$.

**6.76.** **THINK:** The initial height of the truck is $h_0 = 680$ m. The initial speed is $v_0 = 15$ m/s and the final height is $h = 550$ m. Determine the maximum final speed, $v$.

**SKETCH:**




**RESEARCH:** The maximum final speed, $v$, can be determined by neglecting non-conservative forces and using the conservation of mechanical energy, $\Delta K = -\Delta U$. Use $K = \left(mv^2\right)/2$ and $U = mgh$.



**SIMPLIFY:** $K_f - K_i = U_i - U_f \;\Rightarrow\; \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mgh_0 - mgh \;\Rightarrow\; v = \sqrt{2g\left(h_0 - h\right) + v_0^2}$

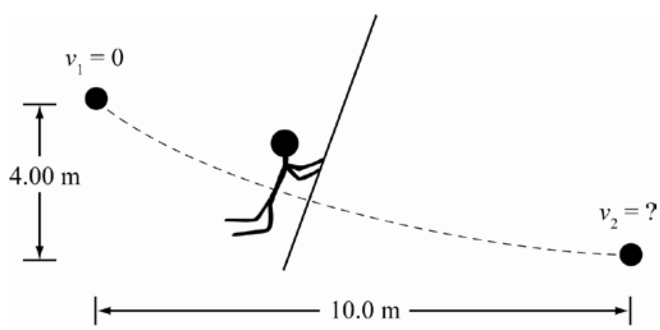
**CALCULATE:** $v = \sqrt{2\left(9.81\text{ m/s}^2\right)\left(680\text{ m} - 550\text{ m}\right) + \left(15.0\text{ m/s}\right)^2} = 52.68\text{ m/s}$

**ROUND:** Each initial value has two significant figures, so the result should be rounded to $v = 53$ m/s.

**DOUBLE-CHECK:** Since the truck is going downhill, its final speed must be greater than its initial speed in the absence of non-conservative forces.

**6.77.** **THINK:** Determine Tarzan's speed when he reaches a limb on a tree. He starts with a speed of $v_0 = 0$ and reaches a limb on a tree which is 10.0 m away and 4.00 m below his starting point. Consider the change in potential energy as he moves to the final point and relate this to the change in kinetic energy. The velocity can be determined from the kinetic energy.

**SKETCH:**




**RESEARCH:** Gravitational potential energy is given by $U = mgh$. The change in potential energy is given by $\Delta U = mgh_2 - mgh_1$. Kinetic energy is given by $K = (mv^2)/2$. The change in kinetic energy is given by $\Delta K = (mv_2^2)/2 - (mv_1^2)/2$.

**SIMPLIFY:** Assume the system is conservative. The change in potential energy must be equal to the negative of the change in kinetic energy:

$$\Delta U = -\Delta K$$
$$mgh_2 - mgh_1 = -\left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right)$$
$$g(h_1 - h_2) = \frac{1}{2}(v_2^2 - v_1^2)$$
$$(v_2^2 - v_1^2) = 2g(h_1 - h_2)$$
$$v_2 = \sqrt{2g(h_1 - h_2) + v_1^2}.$$

**CALCULATE:** $v_2 = \sqrt{2(9.81 \text{ m/s}^2)(4.00 \text{ m})} = 8.86 \text{ m/s}$

**ROUND:** Since the values are given to three significant figures, the result remains $v_2 = 8.86$ m/s.

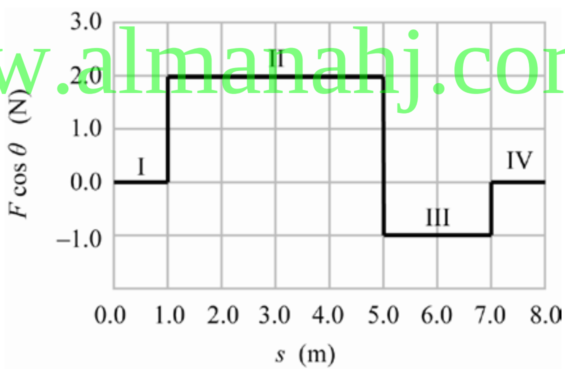
**DOUBLE-CHECK:** This speed is reasonable for swinging on a vine.

**6.78. THINK:**

(a) Determine the net work done on the block given a varying applied force, $F\cos\theta$. The mass of the block is 2.0 kg.

(b) Given an initial speed of zero at $s = 0$, determine the final speed at the end of the trajectory.

**SKETCH:**






**RESEARCH:**

(a) The net work is given by $W_{\text{net}} = \sum_i W_i$ and $W_i = F_i d_i$.

(b) By the work-energy theorem, $W_{\text{net}} = \Delta K$, where $\Delta K = (mv_2^2)/2 - (mv_1^2)/2$.

**SIMPLIFY:**

(a) $W_{\text{net}} = F_{\text{I}}d_{\text{I}} + F_{\text{II}}d_{\text{II}} + F_{\text{III}}d_{\text{III}} + F_{\text{IV}}d_{\text{IV}}$.

(b) $v_2 = \sqrt{\frac{2}{m}\left(W_{\text{net}} + \frac{1}{2}mv_1^2\right)}$

**CALCULATE:**

(a) $W_{\text{net}} = (0.0 \text{ N})(1.0 \text{ m}) + (2.0 \text{ N})(4.0 \text{ m}) + (-1.0 \text{ N})(2.0 \text{ m}) + (0.0 \text{ N})(1.0 \text{ m})$

$$= 8.0 \text{ N m} - 2.0 \text{ N m}$$
$$= 6.0 \text{ N m}$$

(b) $v_2 = \sqrt{\left(\frac{2}{2.0 \text{ kg}}\right)\left(6.0 \text{ N m} + \frac{1}{2}(2.0 \text{ kg})(0.0 \text{ m/s})^2\right)}$

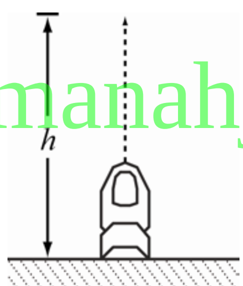
$$= \sqrt{6.0 \frac{\text{N m}}{\text{kg}}}$$
$$= \sqrt{6.0 \frac{\text{m}^2}{\text{s}^2}}$$
$$= 2.4 \text{ m/s}$$

**ROUND:** Since all values are given to two significant figures, the results remain $W_{\text{net}} = 6.0 \text{ N m}$ and $v_2 = 2.4 \text{ m/s}$.

**DOUBLE-CHECK:** An increase of speed of 2.4 m/s after doing $6.0 \text{ N} \cdot \text{m}$ of work is reasonable.

**6.79.** **THINK:** A rocket that has a mass of $m = 3.00$ kg reaches a height of $1.00 \cdot 10^2$ m in the presence of air resistance which takes $8.00 \cdot 10^2$ J of energy away from the rocket, so $W_{\text{air}} = -8.00 \cdot 10^2$ J. Determine the height the rocket would reach if air resistance could be neglected.

**SKETCH:**






**RESEARCH:** Air resistance performs $-8.00 \cdot 10^2$ J of work on the rocket. The absence of air resistance would then provide an extra $8.00 \cdot 10^2$ J of energy to the system. If this energy is converted into potential energy, the increase in height of the rocket can be determined.

**SIMPLIFY:** $U_{\text{t}} = -W_{\text{air}} \Rightarrow mgh_{\text{t}} = -W_{\text{air}} \Rightarrow h_{\text{t}} = \frac{-W_{\text{air}}}{mg}$, where $h_{\text{t}}$ is the added height.

**CALCULATE:** $h_{\text{t}} = \frac{-(-8.00 \cdot 10^2 \text{ J})}{(3.00 \text{ kg})(9.81 \text{ m/s}^2)} = 27.183 \text{ J/kg} \cdot \text{m/s}^2 = 27.183 \text{ m}$

Therefore, the total height reached by the rocket in the absence of air resistance is

$$h_{\text{tot}} = h_0 + h_{\text{t}} = 1.00 \cdot 10^2 \text{ m} + 0.27183 \cdot 10^2 \text{ m} = 1.27183 \cdot 10^2 \text{ m}.$$

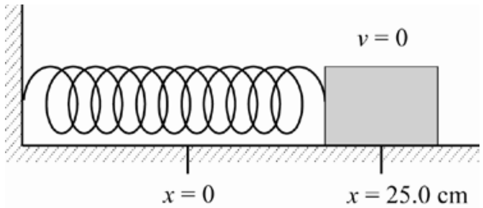
**ROUND:** Since the values are given to three significant figures, the result should be rounded to $h_{\text{tot}} = 1.27 \cdot 10^2$ m.

**DOUBLE-CHECK:** It is reasonable that air resistance will decrease the total height by approximately a fifth.

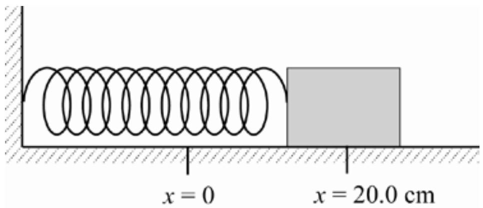
**6.80.** **THINK:** The mass-spring system is frictionless. The spring constant is $k$ = 100. N/m and the mass is 0.500 kg. For a stretch of 25.0 cm, determine (a) the total mechanical energy of the system, (b) the speed of the mass after it has moved 5.0 cm (at $x$ = 20.0 cm) and (c) the maximum speed of the mass.

**SKETCH:**

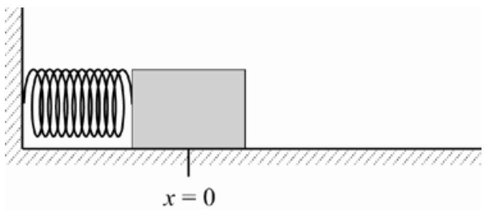
(a)




(b)




(c)




**RESEARCH:**

(a) The total mechanical energy of the system is given by $E_{tot} = K + U$. For a conservative system, it is known that $E_{tot} = \text{constant} = K_{max} = U_{max}$. The maximum potential energy can be calculated so the total mechanical energy can be determined:



$$E_{tot} = U_{max} = \frac{1}{2}kx_{max}^2.$$

(b) The speed at any point can be determined by considering the difference in potential energy and relating this to the kinetic energy. Kinetic energy at $x$ is given by

$$K(x) = -\Delta U = \frac{kx_{max}^2}{2} - \frac{kx^2}{2}, \text{ and } K = \frac{mv^2}{2}.$$

(c) Speed, and therefore kinetic energy, is at its maximum when potential energy is zero, i.e., at the equilibrium position $x = 0$. Since $K_{max} = U_{max}$, $\left(mv_{max}^2\right)/2 = \left(kx_{max}^2\right)/2$.

**SIMPLIFY:**

(a) $E_{tot} = \frac{1}{2}kx_{max}^2$

(b) $K(x) = \frac{1}{2}mv_x^2 = \frac{1}{2}kx_{max}^2 - \frac{1}{2}kx^2 \Rightarrow v_x = \sqrt{\frac{k}{m}\left(x_{max}^2 - x^2\right)}$

(c) $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kx_{max}^2 \Rightarrow v_{max} = \sqrt{\frac{k}{m}x_{max}^2} = x_{max}\sqrt{\frac{k}{m}}$

**CALCULATE:**

(a) $E_{tot} = \frac{1}{2}(100.\text{ N/m})\left(2.50 \cdot 10^{-1}\text{ m}\right)^2 = 3.125\text{ J}$

(b) $v_x = \sqrt{\left(\frac{100.\text{ N/m}}{0.500\text{ kg}}\right)\left[\left(2.50\cdot 10^{-1}\text{ m}\right)^2 - \left(2.00\cdot 10^{-1}\text{ m}\right)^2\right]} = \sqrt{4.5\text{ m}^2/\text{s}^2} = 2.1213\text{ m/s}$

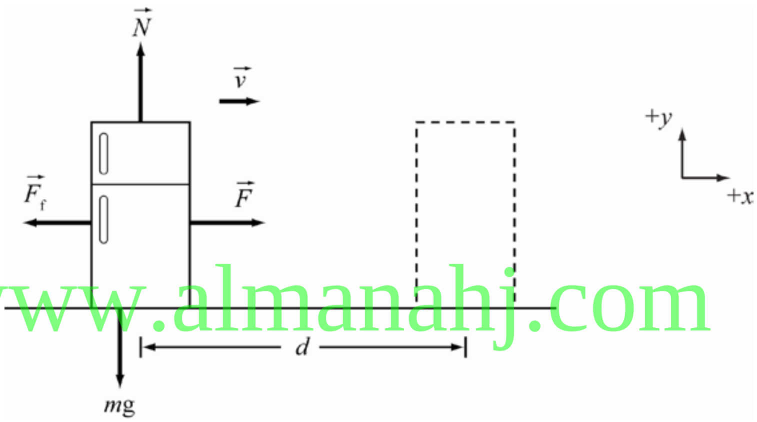
(c) $v_{\text{max}} = \left(2.50\cdot 10^{-1}\text{ m}\right)\sqrt{\frac{(100.\text{ N/m})}{0.500\text{ kg}}} = 3.5355\text{ m/s}$

**ROUND:** The results should be rounded to three significant figures: $E_{\text{tot}} = 3.13\text{ J}$, $v_x = 2.12\text{ m/s}$ and $v_{\text{max}} = 3.54\text{ m/s}$.

**DOUBLE-CHECK:** A total mechanical energy of 3 J is reasonable for this system, based on the given values. A speed anywhere other than at $x = 0$ must be less than at $x = 0$. In this case, $v_x$ must be less than $v_{\text{max}}$. At $x = 0$ the potential energy is zero. Therefore, all of the energy is kinetic energy, so the velocity is maximized. This value is greater than the value found in part (b), as expected.

**6.81.** **THINK:** The mass of a refrigerator is $m$ = 81.3 kg. The displacement is $d$ = 6.35 m. The coefficient of kinetic friction is $\mu_{\text{k}} = 0.437$.

**SKETCH:**






**RESEARCH:** The force of friction is given by $F_{\text{f}} = \mu_{\text{k}}N$. Use Newton's second law and $W = Fd$. This net mechanical work is the work done by you. The net mechanical work done by the roommate is zero, since he/she lifts the refrigerator up and then puts it back down. Therefore, $\Delta E = 0$.

**SIMPLIFY:** $\sum F_y = 0 \Rightarrow N - mg = 0 \Rightarrow N = mg$, $\sum F_x = 0 \Rightarrow F - F_{\text{f}} = 0 \Rightarrow F = \mu_{\text{k}}N \Rightarrow F = \mu_{\text{k}}mg$

The work done is given by $W = Fd = \mu_{\text{k}}mgd$.

**CALCULATE:** $W = (0.437)(81.3\text{ kg})(9.81\text{ m/s}^2)(6.35\text{ m}) = 2213.17\text{ J}$

**ROUND:** Rounding to three significant figures, $W = 2.21\text{ kJ}$.

**DOUBLE-CHECK:** Joules are a usual unit for work. One kilogram is equivalent to about 10 Newtons on Earth, and the fridge weighs about 100 kilograms. The fridge is being moved about 5 meters with a coefficient of friction around a half, so the work should be roughly $0.5\cdot 100\cdot 10\cdot 5 = 2500$ J. The calculated value is reasonable close to this approximation, so the calculated value is reasonable.

**6.82.** **THINK:** A 1.00 kg block is moving between two springs with constants $k_1 = 100.\text{ N/m}$ and $k_2 = 50.0\text{ N/m}$. If the block is compressed against spring 1 by 20.0 cm, determine

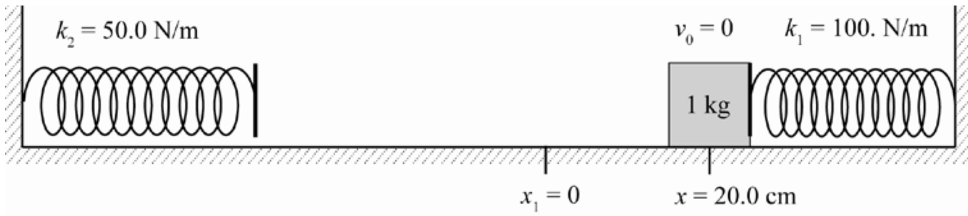
(a) the total energy in the system,

(b) the speed of the block as it moves from one spring to the other and

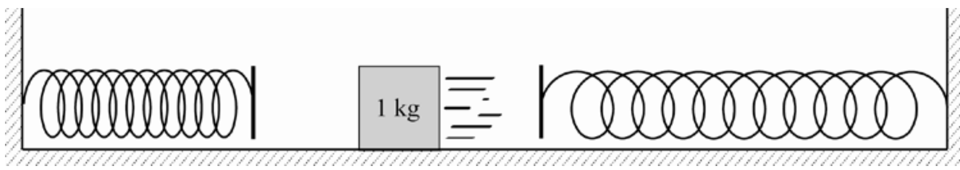
(c) the maximum compression on spring 2.

**SKETCH:**

(a)




(b)




(c)


**RESEARCH:**



(a) The total mechanical energy can be determined by recalling that in a conservative system $E_{tot} = \text{constant} = U_{max} = K_{max}$. $U_{max}$ can be determined from spring 1: $U_{max} = \frac{1}{2}k_1 x_{max}^2 = E_{tot}$.

(b) $K_{max} = U_{max} \Rightarrow \left(mv_{max}^2\right)/2 = \left(k_1 v_{max,1}^2\right)/2$. Since the system is conservative, the speed of the block is $v_{max}$ anytime it is not touching a spring.

(c) The compression on spring 2 can be determined by the following relation:

$$U_{max,2} = K_{max} \Rightarrow \frac{1}{2}k_2 v_{max,\,2}^2 = K_{max}.$$

**SIMPLIFY:**

(a) $E_{tot} = \frac{1}{2}k_1 x_{max,1}^2$

(b) $v_{max} = \sqrt{\frac{k_1}{m}x_{max,1}^2} = x_{max,1}\sqrt{\frac{k_1}{m}}$

(c) $x_{max,2} = \sqrt{\frac{2K_{max}}{k_2}}$

**CALCULATE:**

(a) $E_{tot} = \frac{1}{2}(100.\text{ N/m})\left(20.0 \cdot 10^{-2}\text{ m}\right)^2 = 2.00\text{ J}$

(b) $v_{max} = \left(20.0 \cdot 10^{-2}\text{ m}\right)\sqrt{\frac{(100.\text{ N/m})}{1.00\text{ kg}}} = 2.00\text{ m/s}$

(c) $x_{max,2} = \sqrt{\frac{2(2.00\text{ J})}{50.0\text{ N/m}}} = 2.83 \cdot 10^{-1}\text{ m} = 28.3\text{ cm}$

**ROUND:** Since the least number of significant figures in the given values is three, so the results should be rounded to $E_{\text{tot}} = 2.00$ J, $v_{\text{max}} = 2.00$ m/s and $x_{\text{max},2} = 28.3$ cm.

**DOUBLE-CHECK:** It can be seen that $U_{\text{max},1} = U_{\text{max},2} = K_{\text{max}}$

$$U_{\text{max},1} = \frac{1}{2}k_1 x_{\text{max},1}^2 = \frac{1}{2}(100.\text{ N/m})\left(20.0\cdot 10^{-2}\text{ m}\right)^2 = 2.00\text{ J}$$

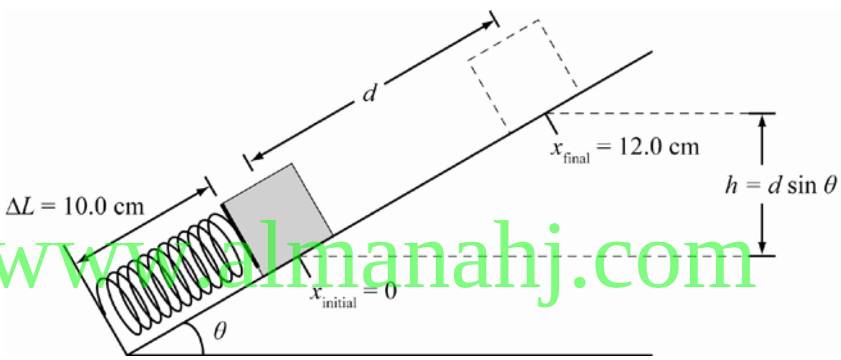
$$U_{\text{max},2} = \frac{1}{2}k_1 x_{\text{max},1}^2 = \frac{1}{2}(50.\text{ N/m})\left(28.3\cdot 10^{-2}\text{ m}\right)^2 = 2.00\text{ J}$$

$$K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}(1.00\text{ kg})(2.00\text{ m/s})^2 = 2.00\text{ J}$$

and all results are reasonable for the given values.

**6.83.** **THINK:** A block of mass, $m$ = 1.00 kg is against a spring on an inclined plane of angle, $\theta = 30.0°$. The coefficient of kinetic friction is $\mu_{\text{k}} = 0.100$. The spring is initially compressed 10.0 cm and the block moves to 2.00 cm beyond the springs normal length after release (therefore the block moves $d$ = 12.0 cm after it is released). Determine (a) the change in the total mechanical energy and (b) the spring constant.

**SKETCH:**






**RESEARCH:**

(a) Since this is not a conservative system, the change in the total mechanical energy can be related to the energy lost due to friction. This energy can be determined by calculating the work done by the force of friction: $W_{\text{friction}} = F_{\text{friction}}d = \mu_{\text{k}}mg(\cos\theta)d$, and $\Delta E_{\text{tot}} = -W_{\text{friction}} = -\mu_{\text{k}}mg(\cos\theta)d$.

(b) From conservation of energy, the change in total energy, $\Delta E_{\text{tot}}$ determined in (a), is equal to $\Delta K + \Delta U$. Since $K = 0$ at both the initial and final points it follows that

$$\Delta E_{\text{tot}} = U_{\text{final}} - U_{\text{initial}} = mgd\sin\theta - \frac{1}{2}k\Delta L^2.$$

**SIMPLIFY:**

(a) $\Delta E_{\text{tot}} = -\mu_k mg(\cos\theta)d$

(b) $k = 2\dfrac{(mgd\sin\theta - \Delta E_{\text{tot}})}{\Delta L^2}$

**CALCULATE:**

(a) $\Delta E_{\text{tot}} = -(0.100)(1.00\text{ kg})\left(9.81\text{ m/s}^2\right)\cos(30.0°)\left(12.0\cdot 10^{-2}\text{ m}\right) = -0.1019$ J

(b) $k = -2\dfrac{(1.00\text{ kg})\left(9.81\text{ m/s}^2\right)(0.120\text{ m})\sin(30.0°) - (-0.1019\text{ J})}{(0.100\text{ m})^2} = 138.1$ N/m

**ROUND:**

(a) Since the lowest number of significant figures is three, the result should be rounded to $\Delta E_{\text{tot}} = -1.02\cdot 10^{-1}$ J (lost to friction).

(b) Since the mass is given to three significant figures, the result should be rounded to $k = 138$ N/m.

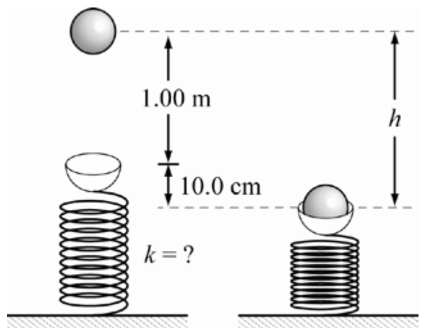
**DOUBLE-CHECK:**

(a) A change of about 0.1 J given away to friction for a distance of 12 cm and with this particular coefficient of friction is reasonable.

(b) The spring constant is in agreement with the expected values.

**6.84.** **THINK:** A 0.100 kg ball is dropped from a height of 1.00 m. If the spring compresses 10.0 cm, determine (a) the spring constant and (b) the percent difference between a spring constant calculated by neglecting a change in $U_{\text{gravity}}$ while compressing the spring, and the result in part (a).

**SKETCH:**




**RESEARCH:**

(a) Determine the spring constant by relating the gravitational potential energy, given to the system, to the elastic potential energy stored by the spring: $U_{\text{gravity}} = U_{\text{spring}} \;\Rightarrow\; mgh = (1/2)kx^2$.

(b) If the change in gravitational potential energy is ignored during the compression:

$$mg(h-x) = \frac{1}{2}kx^2.$$



To calculate the percent difference, use $\%\text{ difference} = \dfrac{|k_1 - k_2|}{(k_1 + k_2)/2}(100\%)$.

**SIMPLIFY:**

(a) $mgh = \dfrac{1}{2}k_1x^2 \;\Rightarrow\; k_1 = \dfrac{2mgh}{x^2}$

(b) $mg(h-x) = \dfrac{1}{2}k_2x^2 \;\Rightarrow\; k_2 = \dfrac{2mg(h-x)}{x^2}$

Therefore,

$$\%\text{ difference} = \frac{\left|\dfrac{2mgh}{x^2} - \dfrac{2mg(h-x)}{x^2}\right|}{\left(\dfrac{2mgh}{x^2} + \dfrac{2mg(h-x)}{x^2}\right)/2} = \frac{|h-(h-x)|}{(h+h-x)/2} = \frac{2x}{2h-x}.$$

**CALCULATE:**

(a) $k_1 = \dfrac{2(0.100\text{ kg})(9.81\text{ m/s}^2)(1.10\text{ m})}{(0.100\text{ m})^2} = 215.82$ N/m

(b) $\%\text{ difference} = \dfrac{2(0.100\text{ m})}{2(1.10\text{ m}) - 0.100\text{ m}} = 9.52\%$

**ROUND:** Rounding to three significant figures, $k_1 = 216$ N/m and the % difference is 9.52 %.

**DOUBLE-CHECK:** The percent difference is reasonable.

**6.85.** **THINK:** The mass is $m$ = 1.00 kg, $k$ = 100. N/m, the amplitude is $A$ = 0.500 m and $x_1 = 0.250$ m. Determine:

(a) the total mechanical energy,

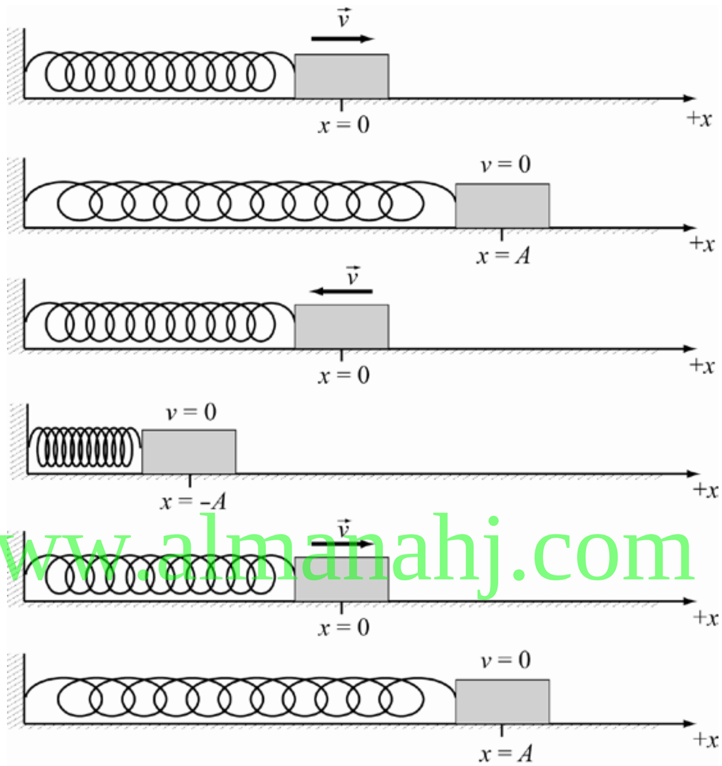
(b) the potential energy for the system and the kinetic energy of the mass at $x_1$,

(c) the kinetic energy of the mass at $x = 0$, that is $K_{\max}$,

(d) the change in kinetic energy of the mass if the amplitude is cut in half due to friction, and

(e) the change in potential energy if the amplitude is cut in half due to friction.

**SKETCH:**






**RESEARCH:**

(a) Assume a frictionless table and write $E_{tot} = U_{\max} = K_{\max}$ and calculate $U_{\max} = \left(kA^2\right)/2$.

(b) At $x_1$, the potential energy is $U_{x_1} = \left(\mathrm{k}x_1^{\ 2}\right)/2$ and the kinetic energy will be given by:

$$K_{x_1} = U_{\max} - U_{x_1}.$$

(c) At $x = 0$, all the energy is in the form of kinetic energy, therefore $K_{x=0} = K_{\max} = U_{\max}$.

(d) Let $K_{\max}$ * denote that the maximum kinetic energy of the mass if there was friction between the mass and the table. At the moment when the amplitude is cut in half, the maximum kinetic energy is obtained by the maximum potential energy:

$$K_{\max} = U_{\max} = \frac{1}{2}kA^2 \Rightarrow K_{\max}{}^* = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}kA^2\right).$$

(e) As described in part (d), $U_{\max}{}^* = \frac{1}{4}U_{\max}$.

**SIMPLIFY:**

(a) $E_{\text{tot}} = \frac{1}{2}kA^2$

(b) $K_{x_1} = U_{\max} - U_{x_1} = \frac{1}{2}kA^2 - \frac{1}{2}kx_1^{\ 2} = \frac{1}{2}k\left(A^2 - x_1^{\ 2}\right)$

(c) $K_{max} = U_{max} = \frac{1}{2}kA^2$

(d) $K_{max}{}^* = \frac{1}{4}K_{max}$

(e) $U_{max}{}^* = \frac{1}{4}U_{max}$

**CALCULATE:**

(a) $E_{tot} = \frac{1}{2}(100.\text{ N/m})(0.500\text{ m})^2 = 12.5\text{ J}$

(b) $U_{x_1} = \frac{1}{2}(100.\text{N/m})(0.250\text{m})^2 = 3.125\text{J},\ K_{x_1} = \frac{1}{2}(100.\text{ N/m})\left[(0.500\text{ m})^2 - (0.250\text{m})^2\right] = 9.375\text{ J}$

(c) $K_{max} = E_{tot} = 12.5\text{ J}$

(d) A factor of ¼.

(e) A factor of ¼.

**ROUND:** Rounding to three significant figures:

(a) $E_{tot} = 12.5\text{ J}$

(b) $U_{x_1} = 3.13\text{J}\ \ K_{x_1} = 9.38\text{ J}$

(c) $K_{max} = 12.5\text{ J}$

(d) $K_{max}$ changes by a factor of ¼.

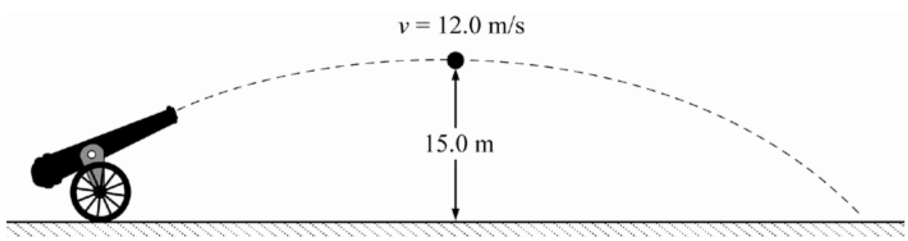
(e) $U_{max}$ changes by a factor of ¼.

**DOUBLE-CHECK:** As expected, the kinetic energy at any point other than $x = 0$ is less than the maximum kinetic energy.

**6.86.** **THINK:** Bolo has a mass of 80.0 kg and is projected from a 3.50 m long barrel. Determine the average force exerted on him in the barrel in order to reach a speed of 12.0 m/s at the top of the trajectory at 15.0 m above the ground.



**SKETCH:**




**RESEARCH:** When Bolo is at the top of the trajectory, his total energy (neglecting air friction) is $E_{tot} = U + K$. This energy can be related to the force exerted by the cannon by means of the work done on Bolo by the cannon: $W = Fd \Rightarrow F = W/d$. Since all the energy was provided by the cannon, $W = E_{tot} \Rightarrow F = E_{tot}/d$.

**SIMPLIFY:** $F = \frac{E_{tot}}{d} = \frac{U+K}{d} = \frac{mgh + \frac{1}{2}mv^2}{d} = \frac{m}{d}\left(gh + \frac{v^2}{2}\right)$

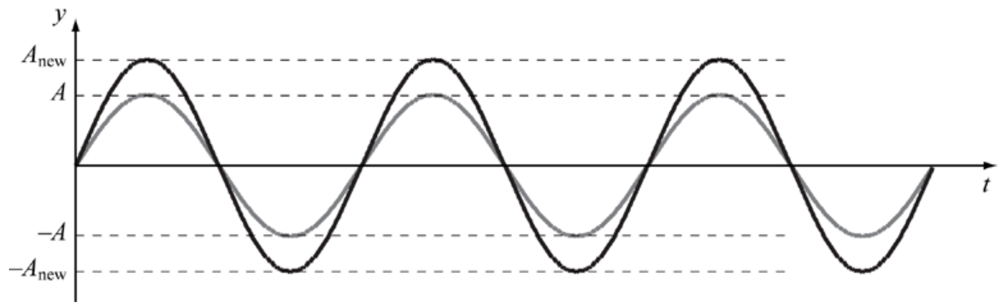
**CALCULATE:** $F = \left(\frac{80.0\text{ kg}}{3.50\text{ m}}\right)\left((9.81\text{ m/s}^2)(15.0\text{ m}) + \frac{(12.0\text{ m/s})^2}{2}\right) = 5009.14\text{ N}$

**ROUND:** Since the number of significant figures in the calculation is three, the result rounds to $F = 5010\text{ N}$.

**DOUBLE-CHECK:** That a force of about 5000 N is required to propel an 80 kg object through such a distance is reasonable.

**6.87.** **THINK:** The mass hanging vertically from a spring can be treated using a method that is independent of gravitational effects on the mass (see page 185 in the text). The mechanical energy of the mass on a spring is defined in terms of the amplitude of the oscillation and the spring constant. When the mass is pushed, the system gains mechanical energy. This new mechanical energy can be used to calculate the new velocity of the mass at the equilibrium position (b) and the new amplitude (c).

**SKETCH:** Before the mass is hit, the amplitude of the oscillation is $A$. After the mass is hit, the amplitude of the oscillation is $A_{\text{new}}$.




**RESEARCH:** The total mechanical energy before the hit is $E=\frac{1}{2}kA^2$. After the hit, the total mechanical energy is given by $E_{\text{new}}=\frac{1}{2}kA^2+\frac{1}{2}mv_{\text{push}}^2$ where $v_{\text{push}}$ is the speed with which the mass is pushed. The new speed at equilibrium is given by $\frac{1}{2}mv_{\text{new}}^2=E_{\text{new}}$ and the new amplitude of oscillation is given by $\frac{1}{2}kA_{\text{new}}^2=E_{\text{new}}$.



**SIMPLIFY:**

(a) $E_{\text{new}}=\frac{1}{2}kA^2+\frac{1}{2}mv_{\text{push}}^2$

(b) $v_{\text{new}}=\sqrt{\frac{2E_{\text{new}}}{m}}$

(c) $A_{\text{new}}=\sqrt{\frac{2E_{\text{new}}}{k}}$

**CALCULATE:**

(a) $E_{\text{new}}=\frac{1}{2}kA^2+\frac{1}{2}mv_{\text{push}}^2=\frac{1}{2}(100.\text{ N/m})(0.200\text{ m})^2+\frac{1}{2}(1.00\text{ kg})(1.00\text{ m/s})^2=2.50\text{ J}$

(b) $v_{\text{new}}=\sqrt{\frac{2E_{\text{new}}}{m}}=\sqrt{\frac{2(2.50\text{ J})}{1.00\text{ kg}}}=2.236\text{ m/s}$

(c) $A_{\text{new}}=\sqrt{\frac{2E_{\text{new}}}{k}}=\sqrt{\frac{2(2.50\text{ J})}{100.\text{ N/m}}}=0.2236\text{ m}$

**ROUND:** Rounding to three significant figures: $E_{\text{new}}=2.50\text{ J}$, $v_{\text{max},2}=2.24\text{ m/s}$ and $A_2=22.4$ cm.

**DOUBLE-CHECK:** The mechanical energy before the hit was

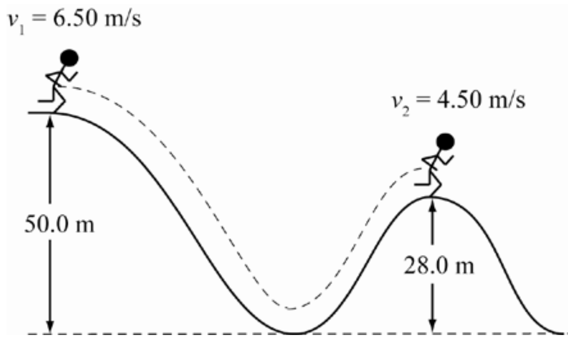
$$E=(1/2)kA^2=(1/2)(100.\text{ N/m})(0.200\text{ m})^2=2.00\text{ J}.$$

The speed of the mass passing the equilibrium point before the hit was $v=\sqrt{\frac{2E}{m}}=\sqrt{\frac{2(2.00\text{ J})}{1.00\text{ kg}}}=2.00$ m/s.

It is reasonable that adding 0.5 J to the total energy by means of a hit results in an increase of the speed of the mass at the equilibrium point of 0.24 m/s and an increase of about 2.4 cm to the amplitude.

**6.88.** **THINK:** Determine the total work done by a runner on a track where the initial speed is $v_1 = 6.50$ m/s at a height of 50.0 m and the final speed is $v_2 = 4.50$ m/s at a different hill with a height of 28.0 m. The runner has a mass of 83.0 kg, there is a constant resistance of 9.00 N and the total distance covered is 400. m.

**SKETCH:**




**RESEARCH:** Let the force of resistance be denoted $F_r$ The total work done by the runner can be determined by considering the change in kinetic and potential energy and by considering the work done by the resistance force: $W_1 = \Delta K$, $W_2 = \Delta U$ and $W_3 = F_r d$.

**SIMPLIFY:** $W_1 = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = \frac{1}{2}m\left(v_1^2 - v_2^2\right)$, $W_2 = mg\left(h_1 - h_2\right)$, and $W_3 = F_r d$. The total energy at point 1: $E_{\text{tot},1} = \frac{1}{2}mv_1^2 + mgh_1$. The total energy at point 2: $E_{\text{tot},2} = \frac{1}{2}mv_2^2 + mgh_2$.

**CALCULATE:** $E_{\text{tot},1} = \frac{1}{2}(83.0 \text{ kg})(6.50 \text{ m/s})^2 + (83.0 \text{ kg})(9.81 \text{ m/s}^2)(50.0 \text{ m}) = 4.25 \cdot 10^4 \text{ J}$

$E_{\text{tot},2} = \frac{1}{2}(83.0 \text{ kg})(4.50 \text{ m/s})^2 + (83.0 \text{ kg})(9.81 \text{ m/s}^2)(28.0 \text{ m}) = 2.36 \cdot 10^4 \text{ J}$



Therefore, $\Delta E_{\text{tot}} = 4.25 \cdot 10^4 \text{ J} - 2.36 \cdot 10^4 \text{ J} = 1.89 \cdot 10^4 \text{ J}$.

$W_{\text{friction}} = (9.00 \text{ N})(400. \text{ m}) = 3600 \text{ J}$.

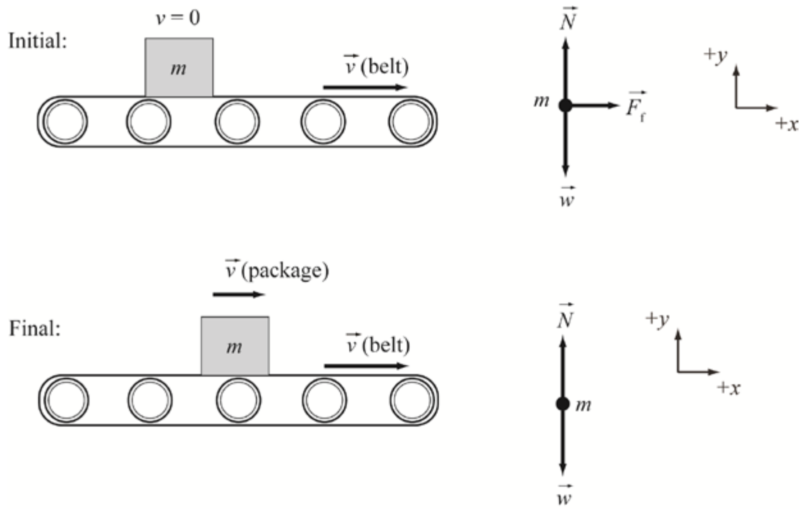
Therefore, $E_{\text{lost}} = \Delta E_{\text{tot}} + W_{\text{friction}} = 1.89 \cdot 10^4 \text{ J} + 3.60 \cdot 10^3 \text{ J} = 2.25 \cdot 10^4 \text{ J}$.

**ROUND:** Rounding to three significant figures, $E_{\text{lost}} = 2.25 \cdot 10^4$ J.

**DOUBLE-CHECK:** This is a reasonable value for the energy exerted by a runner with the given values.

**6.89.** **THINK:** Once the package is dropped on the left, the only horizontal force acting on the package is friction. The speed the package is moving relative to the belt is known, so the constant acceleration expressions can be used to determine the time taken for the package to stop sliding on the belt, i.e. the time it takes for the package to stop moving relative to the belt (part (a)). For the remaining problems, the principles of work and conservation of energy can be used to determine the required values. The known quantities are: $v$ (the speed of the belt relative to the package), $m$ (the mass of the package), $\mu_k$ (the coefficient of kinetic friction).

**SKETCH:**




**RESEARCH:** Work is given by $W = Fd$ ($\vec{F}$ is parallel to $\vec{d}$). Kinetic energy is given by $K = \left(mv^2\right)/2$. The constant acceleration equations are: $v_f = v_i + at$ and $v_f^2 = v_i^2 + 2ax$.

**SIMPLIFY:**

(a) $v_f = v_i + at$, $v_i = 0 \;\Rightarrow\; t = \dfrac{v_f}{a}$, $\quad v_f = v$, $\quad ma = F_f = \mu_k mg \;\Rightarrow\; a = \mu_k g$, and $t = \dfrac{v_f}{a} = \dfrac{v}{\mu_k g}$.

(b) $v_f^2 = v_i^2 + 2ax$, $\quad v_i = 0$, $\quad v_f = v$, $\quad a = \mu_k g$, and $x = \dfrac{v_f^2}{2a} = \dfrac{v^2}{2\mu_k g}$.



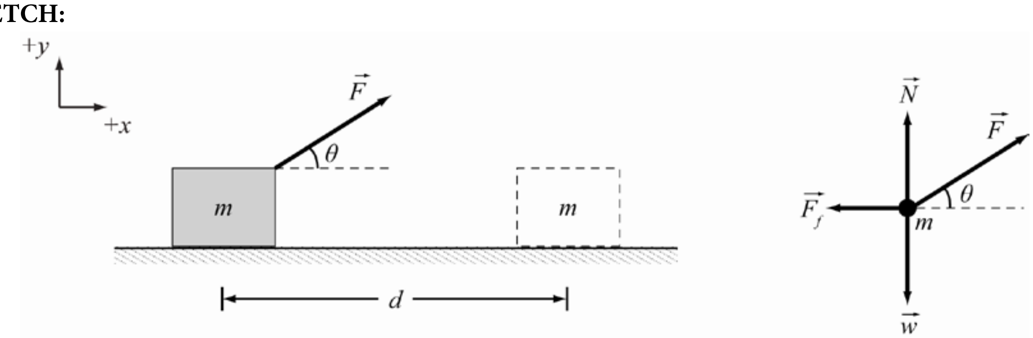
(c) The energy dissipated is equal to the work done by the belt minus the change in kinetic energy:
$W - \Delta E = Fd - \left(mv^2\right)/2 = \left(\mu_k mg\right)\left(vt\right) - \left(mv^2\right)/2 = \left(\mu_k mg\right)\left(v^2/\mu_k g\right) - \left(mv^2\right)/2 = \left(mv^2\right)/2$

(d) The total work done by the belt is $W = Fd = \left(\mu_k mg\right)\left(vt\right) = \left(\mu_k mg\right)\left(v^2/\mu_k g\right) = mv^2$

**CALCULATE:** It is not necessary to calculate any values.
**ROUND:** This step is not necessary.
**DOUBLE-CHECK:** Of the work done by the conveyor belt, half has ended up as kinetic energy of the package and the other half has been dissipated as friction heat. This seems reasonable, since the package transitioned steadily from a state ($v_i = 0$) where all the belt work was being dissipated as friction to a state ($v_f = v$) where none of it was.

**6.90.** **THINK:** There is enough information to determine all the forces. From the forces, the work can be determined. The given values are as follows: $m$ = 85.0 kg, $d$ = 8.00 m, $\theta = 20.0°$, $\left|\vec{F}\right| = 2.40 \cdot 10^2$ N and $\mu_k = 0.200$.

**SKETCH:**



**RESEARCH:** $W = \vec{F} \cdot \vec{d} = Fd\cos\theta \;\Rightarrow\; W_{\text{tot}} = F_{\text{net}} d\cos\theta$

**SIMPLIFY:**

(a) $W_{\text{father}} = F_{\text{father}} d\cos\theta$

(b) $W_{\text{friction}} = F_{\text{friction}} d$ (the force is parallel to the displacement), $F_{\text{friction}} = \mu_{\text{k}}(mg - F\sin\theta)$

(c) $W_{\text{total}} = W_{\text{father}} + W_{\text{friction}}$

**CALCULATE:**

(a) $W_{\text{father}} = \left(2.40 \cdot 10^2 \text{ N}\right)\left(8.00 \text{ m}\right)\cos\left(20.0°\right) = 1.8042 \cdot 10^3 \text{ J}$

(b) $F_{\text{friction}} = \left(0.200\right)\left(\left(85.0 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right) - \left(2.40 \cdot 10^2 \text{ N}\right)\sin\left(20.0°\right)\right) = 150.35 \text{ N}$

$W_{\text{friction}} = \left(1.5035 \cdot 10^2 \text{ N}\right)\left(8.00 \text{ m}\right)\cos\left(180°\right) = -1.2028 \cdot 10^3 \text{ J}$

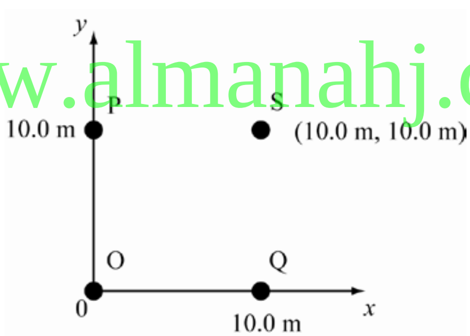
(c) $W_{\text{total}} = \left(1.8042 \cdot 10^3 \text{ J}\right) - \left(1.2028 \cdot 10^3 \text{ J}\right) = 6.014 \cdot 10^2 \text{ J}$

**ROUND:** The given quantities have three significant figures, so the results should be rounded to $W_{\text{father}} = 1.80 \cdot 10^3 \text{ J}$, $W_{\text{friction}} = -1.20 \cdot 10^3 \text{ J}$ and $W_{\text{total}} = 601 \text{ J}$.

**DOUBLE-CHECK:** Note also that the total work can be calculated using the net force, $W_{\text{tot}} = F_{\text{net}} d\cos\theta$, which gives the same result.

**6.91.** **THINK:** The total work can be determined if the path taken and the force applied are known. These are both given as follows: $\vec{F}(x,y) = \left(x^2\hat{x} + y^2\hat{y}\right)$ N and the points are S(10.0 m,10.0 m), P(0 m,10.0 m), Q(10.0 m,0 m) and O(0 m,0 m).

**SKETCH:**






**RESEARCH:** Work is given by:

$$W = \int_a^b d\vec{l} \bullet \vec{F} = \int_a^b \left(x^2 dx + y^2 dy\right).$$

The equations of the paths are: along OP, $x = 0$, $dx = 0$; along OQ, $y = 0$, $dy = 0$; along OS, $y = x$, $dy = dx$; along PS, $y = 10$, $dy = 0$; along QS, $x = 10$, $dx = 0$.

**SIMPLIFY:**

(a) OPS: $W = \int_O^P \left(x^2 dx + y^2 dy\right) + \int_P^S \left(x^2 dx + y^2 dy\right)$

$= \int_0^{10} y^2 dy + \int_0^{10} x^2 dx = \frac{1}{3} y^3\Big|_0^{10} + \frac{1}{3} x^3\Big|_0^{10}$

$= \frac{1}{3}(10)^3 + \frac{1}{3}(10)^3 = \frac{2}{3}(10)^3$

$= W_{\text{OP}} + W_{\text{PS}}$

(b) OQS: $W = \int_O^Q d\vec{l}\bullet\vec{F} + \int_Q^S d\vec{l}\bullet\vec{F}$

$= \int_0^{10} x^2dx + \int_0^{10} y^2dy$

$= W_{OQ} + W_{QS}$

$= W_{PS} + W_{OP}$

$= \left(\frac{2}{3}\right)10^3$

(c) OS: $W = W_{OS} = \int_O^S \left(x^2dx + y^2dy\right) \Rightarrow \int_O^S \left(x^2dx + x^2dx\right) = \int_0^{10} 2x^2dx = 2W_{PS} = \frac{2}{3}\left(10^3\right)$

(d) OPSQO: $W = W_{OP} + W_{PS} + W_{SQ} + W_{QO} = \frac{10^3}{3} + \frac{10^3}{3} + \left(-W_{QS}\right) + \left(-W_{OQ}\right) = \frac{2}{3}\left(10^3\right) - \frac{2}{3}\left(10^3\right) = 0$

(e) OQSPO: $W = W_{OQ} + W_{QS} + W_{SP} + W_{PO} = \frac{10^3}{3} + \frac{10^3}{3} - \frac{10^3}{3} - \frac{10^3}{3} = 0$

**CALCULATE:** $\frac{2}{3}\left(10.0^3\right) = 666.67.$ (a), (b) and (c): $W = 666.67.$ (d) and (e): $W = 0.$

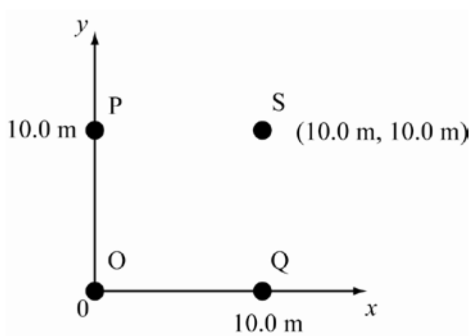
**ROUND:** Rounding to three significant figures, (a), (b) and (c): W = 667 J, and (d), (e): W = 0 J.

**DOUBLE-CHECK:** The force is conservative and it should not depend on the path. It is expected that $W_{OS} = W_{OP} + W_{PS} = W_{OQ} + W_{QS}$, which is shown to be true in the calculation. It is also expected that the work along a closed path is zero, which is also shown to be true in the calculations.

**6.92.** **THINK:** The net work done is the sum of the work done by the applied force, calculated in the previous problem, and the work done by the frictional force.



**SKETCH:**




**RESEARCH:** The force of friction is constant, $F_f = \mu_k mg$, and always points opposite to the direction of motion. First determine the work done by friction, $W_f$, and then calculate $W_{net} = W_{applied}$ (from 6.87) + $W_f$. Refer to the constraints on $x$, $y$, $dx$, and $dy$ determined in 6.87.

Along OP: $\vec{F}_f = -F_f\hat{y}$, $\vec{F}_f \cdot dl = -\mu_k mg dy \Rightarrow W_f = -\mu_k mg\int_0^{10} dy = -10\mu_k mg$

Along OQ: $\vec{F}_f = -F_f\hat{x}$, $\vec{F}_f \cdot dl = -\mu_k mg dx \Rightarrow W_f = -10\mu_k mg$

Along OS: $\vec{F}_f = -F_f\frac{\left(\hat{x}+\hat{y}\right)}{\sqrt{2}}$, $\vec{F} \cdot dl = -\frac{\mu_k mg}{\sqrt{2}}\left(dx+dy\right) = -\frac{\mu_k mg}{\sqrt{2}}\left(dx+dx\right) = -\sqrt{2}\mu_k mg dx$

$\Rightarrow W_f = -10\sqrt{2}\mu_k mg$

Along PS: $\vec{F}_f = -F_f\hat{x}$, $\vec{F} \cdot dl = -\mu_k mg dx \Rightarrow W_f = -10\mu_k mg$

Along QS: $\vec{F}_f = -F_f\hat{y}$, $\vec{F} \cdot dl = -\mu_k mg dy \Rightarrow W_f = -10\mu_k mg$

**SIMPLIFY:**

(a) Friction: $W_{\text{OPS,f}} = W_{\text{OP,f}} + W_{\text{PS,f}} = -10\mu_k mg - 10\mu_k mg = -20\mu_k mg$;

$$\Rightarrow \text{Net work: } W_{\text{OPS}} = W_{\text{OPS,applied}} + W_{\text{OPS,f}} = \frac{2}{3}\left(10^3\right) - 20\mu_k mg$$

(b) Net work: $W_{\text{OQS}} = W_{\text{OQS,applied}} + W_{\text{OQS,f}} = \frac{2}{3}\left(10^3\right) - 20\mu_k mg = W_{\text{OPS}}$

(c) Net work: $W_{\text{OS}} = W_{\text{OS,applied}} + W_{\text{OS,f}} = \frac{2}{3}\left(10^3\right) - 10\sqrt{2}\mu_k mg$

(d) Net work: $W_{\text{OPQSO}} = W_{\text{OPQSO,applied}} + W_{\text{OPQSO,f}} = 0 - 40\mu_k mg$

(e) Net work: $W_{\text{OQSPO}} = W_{\text{OPQSO}} = -40\mu_k mg$

**CALCULATE:**

(a) and (b) $W_{\text{net}} = \frac{2}{3}\left(10.0^3\right) - (20.0)(0.100)(0.100 \text{ kg})\left(9.81 \text{ m/s}^2\right) = 664.7 \text{ J}$

(c) $W_{\text{net}} = \frac{2}{3}\left(10.0^3\right) - (10.0)\left(\sqrt{2}\right)(0.100)(0.100 \text{ kg})\left(9.81 \text{ m/s}^2\right) = 665.3 \text{ J}$

(d) and (e) $W_{\text{net}} = -40(0.100)(0.100 \text{ kg})\left(9.81 \text{ m/s}^2\right) = -3.924 \text{ J}$.

**ROUND:** Rounding to three significant figures, (a), (b) and (c): $W = 665$ J, and (d), (e): $W = -3.92$ J.

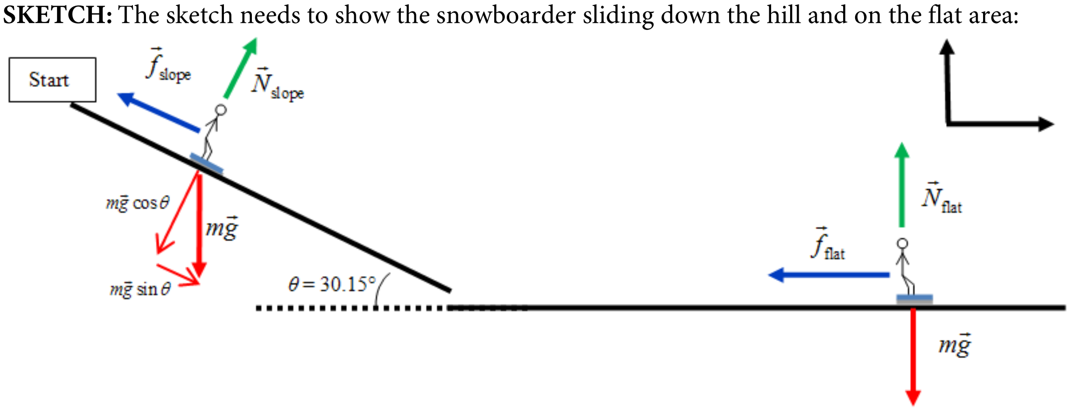
**DOUBLE-CHECK:** The work is slightly reduced but within the significant figures quoted in the question, friction only changes the result for (d) and (e) where the path is the longest. As expected, the net work is path dependent because friction is a non-conservative force.

## Multi-Version Exercises



**6.93.** **THINK:** The gravitational potential energy that the snowboarder has at her highest point is dissipated by friction as she rides down the hill and across the flat area. Think of her motion in two parts: riding down the slope and riding across the flat area.

**SKETCH:** The sketch needs to show the snowboarder sliding down the hill and on the flat area:




**RESEARCH:** The energy dissipated by friction must equal the change in gravitational potential energy from her highest point (at the start) to her final position. The work-energy theorem gives $mgh = f_{\text{slope}}d_{\text{slope}} + f_{\text{flat}}d_{\text{flat}}$, where $d_{\text{flat}}$ is the distance she travels on the flat snow and $d_{\text{slope}}$ is the distance she travels down the slope. Her original starting height is given by $h = d_{\text{slope}}\sin\theta$. The friction force on the slope is given by $f_{\text{slope}} = \mu_k mg\cos\theta$ and the friction force on the flat snow is given by $f_{\text{flat}} = \mu_k mg$.

**SIMPLIFY:** Since the mass of the snowboarder is not given in the question, it is necessary to find an expression for the distance traveled on the flat snow $d_{\text{flat}}$ that does not depend on the mass $m$ of the snowboarder. Substitute the frictional forces $f_{\text{slope}} = \mu_k mg\cos\theta$ and $f_{\text{flat}} = \mu_k mg$ into the work-energy theorem to get

$$mgh = (\mu_k mg\cos\theta)\cdot d_{\text{slope}} + (\mu_k mg)d_{\text{flat}}$$
$$mgh = mg(\mu_k\cos\theta\cdot d_{\text{slope}} + \mu_k d_{\text{flat}})$$
$$h = \mu_k\cos\theta\cdot d_{\text{slope}} + \mu_k d_{\text{flat}}$$

Finally, substitute in $h = d_{\text{slope}}\sin\theta$ for the height $h$ and solve for $d_{\text{flat}}$ to get:

$$\mu_k\cos\theta\cdot d_{\text{slope}} + \mu_k d_{\text{flat}} = h$$
$$\mu_k\cos\theta\cdot d_{\text{slope}} + \mu_k d_{\text{flat}} = d_{\text{slope}}\sin\theta$$
$$\mu_k d_{\text{flat}} = d_{\text{slope}}\sin\theta - \mu_k\cos\theta\cdot d_{\text{slope}}$$
$$d_{\text{flat}} = \frac{d_{\text{slope}}\sin\theta - \mu_k\cos\theta\cdot d_{\text{slope}}}{\mu_k}$$

**CALCULATE:** The question states that the distance the snowboarder travels down the slope is $d_{\text{slope}} = 38.09\text{ m}$, the coefficient of friction between her and the snow is 0.02501, and the angle that the hill makes with the horizontal is $\theta = 30.15°$. Plugging these into the equation gives:

$$\begin{aligned} d_{\text{flat}} &= \frac{d_{\text{slope}}\sin\theta - \mu_k\cos\theta\cdot d_{\text{slope}}}{\mu_k} \\ &= \frac{38.09\text{ m}\cdot\sin 30.15° - 0.02501\cdot\cos 30.15°\cdot 38.09\text{ m}}{0.02501} \\ &= 732.008853\text{ m} \end{aligned}$$

**ROUND:** The quantities in the problem are all given to four significant figures. Even after performing the addition in the numerator, the calculated values have four significant figures, so the snowboarder travels 732.0 m along the flat snow.

**DOUBLE-CHECK:** For those who are frequent snowboarders; this seems like a reasonable answer: travel 38.0 m down a slope of more than 30°, and you go quite far: almost three quarters of a kilometer. Working backwards from the answer, the snowboarder traveled 732.0 m along the flat snow and 38.09 m along the slope, so the energy dissipated is

$$f_{\text{slope}}d_{\text{slope}} + f_{\text{flat}}d_{\text{flat}} = 0.02501(mg)\cos(30.15°)38.0\text{ m} + 0.02501(mg)\cdot 732.0\text{ m}, \text{ or } 19.13mg.$$

Since this must equal the loss in gravitational potential, we know $mgh = 19.13mg$, so the start was 19.13 m above the flat area. This agrees with the values given in the problem, where the snowboarder traveled 38.09 m at a slope of 30.15°, so she started $38.09\sin 30.15 = 19.13$ meters above the horizontal area.



**6.94.** $d_{\text{slope}}\sin\theta = \mu_{\text{k}} d_{\text{slope}}\cos\theta + \mu_{\text{k}} d_{\text{flat}}$

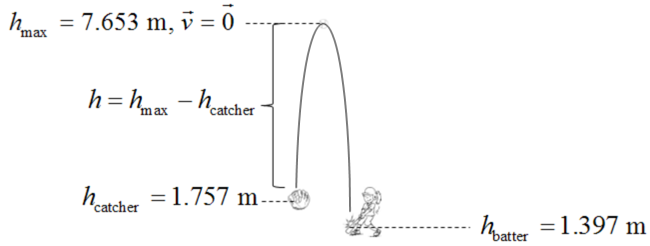
$$\mu_{\text{k}} = \frac{d_{\text{slope}}\sin\theta}{d_{\text{slope}}\cos\theta + d_{\text{flat}}} = \frac{(30.37\text{ m})\sin 30.35°}{(30.37\text{ m})\cos 30.35° + 506.4\text{ m}} = 0.02881$$

**6.95.** $d_{\text{slope}}\sin\theta = \mu_{\text{k}} d_{\text{slope}}\cos\theta + \mu_{\text{k}} d_{\text{flat}}$

$$d_{\text{slope}}\sin\theta - \mu_{\text{k}} d_{\text{slope}}\cos\theta = \mu_{\text{k}} d_{\text{flat}}$$
$$d_{\text{slope}} = d_{\text{flat}}\frac{\mu_{\text{k}}}{\sin\theta - \mu_{\text{k}}\cos\theta} = (478.0\text{ m})\frac{0.03281}{\sin 30.57° - (0.03281)\cos 30.57°} = 32.65\text{ m}$$

**6.96.** **THINK:** At the maximum height, the baseball has no kinetic energy, only gravitational potential energy. We can define zero gravitational potential energy at the point where the catcher gloves the ball. Then the total gravitational potential energy at maximum height equals the total kinetic energy when the ball was caught. The velocity is computed from the kinetic energy.

**SKETCH:** Sketch the path of the baseball, showing the different heights:




**RESEARCH:** The gravitational potential energy is given by $K = mgh$ and the total kinetic energy is given by $KE = \frac{1}{2}mv^2$. In this case, the kinetic energy when the baseball lands in the catcher's mitt is equal to the gravitational potential energy difference from the maximum height to the height at which the catcher caught the baseball.

**SIMPLIFY:** To find the velocity of the baseball when it was caught, it is necessary to note that $K = KE$. This means that $mgh = \frac{1}{2}mv^2$ or $gh = \frac{v^2}{2}$. Since the height $h$ in this problem is really the difference between the maximum height and the height at which the ball was caught ($h = h_{max} - h_{catcher}$), the equation can be solved for the velocity when the ball is caught:

$$\frac{v^2}{2} = gh$$
$$v^2 = 2g\left(h_{max} - h_{catcher}\right)$$
$$v = -\sqrt{2g\left(h_{max} - h_{catcher}\right)}$$

Since the baseball is moving downward when it was caught, we take the negative square root to indicate that the velocity is in the downward direction.



**CALCULATE:** The maximum height of the baseball and the height at which it was caught are given in the problem as 7.653 m and 1.757 m, respectively. The velocity is then calculated to be

$$v = -\sqrt{2g\left(h_{max} - h_{catcher}\right)} = -\sqrt{2 \cdot 9.81 \text{ m/s}^2 \left(7.653 \text{ m} - 1.757 \text{ m}\right)}, \text{ or } -10.75544141 \text{ m/s}$$

**ROUND:** The measured heights are all given to four significant figures, and the height $h$ calculated by taking their difference also has four significant digits. These are the only measured values used in the problem, so the final answer should also have four significant digits. The velocity of the ball when it was caught was 10.76 m/s towards the ground.

**DOUBLE-CHECK:** Normally, the speed of pitches and batted balls in baseball are given in terms of miles per hour. It is not uncommon for pitchers to achieve speeds of around 100 mph, but a pop fly rarely travels that quickly. The baseball was going $10.76 \frac{\text{m}}{\text{s}} \cdot \frac{1 \text{ mile}}{1609.344 \text{ m}} \cdot \frac{3600 \text{ s}}{\text{hour}} = 24.07$ mph when it was caught, which is reasonable in this context.

**6.97.**

$$g\left(h_{max} - h_{catcher}\right) = \frac{1}{2}v^2$$
$$h_{max} - h_{catcher} = \frac{v^2}{2g}$$
$$h_{max} = h_{catcher} + \frac{v^2}{2g} = 1.859 \text{ m} + \frac{(10.74 \text{ m/s})^2}{2\left(9.81 \text{ m/s}^2\right)} = 7.738 \text{ m}$$

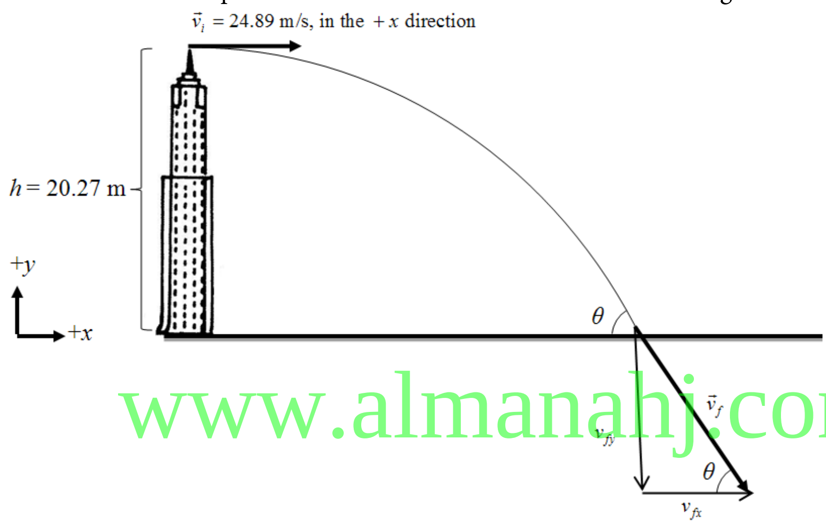
**6.98.** $g\left(h_{\text{max}}-h_{\text{catcher}}\right)=\frac{1}{2}v^2$

$$h_{\text{max}}-h_{\text{catcher}}=\frac{v^2}{2g}$$

$$h_{\text{catcher}}=h_{\text{max}}-\frac{v^2}{2g}=7.777\text{ m}-\frac{(10.73)^2}{2\left(9.81\text{ m/s}^2\right)}=1.909\text{ m}$$

**6.99.** **THINK:** This is a projectile motion problem, where it is possible to ignore air resistance. So, the horizontal velocity stays constant. The vertical component of the velocity can be calculated using energy conservation, and then the angel that the ball strikes the ground can be calculated from the horizontal ($x$-) and vertical ($y$-) components of the velocity.

**SKETCH:** Sketch the path of the ball as it is thrown from the building:






**RESEARCH:** Since the horizontal velocity is constant, the $x$–component of the velocity when the ball is released is equal to the $x$–component of the velocity when the ball lands; $v_{fx}=v_{ix}=v_i$. Since the only change in the velocity is to the $y$–component, the kinetic energy from the $y$–component of the velocity must equal the change in gravitational potential energy, $mgh=\frac{1}{2}m\left(v_{fy}^2\right)$. The angle at which the ball strikes the ground can be computed from the $x$– and $y$– components of the velocity, plus a little trigonometry: $\theta=\tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$.

**SIMPLIFY:** To find the final velocity, it is necessary to eliminate the mass term from the equation $mgh=\frac{1}{2}m\left(v_{fy}^2\right)$ and solve for the final velocity, getting $\sqrt{2gh}=v_{fy}$. Since the horizontal velocity does not change, $v_{fx}=v_i$ can also be used. Substitute these into the equation $\theta=\tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$ to get that $\theta=\tan^{-1}\left(\frac{\sqrt{2gh}}{v_i}\right)$.

**CALCULATE:** The height and initial velocity are given in the problem, and the gravitational acceleration on Earth is about 9.81 m/s$^2$ towards the ground. This means that

$$\theta = \tan^{-1}\left(\frac{\sqrt{2gh}}{v_i}\right) = \tan^{-1}\left(\frac{\sqrt{2\cdot 9.81 \text{ m/s}^2 \cdot 20.27 \text{ m}}}{24.89 \text{ m/s}}\right) = 38.7023859°$$

**ROUND:** The measured values in the question were given to four significant figures, and all of the calculations maintain that degree of accuracy. So the final answer should be rounded to four significant figures. The ball lands at an angle of 38.70° from the horizontal.

**DOUBLE-CHECK:** Working backwards, if the ball lands with a velocity of magnitude $\left|\vec{v}_f\right| = \sqrt{\left|v_{fx}\right|^2 + \left|v_{fy}\right|^2}$, the final velocity has a magnitude $\sqrt{24.89^2 + \sqrt{2gh}^2} = \sqrt{1017.2095}$ m/s. The initial velocity was 24.89 m/s, so the ball gained $\frac{1}{2}m\left(\sqrt{1017.2095}\right)^2 - \frac{1}{2}m(24.89)^2$ J or 198.8487 J in kinetic energy. Since the gravitational potential energy is given by $mgh$, use conservation of energy and algebra to solve for $h$:

$$mgh = \frac{1}{2}m\left(\sqrt{1017.2095}\right)^2 - \frac{1}{2}m(24.89)^2$$

$$9.81mh = m\left(\frac{1}{2}1017.2095 - \frac{1}{2}24.89^2\right)$$

$$h = \frac{m}{9.81m}\left(\frac{1}{2}1017.2095 - \frac{1}{2}24.89^2\right) = \frac{1}{2\cdot 9.81}\left(1017.2095 - 24.89^2\right) = 20.27$$



This height (20.27 m) agrees with the value given in the problem, confirming the calculations.

**6.100.** $\theta = \tan^{-1}\left(\frac{\sqrt{2gh}}{v_i}\right)$

$$\tan\theta = \frac{\sqrt{2gh}}{v_i}$$

$$v_i = \frac{\sqrt{2gh}}{\tan\theta} = \frac{\sqrt{2\left(9.81 \text{ m/s}^2\right)(26.01 \text{ m})}}{\tan 41.86°} = 25.21 \text{ m/s}$$

**6.101.** $\theta = \tan^{-1}\left(\frac{\sqrt{2gh}}{v_i}\right)$

$$\tan\theta = \frac{\sqrt{2gh}}{v_i}$$

$$\sqrt{2gh} = v_i \tan\theta$$

$$h = \frac{v_i^2 \tan^2\theta}{2g} = \frac{(25.51 \text{ m/s})^2 \tan^2(44.37°)}{2\left(9.81 \text{ m/s}^2\right)} = 31.74 \text{ m}$$

# Chapter 7: Momentum and Collisions

## Concept Checks

**7.1.** c **7.2.** b **7.3.** d **7.4.** b **7.5.** b **7.6.** d **7.7.** c **7.8.** d **7.9.** a **7.10.** d **7.11.** b

## Multiple-Choice Questions

**7.1.** b **7.2.** b, c **7.3.** b, d **7.4.** e **7.5.** e **7.6.** b **7.7.** c **7.8.** a, c, and d **7.9.** c **7.10.** a
**7.11.** a, b, and c **7.12.** c **7.13.** a

## Conceptual Questions

**7.14.** She should push object B because it is 10 times more massive than object A. Momentum is conserved here so, after she pushes both she and the object have the same momentum. Since object A has the same mass as the astronaut, it will also have the same speed as the astronaut after she pushes it. Since object B is 10 times more massive than object A, the astronaut will have 10 times the speed of the object.

**7.15.** If the bullet passes through the block then the bullet carries momentum with it. Since momentum is conserved, the block now has less momentum than it did when the bullet remained lodged in the block (in which case it imparted all of its momentum to the block). Since the block now has less momentum, its maximum height is reduced. In contrast, if the bullet bounces off the block, then the maximum height of the block is increased. This is again because momentum is conserved. The block now has a momentum equal to the initial momentum of the block plus an additional momentum equal in magnitude to the bullet's final momentum.



**7.16.** No, this is not a good idea. The steel cable will not gradually absorb energy from the jumper. Because of this, the jumper's kinetic energy will be transferred to the cable very suddenly, leading to a much greater impulse and a higher probability that the jumper will be hurt or the cable breaks. Because the bungee cord stretches, the jumper's kinetic energy and momentum will be transferred much more gradually to the cord, leading to a smaller impulse.

**7.17.** The momentum of the block/ball system is not conserved. The details of the impact are complex, but in simple terms it is like a ball bouncing directly on the ground: the ground remains (ideally) motionless and the ball experiences an impulse that changes its momentum. However, since the impulse from the ice will in this case be straight up, the horizontal components of momentum for the ball and for the block will be equal and opposite, since their sum must be zero. Also, if the impacts between the ball and the block and between the block and the ice are both perfectly elastic, then kinetic energy will be conserved and therefore the total kinetic energy of the block/ball system will be exactly the same before and after--again on the (ideal) assumption that the ice does not move and therefore does not aquire any kinetic energy.

**7.18.** Conservation of momentum is applicable only when there are no external forces acting on the object of interest. In the case of projectiles, gravity acts on the system and will accelerate the objects. We compute the momentum immediately before and after the collision or explosion so that the time interval is very small. In this case, the acceleration due to gravity is negligible and momentum can be considered to be conserved.

**7.19.** (a) The carts exert forces only during the collision. Hence, the curves must go to zero at the beginning and the end of the time shown on the plots. Only #4 and #5 do this. During the collision, cart B exerts a positive force (i.e., a force in the positive $x$-direction) and cart $A$ exerts a negative force. Graph #5 is consistent with this.

(b) Initially, cart A's position is a constant in time (i.e., a horizontal line in the graph) and cart B's position is increasing linearly (i.e., constant positive velocity). After the collision, both A and B move with constant velocity. B's speed is reduced and A's speed is increased. Graph #2 shows this behavior.
(c) All initial and final velocities of the carts are constants (i.e. horizontal lines in the graph). B's final velocity is less than its initial velocity and A's final velocity is increased from its initial velocity of zero. Because B has the larger mass, the velocity change in A is greater than that of B. Graph #7 could describe these properties.
(d) The initial and final accelerations of both carts are zero. During the collision, cart B decelerated (i.e., negative acceleration) and cart A's acceleration is positive. This is as shown in graph #4.
(e) Momentum is conserved; the sum of B's momentum and A's momentum must be a constant at all times. The momenta of both carts are constants before and after the collision. A's momentum increases and B's momentum decreases during the collision, and A's initial momentum is zero. Only graph #6 satisfies all of these constraints.

**7.20.** The air bag is softer than the dashboard and the steering wheel. As the occupant continues to move forward due to inertia immediately after the collision, this momentum will eventually be transferred to the car. In the case of no air bag, the steering column and dashboard absorb the momentum very abruptly and a great impulse causes injury. In the case of the air bag, the momentum transfer is much more gradual; as the occupant compresses the air bag, the forces that the air bag exerts on the passenger is gradually increases due to the increasing pressure of the air in the air bag. Thus the impulse is partially mitigated and injury is reduced.

**7.21.** Momentum is conserved. The total momentum of the rocket-fuel system is always zero. The momentum with which the fuel is expelled from the rocket is equal in magnitude and opposite in direction to the momentum of the rocket itself. The rocket must move in order to conserve the total momentum. Energy is also conserved, if we include the chemical energy stored in the fuel. A chemical reactor converts the fuel's chemical potential energy to mechanical kinetic energy, with the velocity directed out the fuel nozzles.



**7.22.** By riding the punch, the momentum transfer to the boxer's head occurs over a greater time interval than if the boxer stiffens his neck muscles. In the latter case, the momentum transfer is very abrupt and the boxer experiences a greater force resulting in greater damage. By pulling his head back, the boxer lengthens the time interval and thereby reduces the impact force, leading to less injury.

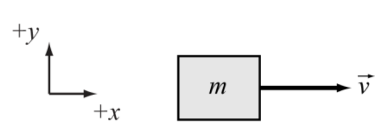
**7.23.** Momentum is conserved. As the car is filled with water, the total mass being transported increases. In order for the momentum to remain constant, the speed of the rail car must decrease.

## Exercises

**7.24.** **THINK:** The masses and the speeds of all the objects are given. The kinetic energy and momentum of each object can be directly computed, and then sorted in decreasing order.

| | $m$ | $v$ |
|---|---|---|
| (a) | $10^6$ kg | 500 m/s |
| (b) | 180,000 kg | 300 km/h |
| (c) | 120 kg | 10 m/s |
| (d) | 10 kg | 120 m/s |
| (e) | $2\cdot 10^{-27}$ kg | $2\cdot 10^{8}$ m/s |

**SKETCH:**



**RESEARCH:** $E = \frac{1}{2}mv^2$, $p = mv$

**SIMPLIFY:** Not applicable.

**CALCULATE:**

(a) $E = \frac{1}{2}(10^6 \text{ kg})(500 \text{ m/s})^2 = 1.3 \cdot 10^{11} \text{ J}$, $p = (10^6 \text{ kg})(500 \text{ m/s}) = 5.0 \cdot 10^8$ kg m/s,

(b) $E = \frac{1}{2}(1.8 \cdot 10^5 \text{ kg})\left((300 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right)\right)^2 = 6.3 \cdot 10^8 \text{ J}$,

$p = (1.8 \cdot 10^5 \text{ kg})\left((300 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right)\right) = 1.5 \cdot 10^7$ kg m/s

(c) $p_y = \sqrt{2(49.5 \text{ J})(0.442 \text{ kg})}\sin 58.0° = 5.610$ kg m/s , $p = (120 \text{ kg})(10 \text{ m/s}) = 1200$ kg m/s

(d) $E = \frac{1}{2}(10 \text{ kg})(120 \text{ m/s})^2 = 7.2 \cdot 10^4 \text{ J}$, $p = (10 \text{ kg})(120 \text{ m/s}) = 1200$ kg m/s

(e) $E = \frac{1}{2}(2 \cdot 10^{-27} \text{ kg})(2 \cdot 10^8 \text{ m/s})^2 = 4 \cdot 10^{-11} \text{ J}$, $p = (2 \cdot 10^{-27} \text{ kg})(2 \cdot 10^8 \text{ m/s}) = 4 \cdot 10^{-19}$ kg m/s

**ROUND:** Rounding to one significant figure:

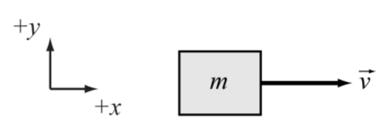
| | $E$ [J] | $p$ [kg m/s] |
|---|---|---|
| (a) | $1 \cdot 10^{11}$ | $5 \cdot 10^8$ |
| (b) | $6 \cdot 10^8$ | $2 \cdot 10^7$ |
| (c) | $6 \cdot 10^3$ | $1 \cdot 10^3$ |
| (d) | $7 \cdot 10^4$ | $1 \cdot 10^3$ |
| (e) | $4 \cdot 10^{-11}$ | $4 \cdot 10^{-19}$ |



**DOUBLE-CHECK:** In order from largest to smallest energy: (a), (b), (d), (c), (e); and momentum: (a), (b), (d) = (c) , (e).

**7.25.** **THINK:** Compute the ratios of the momenta and kinetic energies of the car and SUV. $m_{\text{car}} = 1200.$ kg, $m_{\text{SUV}} = 1.5m_{\text{car}} = \frac{3}{2}m_{\text{car}}$, $v_{\text{car}} = 72.0$ mph, and $v_{\text{SUV}} = \frac{2}{3}v_{\text{car}}$.

**SKETCH:**




**RESEARCH:**

(a) $p = mv$

(b) $K = \frac{1}{2}mv^2$

**SIMPLIFY:**

(a) $\frac{p_{\text{SUV}}}{p_{\text{car}}} = \frac{m_{\text{SUV}}v_{\text{SUV}}}{m_{\text{car}}v_{\text{car}}} = \frac{(3/2)m_{\text{car}}(2/3)v_{\text{car}}}{m_{\text{car}}v_{\text{car}}}$

(b) $\frac{K_{\text{SUV}}}{K_{\text{car}}} = \frac{(1/2)m_{\text{SUV}}v_{\text{SUV}}^2}{(1/2)m_{\text{car}}v_{\text{car}}^2} = \frac{(3/2)m_{\text{car}}\left((2/3)v_{\text{car}}\right)^2}{m_{\text{car}}v_{\text{car}}^2}$

**CALCULATE:**

(a) $\dfrac{p_{\text{SUV}}}{p_{\text{car}}} = \dfrac{(3/2)(2/3)}{1} = 1$

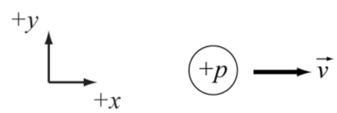
(b) $\dfrac{K_{\text{SUV}}}{K_{\text{car}}} = \dfrac{(3/2)(2/3)^2}{1} = \dfrac{(3/2)(4/9)}{1} = 2/3 = 0.6667$

**ROUND:** (a) $\dfrac{p_{\text{SUV}}}{p_{\text{car}}} = 1.0$ (b) $\dfrac{K_{\text{SUV}}}{K_{\text{car}}} = 0.67$

**DOUBLE-CHECK:** Although the car is lighter, it is moving faster. The changes in mass and speed cancel out for the momentum but not for the kinetic energy because the kinetic energy is proportional to $v^2$.

**7.26.** **THINK:** Both the mass and velocity of the proton are given; $m = 938.3 \text{ MeV}/c^2$, and $v = 17{,}400$ km/s. The velocity of the proton must be converted to units of $c$, the speed of light. $c = 2.998 \cdot 10^8 \text{ m/s} = 2.998 \cdot 10^5 \text{ km/s}$, $v = 17{,}400 \text{ km/s}\left(\dfrac{c}{c}\right) = 17{,}400 \text{ km/s}\left(\dfrac{c}{2.998 \cdot 10^5 \text{ km/s}}\right) = 0.0580387c.$

**SKETCH:**




**RESEARCH:** $p = mv$

**SIMPLIFY:** No simplification is required.

**CALCULATE:** $p = (938.3 \text{ MeV}/c^2)(0.0580387c) = 54.4577 \text{ MeV}/c$

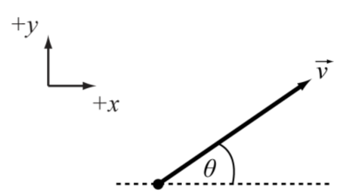
**ROUND:** Round to three significant figures. $p = 54.5 \text{ MeV}/c$

**DOUBLE-CHECK:** For something as small as a proton, moving at large speeds, units in terms of MeV and $c$ are more reasonable than J and m/s.



**7.27.** **THINK:** The ball's velocity can be determined from its kinetic energy. The angle of the ball's velocity is given, so the velocity vector can be determined. The components of the ball's momentum can be computed from the velocity vector and the mass. $m = 442$ g, $\theta = 58.0°$, and $K = 49.5$ J.

**SKETCH:**




**RESEARCH:** $K = \dfrac{1}{2}mv^2$, $v_x = v\cos\theta$, $p_x = mv_x$, $v_y = v\sin\theta$, and $\varphi = \tan^{-1}\left(\dfrac{16.756 \text{ m/s}}{-13.928 \text{ m/s}}\right) = -50.27°.$

**SIMPLIFY:** $v = \sqrt{\dfrac{2K}{m}}$, $p_x = mv_x = mv\cos\theta = m\sqrt{\dfrac{2K}{m}}\cos\theta = \sqrt{2Km}\cos\theta$, $\varphi = 50.3°.$

**CALCULATE:** $p_x = \sqrt{2(49.5 \text{ J})(0.442 \text{ kg})}\cos 58.0° = 3.505$ kg m/s,

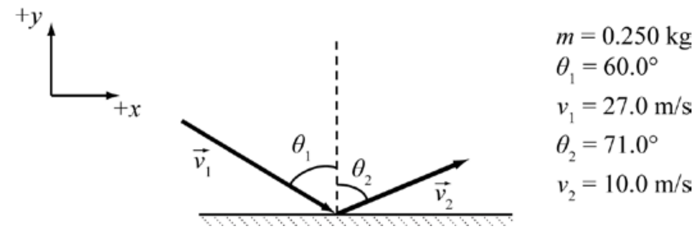
$p_y = \sqrt{2(49.5 \text{ J})(0.442 \text{ kg})}\sin 58.0° = 5.610$ kg m/s

**ROUND:** The answers should be rounded to 3 significant figures: $p_x = 3.51$ kg m/s, and $p_y = 5.61$ kg m/s.

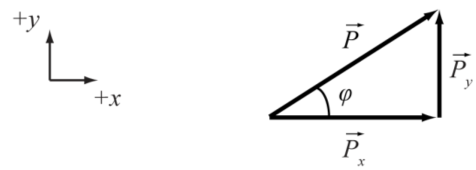
**DOUBLE-CHECK:** The values seem appropriate. Note that $p_y > p_x$. This makes sense because the angle of deflection is greater than $45°$.

**7.28.** **THINK:** The change of momentum is $\overrightarrow{\Delta p} = \vec{p}_2 - \vec{p}_1$. Its magnitude is $\left|\vec{p}_2 - \vec{p}_1\right|$. The magnitude and direction can be calculated by components.

**SKETCH:**




**RESEARCH:** $\vec{p} = m\vec{v}$, $v_{1,x} = v_1 \sin\theta_1$, $v_{1,y} = -v_1\cos\theta_1$, $v_{2,x} = v_2\sin\theta_2$, $v_{2,y} = v_2\cos\theta_2$, $p = \sqrt{p_x^2 + p_y^2}$, and $\varphi = \tan^{-1}\left(\frac{p_y}{p_x}\right)$.




**SIMPLIFY:** $\overrightarrow{\Delta p} = \vec{p}_2 - \vec{p}_1 = m\left(\vec{v}_2 - \vec{v}_1\right)$, $\Delta p_x = m\Delta v_x = m(v_{2,x} - v_{1,x}) = m(v_2\sin\theta_2 - v_1\sin\theta_1)$, $\Delta p_y = m\left(v_2\cos\theta_2 - v_1\cos\theta_1\right)$, $\varphi = \tan^{-1}\left(\frac{\Delta p_y}{\Delta p_x}\right) = \tan^{-1}\left(\frac{v_2\cos\theta_2 - v_1\cos\theta_1}{v_2\sin\theta_2 - v_1\sin\theta_1}\right)$, and finally,

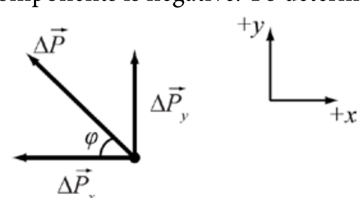
$$\left|\overrightarrow{\Delta p}\right| = \sqrt{\left(\Delta p_x\right)^2 + \left(\Delta p_y\right)^2} = m\left[\left(v_2\sin\theta_2 - v_1\sin\theta_1\right)^2 + \left(v_2\cos\theta_2 - v_1\cos\theta_1\right)^2\right]^{1/2}.$$



**CALCULATE:** $v_x = v_2\sin\theta_2 - v_1\sin\theta_1 = (10.0 \text{ m/s})(\sin 71.0°) - (27.0 \text{ m/s})(\sin 60.0°) = -13.928 \text{ m/s}$,

$v_y = v_2\cos\theta_2 - v_1\cos\theta_1 = (10.0 \text{ m/s})(\cos 71.0°) - (-27.0 \text{ m/s})(\cos 60.0°) = 16.756 \text{ m/s}$,

$\left|\overrightarrow{\Delta p}\right| = (0.250 \text{ kg})\left[(-13.928 \text{ m/s})^2 + (16.756)^2\right]^{1/2} = 5.447 \text{ kg m/s}$, $\varphi = \tan^{-1}\left(\frac{16.756 \text{ m/s}}{-13.928 \text{ m/s}}\right) = -50.27°$. The sign is negative because one of the components is negative. To determine the direction, draw a diagram.

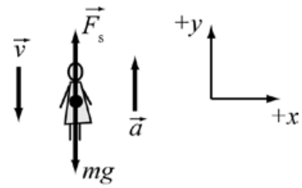



**ROUND:** The answers should be rounded to 3 significant figures: $\left|\overrightarrow{\Delta p}\right| = 5.45$ kg m/s, and $\varphi = 50.3°$. The magnitude is $5.45$ kg m/s. The direction is upwards and to the left $50.3°$ along the horizontal.

**DOUBLE-CHECK:** To get the directions correct, it is far more useful to draw diagrams here than it is to rely on the sign of $\tan^{-1}$.

**7.29.** **THINK:** Lois has a mass of 50.0 kg and speed 60.0 m/s. We need to calculate the force on Lois, $F_s$, when $\Delta t = 0.100$ s. (Subscript *s* means "Superman, mostly, with a small assist from air resistance.") Then we want the value of $\Delta t$ where acceleration is $a = 6.00g$, which when added to the $1.00g$ required to counteract gravity will mean Lois is subjected to 7.00$g$ total. (A person standing motionless on the ground experiences 1 $g$, and any upward acceleration means additional $g$'s.)

**SKETCH:**




**RESEARCH:** The impulse is defined as the change in momentum, $J = \Delta p = F_{net}\Delta t$.

**SIMPLIFY:** Applying Newton's second law and assuming the force exerted is in the positive $y$-direction, $\sum F_y = ma_y$.

$$F_{net} = F_s - mg = ma = \frac{\Delta p}{\Delta t} \Rightarrow F_s = mg + \frac{\Delta p}{\Delta t} = mg + \frac{m(v_f - v_i)}{\Delta t}.\text{ Since } v_f = 0,\ \ \Delta t = 0.75\text{ s.}$$

**CALCULATE:** $v_i = -60.0$ m/s (Note the negative sign as $v$ is in the negative $y$-direction),

$$F_s = (50.0\text{ kg})(9.81\text{ m/s}^2) - \frac{(50.0\text{ kg})(-60.0\text{ m/s})}{0.100\text{ s}} = 30,490.5\text{ N},\quad a = 6.00g \Rightarrow F_{net} = ma = m(6.00g),$$

and

$$F_{net}\Delta t = \Delta p = m(v_f - v_i),\ v_f = 0 \Rightarrow \Delta t = \frac{-mv_i}{m(6.00g)} = \frac{-v_i}{6.00g} = \frac{60.0\text{ m/s}}{6.00(9.81\text{ m/s}^2)} = 1.0194\text{ s.}$$

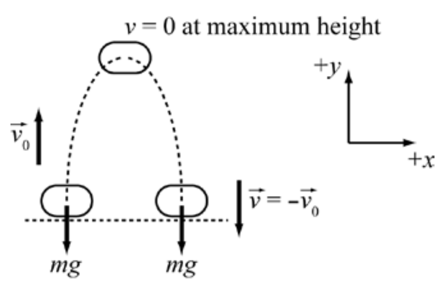
**ROUND:** $F_s = 30,500$ N and $\Delta t = 1.02$ s.

**DOUBLE-CHECK:** The minimal time $\Delta t = 1.02$ s is reasonable.

**7.30. THINK:** A 9.09 kg bag of hay has an initial velocity of 2.7 m/s. I want to calculate the impulse due to gravity.



**SKETCH:**




**RESEARCH:** Impulse is defined as $\vec{J} = F\Delta t = \Delta p$.

**SIMPLIFY:**

(a) $J = \Delta p = m(v_f - v_i),\ v_f = 0,\ v_i = v_0 \Rightarrow J = -mv_0$

(b) $J = m(v_f - v_i),\ v_f = -v_0,\ v_i = 0 \Rightarrow J = -mv_0$

(c) $J_{total} = F\Delta t,\ F = -mg \Rightarrow \Delta t = \frac{J_{total}}{-mg} = -\frac{J_{total}}{mg} = -\frac{-2mv_0}{mg} = \frac{2v_0}{g}$

**CALCULATE:**

(a) $J = (9.09\text{ kg})(-2.7\text{ m/s}) = -24.54$ kg m/s

(b) $J = (9.09\text{ kg})(-2.7\text{ m/s}) = -24.54$ kg m/s

(c) $\Delta t = \frac{2(2.7\text{ m/s})}{(9.81\text{ N})} = 0.55\text{ s}$

**ROUND:** Rounding to two significant digits:

(a) $J = -25$ kg m/s

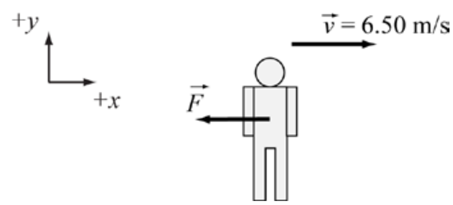
(b) $J = -25$ kg m/s

(c) $\Delta t = 0.55$ s

**DOUBLE-CHECK:** The impulses are expected to be negative since the direction of the force due to gravity is in the negative $y$-direction and $\Delta t$ must always be a positive value.

**7.31.** **THINK:** There is an 83.0-kg running back running with a speed of 6.50 m/s. A 115-kg linebacker applies a force of 900. N on the running back for $\Delta t = 0.750$ s.

**SKETCH:**




**RESEARCH:** We use the definition of impulse, $\vec{J} = \vec{F}_{\text{ave}}\Delta t$ and $\Delta\vec{p} = \vec{J}$.

**SIMPLIFY:** Simplification is not needed here.

**CALCULATE:**

(a) $\vec{J} = \vec{F}_{\text{ave}}\Delta t = (900.\text{ N opposite to } v)(0.750\text{ s}) = 675\text{ Ns opposite to } v.$

(b) The change in momentum is $\Delta\vec{p} = \vec{J} = 675$ Ns opposite to $v$.

(c) The running back's momentum is $\Delta\vec{p} = \vec{J} \Rightarrow \vec{p}_{\text{f}} - \vec{p}_{\text{i}} = \vec{J} \Rightarrow \vec{p}_{\text{f}} = \vec{J} + \vec{p}_{\text{i}}$.



$\vec{p}_{\text{f}} = \vec{J} + m\vec{v} = -675\text{ kg m/s } + (83.0\text{ kg})(6.50\text{ m/s}) = -135.5\text{ kg m/s} = 135.5\text{ kg m/s opposite to } \vec{v}$

(d) No, because the running back's feet have touched the ground. There will be friction between their feet and the ground.

**ROUND:**

(a) $\vec{J} = 675$ N s opposite to $\vec{v}$

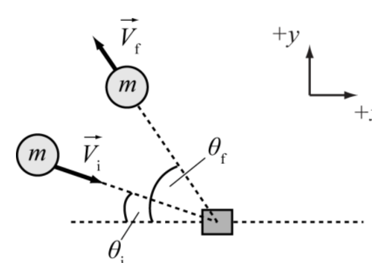
(b) $\Delta\vec{p} = \vec{J} = 675$ N s opposite to $\vec{v}$

(c) $\vec{p}_{\text{f}} = 136$ kg m/s opposite to $\vec{v}$

**DOUBLE-CHECK:** The speed of the running back when his feet touch the ground is $v_{\text{f}} = p_{\text{f}} / m = (135.5\text{ kg m/s opposite to } v)/(83.0\text{ kg}) = 1.63\text{ m/s opposite to } v$ (rounding to three significant figures). So the force on the running back changed his direction in mid air.

**7.32.** **THINK:** The initial speed and angle of the baseball are $v_{\text{i}} = 88.5\text{ mph} = 39.6\text{ m/s}$ and $\theta_{\text{i}} = 7.25°$. Its final speed and angle are $102.7\text{ mph} = 45.9\text{ m/s}$ and $\theta_f = 35.53°$. The mass of the ball is $m = 0.149$ kg. I want to calculate the magnitude of the impulse.

**SKETCH:**



**RESEARCH:** The vector form of impulse and momentum relation must be used in this problem: $\vec{J} = \Delta\vec{p}$. So, in terms of components: $J_x = \Delta p_x = p_{fx} - p_{ix}$, $J_y = \Delta p_y = p_{fy} - p_{iy}$, where the magnitude of $J$ is $J = \sqrt{J_x^2 + J_y^2}$.

**SIMPLIFY:** $J_x = m(v_{fx} - v_{ix}) = m(-v_f\cos\theta_f - v_i\cos\theta_i) = -m(v_f\cos\theta_f + v_i\cos\theta_i)$, and $J_y = m(v_{fy} - v_{iy}) = m(v_f\sin\theta_f + v_i\sin\theta_i)$. The magnitude of impulse is:

$$\begin{aligned} J = \sqrt{J_x^2 + J_y^2} &= \sqrt{m^2(v_f\cos\theta_f + v_i\cos\theta_i)^2 + m^2(v_f\sin\theta_f + v_i\sin\theta_i)^2} \\ &= m\sqrt{(v_f\cos\theta_f + v_i\cos\theta_i)^2 + (v_f\sin\theta_f + v_i\sin\theta_i)^2} \\ &= m\sqrt{v_f^2 + v_i^2 + 2v_iv_f(\cos\theta_i\cos\theta_f + \sin\theta_i\sin\theta_f)} \\ &= m\sqrt{v_f^2 + v_i^2 + 2v_iv_f\cos(\theta_f - \theta_i)} \end{aligned}$$

**CALCULATE:** $J = (0.149\text{ kg})\sqrt{(45.9\text{ m/s})^2 + (39.6\text{ m/s})^2 + 2(45.9\text{ m/s})(39.6\text{ m/s})\cos(35.53° - 7.25°)}$
$= 12.356\text{ kg m/s}$

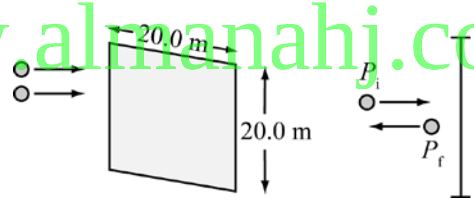
**ROUND:** $J = 12.4\text{ kg m/s}$

**DOUBLE-CHECK:** The result should be less than $J_{max} = m(v_i + v_f) = 12.7\text{ kg m/s}$.

**7.33.** **THINK:** The momentum of a photon is given to be $1.30\cdot10^{-27}$ kg m/s. The number of photons incident on a surface is $\rho = 3.84\cdot10^{21}$ photons per square meter per second. A spaceship has mass $m = 1000.$ kg and a square sail 20.0 m wide.

**SKETCH:**






**RESEARCH:** Using impulse, $\vec{J} = F\Delta t = \Delta p = p_f - p_i$. Also, $v = at$.

**SIMPLIFY:** In $\Delta t = 1$ s, the number of protons incident on the sail is $N = \rho A\Delta t$. The change in momentum in $\Delta t$ is $\Delta p = N(p_f - p_i) \Rightarrow \Delta p = \rho A\Delta t(p_f - p_i)$. Using $p_f = -p_i$, $F\Delta t = \Delta p = \rho A\Delta t(-p_i - p_i) \Rightarrow F = -2\rho Ap_i$.

The actual force on the sail is $F_s = -F = 2\rho Ap_i$, so the acceleration is:

$$a = \frac{F_s}{m_s} = \frac{2\rho Ap_i}{m_s}.$$

**CALCULATE:** $t_{hour} = (1\text{ hr})\left(\frac{3600\text{ s}}{1\text{ hr}}\right) = 3600\text{ s}$,

$t_{week} = (1\text{ week})\left(\frac{24\text{ hours}}{1\text{ day}}\right)\left(\frac{7\text{ days}}{1\text{ week}}\right)\left(\frac{3600\text{ s}}{1\text{ hr}}\right) = 6.048\cdot10^5\text{ s}$,

$t_{month} = (1\text{ month})\left(\frac{3600\text{ s}}{1\text{ hour}}\right)\left(\frac{24\text{ hours}}{1\text{ day}}\right)\left(\frac{365\text{ days}}{1\text{ year}}\right)\left(\frac{1/12\text{ year}}{1\text{ month}}\right) = 2.628\cdot10^6\text{ s}$,

$a = \frac{2\rho Ap_i}{m_s} = \frac{2(3.84\cdot10^{21}\text{ /(m}^2\text{ s))}(20.0\text{ m}\cdot20.0\text{ m})(1.30\cdot10^{-27}\text{ kg m/s})}{1000.\text{ kg}} = 3.994\cdot10^{-6}\text{ m/s}^2$,

$v_{hour} = (3.994\cdot10^{-6}\text{ m/s}^2)(3600\text{ s}) = 0.0144\text{ m/s}$, $v_{week} = (3.994\cdot10^{-6}\text{ m/s}^2)(6.048\cdot10^5\text{ s}) = 2.416\text{ m/s}$,

$v_{\text{month}} = (3.994 \cdot 10^{-6} \text{ m/s}^2)(2.628 \cdot 10^6 \text{ s}) = 10.496 \text{ m/s},$

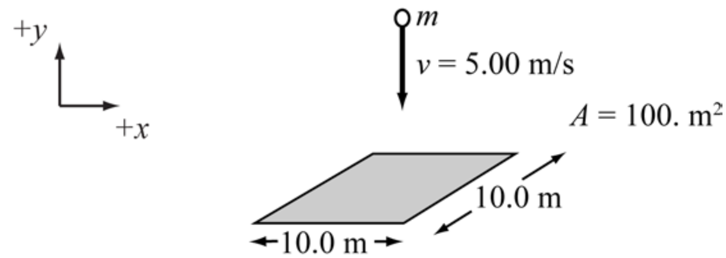
$t = \dfrac{8000. \text{ m/s}}{3.994 \cdot 10^{-6} \text{ m/s}^2} = 2.003 \cdot 10^9 \text{ s} = 762.2 \text{ months}.$

**ROUND:** $v_{\text{hour}} = 0.0144 \text{ m/s}$, $v_{\text{week}} = 2.42 \text{ m/s}$, $v_{\text{month}} = 10.5 \text{ m/s}$, and $t = 762$ months.

**DOUBLE-CHECK:** The answer for velocities and time are understandable since the acceleration is very small.

**7.34.** **THINK:** In a time of $\Delta t = 30.0 \text{ min} = 1.80 \cdot 10^3 \text{ s}$, 1.00 cm of rain falls with a terminal velocity of $v = 5.00 \text{ m/s}$ on a roof. The area of the roof is $100. \text{ m}^2$. Note that mass is density times volume.

**SKETCH:**




**RESEARCH:** Use $F = \dfrac{\Delta p}{\Delta t} = \dfrac{m(v_{\text{f}} - v_{\text{i}})}{\Delta t}$, where $v_{\text{f}} = 0$.

**SIMPLIFY:** $F = -mv_{\text{i}}/\Delta t$. The mass of the rain is $\rho_w V$, where $V = Ah$ is the volume of the water for a depth $h$ of rainfall. $F = -\rho_w A h v_{\text{i}} / \Delta t$.

**CALCULATE:** From a table in the textbook, $\rho_w = 1.00 \cdot 10^3 \text{ kg/m}^3$. $h = 1.00 \text{ cm} = 1.00 \cdot 10^{-2} \text{ m}$, $v = -5.00 \text{ m/s}$, and $\Delta t = 1.80 \cdot 10^3 \text{ s}$.



$$F = -\frac{(1.00 \cdot 10^3 \text{ kg/m}^3)(100. \text{ m}^2)(1.00 \cdot 10^{-2} \text{ m})(-5.00 \text{ m/s})}{1.80 \cdot 10^3 \text{ s}} = -2.777778 \text{ N}$$

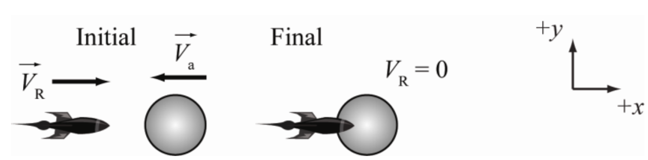
**ROUND:** Round to three significant figures: $F = -2.78$ N

**DOUBLE-CHECK:** This result looks reasonable. It is the equivalent of an approximately half-pound object sitting on the roof.

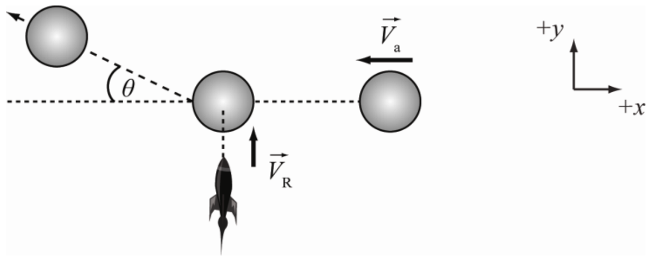
**7.35.** **THINK:** An asteroid has mass $m = 2.10 \cdot 10^{10}$ kg and speed $v_a = 12.0 \text{ km/s}$, and a rocket has mass $8.00 \cdot 10^4$ kg. I want to calculate the speed of the rocket necessary to a. stop the asteroid, and b. divert it from its path by 1.00°.

**SKETCH:**

(a)



(b)

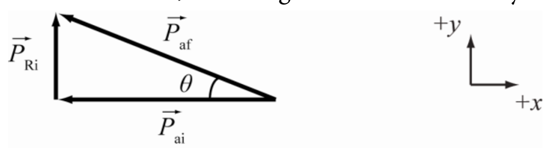



**RESEARCH:** Use conservation of momentum: $\vec{p}_{\text{i}} = \vec{p}_{\text{f}}$.

**SIMPLIFY:**

(a) The rocket and asteroid collide head on. $p_{\text{i}} = p_{\text{f}} \Rightarrow m_{\text{a}}v_{\text{ai}} + m_{\text{R}}v_{\text{Ri}} = m_{\text{a}}v_{\text{af}} + m_{\text{R}}v_{\text{Rf}}$. The final velocities of the rocket and the asteroid are $v_{\text{Rf}} = 0$ and $v_{\text{af}} = 0$. $m_{\text{R}}v_{\text{Ri}} = -m_{\text{a}}v_{\text{ai}} \Rightarrow v_{\text{Ri}} = -\frac{m_{\text{a}}}{m_{\text{R}}}v_{\text{ai}}$

(b) I draw a vector diagram for this collision, assuming that the final velocity of the rocket is $v_{\text{Rf}} = 0$.




Therefore, $\tan\theta = \frac{\left|\overrightarrow{p_{Ri}}\right|}{\left|\overrightarrow{p_{ai}}\right|} \Rightarrow \left|\overrightarrow{p_{Ri}}\right| = \left|\overrightarrow{p_{ai}}\right|\tan\theta.$

$p_{Ri} = p_{ai}\tan\theta \Rightarrow m_R v_{Ri} = m_a v_{ai}\tan\theta \Rightarrow v_{Ri} = \frac{m_a}{m_R}v_{ai}\tan\theta$



**CALCULATE:**

(a) $v_{\text{ai}} = -12.0 \cdot 10^3$ m/s, $v_{\text{Ri}} = \frac{\left(-2.10 \cdot 10^{10}\text{ kg}\right)\left(-12.0 \cdot 10^3\text{ m/s}\right)}{8.00 \cdot 10^4\text{ kg}} = 3.15 \cdot 10^9\text{ m/s}$

(b) $v_{\text{Ri}} = \frac{\left(2.10 \cdot 10^{10}\text{ kg}\right)\left(12.0 \cdot 10^3\text{ m/s}\right)\tan 1.00°}{8.00 \cdot 10^4\text{ kg}} = 5.498 \cdot 10^7\text{ m/s}$

**ROUND:**

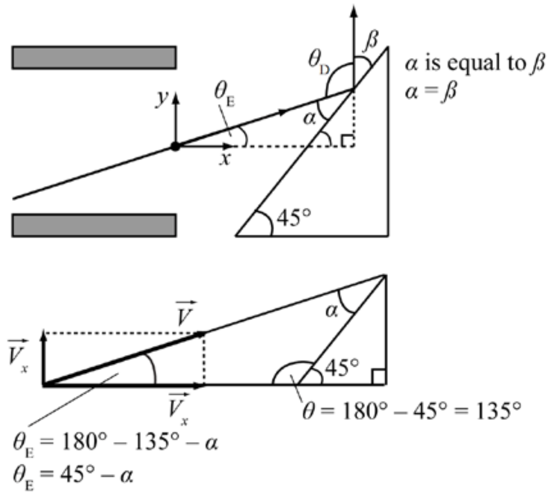
(a) $v_{\text{Ri}} = 3.15 \cdot 10^9$ m/s

(b) $v_{\text{Ri}} = 5.50 \cdot 10^7$ m/s

**DOUBLE-CHECK:** For comparison, the speed of the rocket cannot exceed the speed of light, which is about $3.00 \cdot 10^8$ m/s. The speed of the rocket in (a) is greater than the speed of light, which would be impossible. This means the rocket could not stop the asteroid. The result in (b) is large but is still less than the speed of light.

**7.36.** **THINK:** An electron has velocity $v_x = 1.00 \cdot 10^5$ m/s. The vertical force is $8.0 \cdot 10^{-13}$ N. If $v_y = 0$ and the wall is at 45°, the deflection angle $\theta_{\text{D}}$ is 90°. I want to calculate $\Delta t$ such that the deflection angle $\theta_{\text{D}}$ is 120.0°.

**SKETCH:**




**RESEARCH:** I first need to calculate the angle of the electron velocity after the vertical force has been applied. I need to calculate the angle $\theta_{\text{E}}$ in the above diagram. $\theta_{\text{E}} = 45° - \alpha$, $\alpha + \beta + \theta_{\text{D}} = 180°$, and $\alpha = \beta$.

**SIMPLIFY:** Since it is a reflection condition, the angle of incidence is equal to the angle of reflection. Thus, $2\alpha = 180° - \theta_{\text{D}} \Rightarrow \alpha = 90° - \theta_{\text{D}}/2$. $\theta_{\text{E}} = 45° - (90° - \theta_{\text{D}}/2) = \theta_{\text{D}}/2 - 45°$. Using impulse, $J = F\Delta t = \Delta p_y = m(v_{y\text{f}} - v_{y\text{i}})$, where $v_{y\text{i}} = 0$ and $v_{y\text{f}} = v_x \tan\theta_{\text{E}}$. $\Delta t = \dfrac{mv_x \tan\theta_{\text{E}}}{F} = \dfrac{mv_x \tan(\theta_{\text{D}}/2 - 45°)}{F}$.

**CALCULATE:** $m_{\text{E}} = 9.1 \cdot 10^{-31}$ kg, $\Delta t = \dfrac{(9.1 \cdot 10^{-31} \text{ kg})(1.00 \cdot 10^5 \text{ m/s})\tan(120°/2 - 45°)}{8.0 \cdot 10^{-13} \text{ N}} = 30.48$ fs

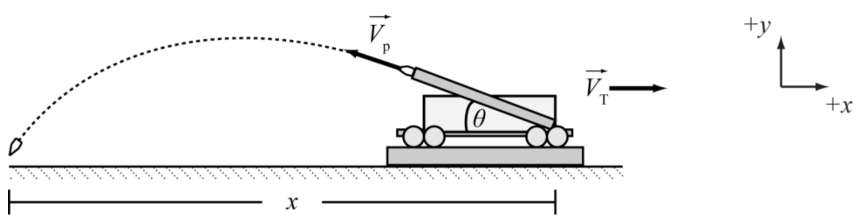


**ROUND:** Rounding to two significant figures: $\Delta t = 30.5$ fs.

**DOUBLE-CHECK:** As a comparison, compute the time taken for an electron with speed $v = 1.00 \cdot 10^5$ m/s to move a distance of 1 nm $= 10^9$ m: $t = 10^{-9} \text{ m}/1.00 \cdot 10^5 \text{ m/s} = 1.0 \cdot 10^{-14} \text{ s} = 10$ fs. The result $\Delta t = 30.$ fs is reasonable.

**7.37.** **THINK:** A projectile with mass 7502 kg is fired at an angle of $20.0°$. The total mass of the gun, mount and train car is $1.22 \cdot 10^6$ kg. The speed of the railway gun is initially zero and $v = 4.68$ m/s after finishing. I want to calculate the initial speed of the projectile and the distance it travels.

**SKETCH:**




**RESEARCH:** Use the conservation of momentum. $p_{x\text{i}} = p_{x\text{f}}$ and $p_{x\text{i}} = 0$, so $p_{x\text{f}} = 0$.

**SIMPLIFY:** $m_{\text{p}} v_{\text{p}} \cos\theta + m_{\text{T}} v_{\text{T}} = 0 \Rightarrow v_{\text{p}} = -\dfrac{m_{\text{T}} v_{\text{T}}}{m_{\text{p}} \cos\theta}$

$x = v_{\text{p}x} t$, where $t$ is twice the time it takes to reach the maximum height. $t_0 = v_{\text{p}y}/g$, and $t = 2t_0$.

$$x = v_{px}(2t_0) = v_{px}\left(\frac{2v_{py}}{g}\right) = \frac{2v_p^2 \sin\theta\cos\theta}{g} = \frac{v_p^2 \sin 2\theta}{g}$$

**CALCULATE:** $v_p = -\dfrac{(1.22\cdot 10^6 \text{ kg})(4.68 \text{ m/s})}{(7502 \text{ kg})\cos 20.0°} = -809.9 \text{ m/s}$

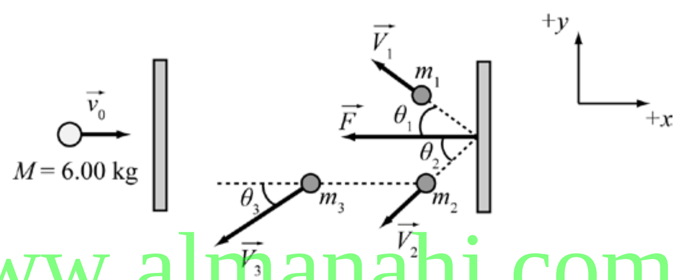
$$x = \frac{(-809.9 \text{ m/s})^2 \sin(2\cdot 20.0°)}{9.81 \text{ m/s}^2} = 42979 \text{ m}$$

**ROUND:** $v_p = -810. \text{ m/s}$ and $x = 43.0 \text{ km}$.

**DOUBLE-CHECK:** The documented muzzle velocity for Gustav was 820 m/s, and its maximum range was approximately 48 km.

**7.38.** **THINK:** A 6.00-kg clay ball collides with a wall and then shatters into three pieces with masses $m_1 = 2.00 \text{ kg}$, $m_2 = 1.00 \text{ kg}$ and $m_3 = 3.00 \text{ kg}$, and velocities $v_1 = 10.0 \text{ m/s}$ at an angle of $32.0°$ above the horizontal, $v_2 = 8.00 \text{ m/s}$ at an angle of $28.0°$ below the horizontal and $v_3$. I need to calculate the velocity of the third mass. The wall exerts a force on the ball of 2640 N for 0.100 s.

**SKETCH:**






**RESEARCH:** To solve this problem, use the definition of impulse, $\vec{J} = \vec{F}\Delta t = \Delta\vec{p}$, or, in component form, $F_x\Delta t = p_{xf} - p_{xi}$ and $p_{yi} = p_{yf}$ since $F_y = 0$.

**SIMPLIFY:** $-F\Delta t = -m_1v_1\cos\theta_1 - m_2v_2\cos\theta_2 - m_3v_{3x} - Mv_0$ and $0 = m_1v_1\sin\theta_1 + m_2v_2\sin\theta_2 + m_3v_{3y}$. Rearranging these expressions gives:

$$v_{3x} = \frac{F\Delta t - m_1v_1\cos\theta_1 - m_2v_2\cos\theta_2 - Mv_0}{m_3}, \quad \text{and} \quad v_{3y} = \frac{-m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2}{m_3}.$$

Use $v_3 = \sqrt{v_{3x}^2 + v_{3y}^2}$ and $\tan\theta_3 = v_{3y}/v_{3x}$ to get the speed and the angle.

**CALCULATE:** $v_{3y} = \dfrac{-(2.00 \text{ kg})(10.0 \text{ m/s})\sin 32.0° - (1.00 \text{ kg})(8.00 \text{ m/s})\sin(-28.0°)}{3.00 \text{ kg}} = -2.281 \text{ m/s},$

$$v_{3x} = \frac{(2640 \text{ N})(0.100 \text{ s}) - (2.00 \text{ kg})(10.0 \text{ m/s})\cos 32.0° - (1.00 \text{ kg})(8.00 \text{ m/s})\cos 28.0° - (6.00 \text{ kg})(22.0 \text{ m/s})}{3.00 \text{ kg}}$$

$= 35.992 \text{ m/s},$

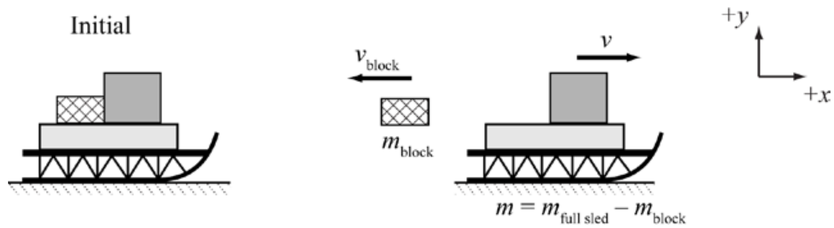
$v_3 = \sqrt{(35.992 \text{ m/s})^2 + (2.281 \text{ m/s})^2} = 36.064 \text{ m/s},$ and $\theta_3 = \tan^{-1}\left(\dfrac{-2.281 \text{ m/s}}{35.992 \text{ m/s}}\right) = -3.6263°.$

**ROUND:** Rounding to three significant figures: $v_3 = 36.0 \text{ m/s}$, $\theta_3 = 3.63°$ below the horizontal

**DOUBLE-CHECK:** The angle $\theta_3$ is expected to be negative or below the horizontal.

**7.39.** **THINK:** The mass of a sled and its contents is $m_{\text{full sled}} = 52.0 \text{ kg}$. A block of mass $m_{\text{block}} = 13.5 \text{ kg}$ is ejected to the left with velocity $v_{\text{block}} = -13.6 \text{ m/s}$. I need to calculate the speed of the sled and remaining contents.

**SKETCH:**




**RESEARCH:** Use the conservation of momentum. $p_{\text{i}} = p_{\text{f}}$, and $p_{\text{i}} = 0$ since the sled and its contents are initially at rest.

**SIMPLIFY:** $p_{\text{f}} = m_{\text{block}} v_{\text{block}} + mv = 0 \quad \Rightarrow \quad v = -\dfrac{m_{\text{block}} v_{\text{block}}}{m_{\text{full sled}} - m_{\text{block}}}$

**CALCULATE:** $v = -\dfrac{(13.5 \text{ kg})(-13.6 \text{ m/s})}{52.0 \text{ kg} - 13.5 \text{ kg}} = 4.7688 \text{ m/s}$

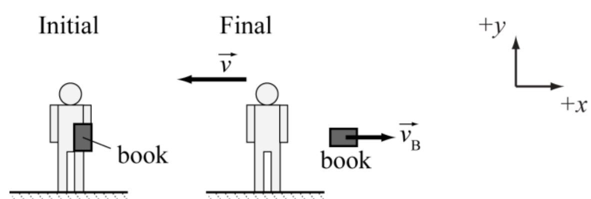
**ROUND:** $v = 4.77 \text{ m/s}$

**DOUBLE-CHECK:** Because the sled and its remaining contents have a mass larger than the mass of the block, it is expected that the speed of the sled and the remaining contents is less than the block's speed, i.e. $v < v_{\text{block}}$.

**7.40.** **THINK:** The mass of the book is $m_{\text{B}} = 5.00 \text{ kg}$ and the mass of the person is $m = 62.0 \text{ kg}$. Initially the book and the person are at rest, and then the person throws the book at 13.0 m/s. I need to calculate speed of the person on the ice after throwing the book.



**SKETCH:**




**RESEARCH:** We use conservation of momentum. $p_{\text{i}} = p_{\text{f}}$ and $p_{\text{i}} = 0$ since the speed is initially zero.

**SIMPLIFY:** $p_{\text{f}} = 0 = mv + m_{\text{B}} v_{\text{B}} \quad \Rightarrow \quad v = -\dfrac{m_{\text{B}} v_{\text{B}}}{m}$

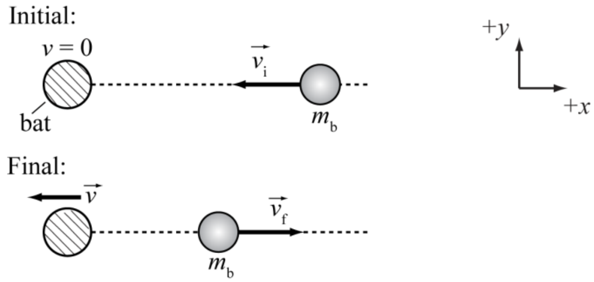
**CALCULATE:** $v = -\dfrac{(5.00 \text{ kg})(13.0 \text{ m/s})}{62.0 \text{ kg}} = -1.0484 \text{ m/s}$

**ROUND:** $v = -1.05 \text{ m/s}$

**DOUBLE-CHECK:** The direction of the person's motion should be in the direction opposite to the direction of the book.

**7.41.** **THINK:** The astronaut's mass is $m_A = 50.0$ kg and the baseball's mass is $m_b = 0.140$ kg. The baseball has an initial speed of 35.0 m/s and a final speed of 45.0 m/s.

**SKETCH:**




**RESEARCH:** Use the conservation of momentum. $p_i = p_f$.

**SIMPLIFY:** $p_i = p_f \Rightarrow m_b v_i + 0 = m_b v_f + m_A v_A \Rightarrow v_A = \dfrac{m_b(v_i - v_f)}{m_A}$

**CALCULATE:** $v_i = -35.0$ m/s, $v_f = 45.0$ m/s, and $v_A = \dfrac{(0.140 \text{ kg})(-35.0 \text{ m/s} - 45.0 \text{ m/s})}{50.0 \text{ kg}} = -0.224$ m/s.

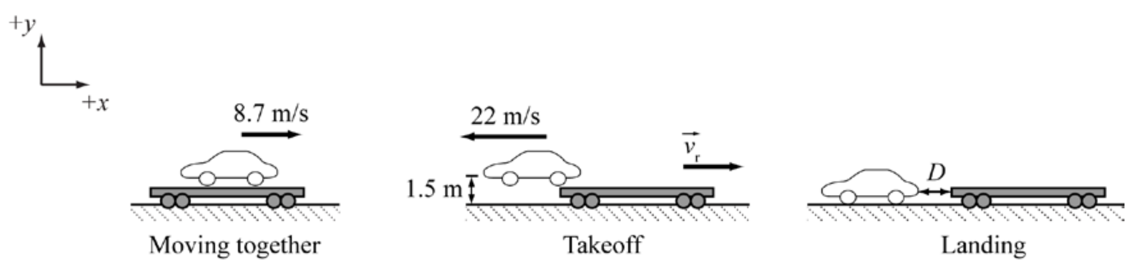
**ROUND:** Three significant figures: $v_A = -0.224$ m/s.

**DOUBLE-CHECK:** The magnitude of $v_A$ is proportional to $m_b/m_A$, which is about $10^{-3}$ so it would be expected to find the velocity of the astronaut as relatively small.

**7.42.** **THINK:** The mass of an automobile is $m_a = 1450$ kg and the mass of a railcar is $m_r = 38,500$ kg. Initially, both are moving at $v_i = +8.7$ m/s. The automobile leaves the railcar at a speed of $v_{af} = -22$ m/s. I need to determine the distance $D$ between the spot where it lands and the left end of the railcar. Call the $x$-component of the velocity of the railcar $v_r$ and that of the automobile $v_a$.



**SKETCH:**




**RESEARCH:** I need first to calculate the speed of the railcar just after the automobile leaves and then I need to find the amount of time it takes for the automobile to reach the ground. Conservation of momentum leads to the two equations $p_i = p_f \Rightarrow m_a v_{ai} + m_r v_{ri} = m_a v_{af} + m_r v_{rf}$ and $v_{ai} = v_{ri} = v_i \Rightarrow v_{rf} = \dfrac{(m_a + m_r)v_i - m_a v_{af}}{m_r}$. The final relative velocity between the automobile and the railcar is $\Delta v = v_{rf} - v_{af}$. The time to reach the ground is determined using $h = gt^2/2 \Rightarrow t = \sqrt{2h/g}$. The separation distance is the product of time and the relative velocity, $D = t\Delta v$.

**SIMPLIFY:** Insert the expression for the time and the relative velocity into the distance equation and obtain:

$$D = t\Delta v = \sqrt{2h/g}\,(v_{\text{rf}} - v_{\text{af}})$$
$$= \sqrt{2h/g}\left(\frac{(m_{\text{a}} + m_{\text{r}})v_{\text{i}} - m_{\text{a}}v_{\text{af}}}{m_{\text{r}}} - v_{\text{af}}\right)$$
$$= \sqrt{2h/g}\left(\frac{m_{\text{a}} + m_{\text{r}}}{m_{\text{r}}}\right)(v_{\text{i}} - v_{\text{af}})$$

**CALCULATE:** $D = \sqrt{2(1.5\text{ m})/(9.81\text{ m/s}^2)}\left(\frac{1450\text{ kg} + 38{,}500\text{ kg}}{38{,}500\text{ kg}}\right)(8.7\text{ m/s} + 22\text{ m/s}) = 17.6165\text{ m}$

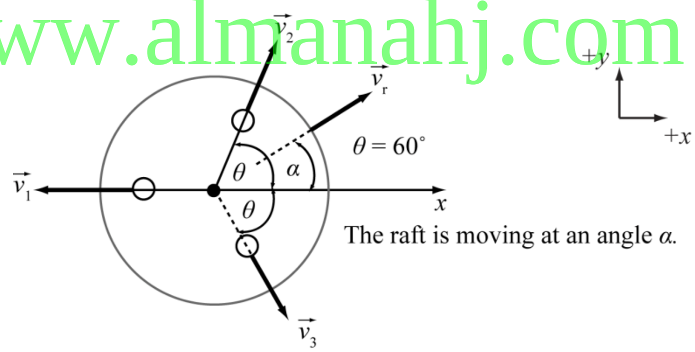
**ROUND:** Rounding to three significant figures, $D = 17.6$ m.

**DOUBLE-CHECK:** The value of $D$ is reasonable. If the mass of the automobile relative to that of the railcar is neglected, then the railcar's velocity is not changed due to the recoil from the car, and the relative velocity between the two is simply $(8.7 + 22)$ m/s = 30.7 m/s. If something moves at a speed of 30.7 m/s for $\sqrt{2(1.5\text{ m})/(9.81\text{ m/s}^2)}$ = 0.553 s, then it moves a distance of 16.977 m. The actual answer is close to this estimate. The actual answer has to be slightly bigger than this estimate because the railcar receives a small velocity boost forward due to the car jumping off in the backwards direction.

**7.43.** **THINK:** The raft is given to be of mass $m_{\text{r}} = 120.$ kg and the three people of masses $m_1 = 62.0$ kg, $m_2 = 73.0$ kg and $m_3 = 55.0$ kg have speeds $v_1 = 12.0$ m/s, $v_2 = 8.00$ m/s, and $v_3 = 11.0$ m/s. I need to calculate the speed of the raft.

**SKETCH:**






**RESEARCH:** Because of the conservation of momentum, $\vec{p}_{\text{i}} = \vec{p}_{\text{f}}$, or $p_{x\text{i}} = p_{x\text{f}}$ and $p_{y\text{i}} = p_{y\text{f}}$. The question provides the information that $\vec{p}_{\text{i}} = 0$, i.e. $p_{x\text{i}} = 0$ and $p_{y\text{i}} = 0$.

**SIMPLIFY:** $v_{\text{r}} = \sqrt{v_{\text{r}x}^2 + v_{\text{r}y}^2}$.

$$p_{x\text{f}} = 0 \;\Rightarrow\; -m_1v_1 + m_2v_2\cos\theta + m_3v_3\cos\theta + m_{\text{r}}v_{\text{r}x} = 0 \;\Rightarrow\; v_{\text{r}x} = \frac{m_1v_1 - m_2v_2\cos\theta - m_3v_3\cos\theta}{m_{\text{r}}}.$$

$$p_{y\text{f}} = 0 \;\Rightarrow\; 0 + m_2v_2\sin\theta - m_3v_3\sin\theta + m_{\text{r}}v_{\text{r}y} = 0 \;\Rightarrow\; v_{\text{r}y} = \frac{-m_2v_2\sin\theta + m_3v_3\sin\theta}{m_{\text{r}}}.$$

**CALCULATE:**

$$v_{\text{r}x} = \frac{(62.0\text{ kg})(12.0\text{ m/s}) - (73.0\text{ kg})(8.00\text{ m/s})\cos 60.0° - (55.0\text{ kg})(11.0\text{ m/s})\cos 60.0°}{120.\text{ kg}} = 1.2458\text{ m/s}$$

$$v_{\text{r}y} = \frac{-(73.0\text{ kg})(8.00\text{ m/s})\sin 60.0° + (55.0\text{ kg})(11.0\text{ m/s})\sin 60.0°}{120.\text{ kg}} = 0.1516\text{ m/s}$$

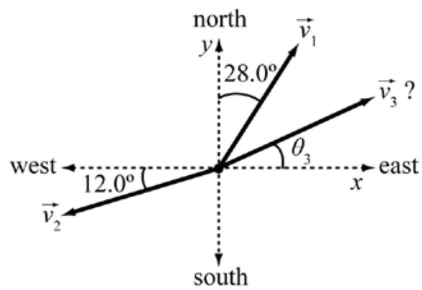
$v_r = \sqrt{(1.2458 \text{ m/s})^2 + (0.1516 \text{ m/s})^2} = 1.2550 \text{ m/s}$

**ROUND:** $v_r = 1.26$ m/s

**DOUBLE-CHECK:** Due to the large mass of the raft, $v_r$ is expected to be small, and it is smaller than 8.00 m/s.

**7.44.** **THINK:** A missile that breaks into three pieces of equal mass $m_1 = m_2 = m_3 = m$. The first piece has a speed of 30.0 m/s in the direction $28.0°$ east of north. The second piece has a speed of 8.00 m/s and is in the direction $12.0°$ south of west. I want to calculate the speed and direction of the third piece.

**SKETCH:**




**RESEARCH:** Use the conservation of momentum. $\vec{p}_i = \vec{p}_f$, and in component form $p_{xi} = p_{xf}$ and $p_{yi} = p_{yf}$. Also, $\vec{p}_i = 0$.

**SIMPLIFY:** $v_3 = \sqrt{v_{3x}^2 + v_{3y}^2}$

$p_{xi} = 0 = p_{xf}$, $mv_1 \sin\theta_1 - mv_2 \cos\theta_2 + mv_{3x} = 0 \Rightarrow v_{3x} = -v_1 \sin\theta_1 + v_2 \cos\theta_2$

$p_{yi} = 0 = p_{yf}$, $mv_1 \cos\theta_1 - mv_2 \sin\theta_2 + mv_{3y} = 0 \Rightarrow v_{3y} = -v_1 \cos\theta_1 + v_2 \sin\theta_2$



**CALCULATE:** $v_{3x} = -(30.0 \text{ m/s})\sin 28.0° + (8.00 \text{ m/s})\cos 12.0° = -6.26$ m/s,

$v_{3y} = -(30.0 \text{ m/s})\cos 28.0° + (8.00 \text{ m/s})\sin 12.0° = -24.83$ m/s,

$v = \sqrt{(-6.26 \text{ m/s})^2 + (-24.83 \text{ m/s})^2} = 25.61$ m/s.

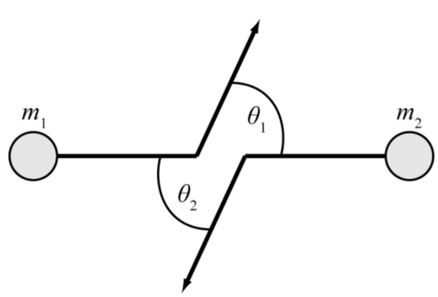
$\theta = \tan^{-1}\left(\dfrac{-24.83 \text{ m/s}}{-6.26 \text{ m/s}}\right) = 75.8498°$. Because both $v_{3x}$ and $v_{3y}$ are negative, we need to add $180°$ to the angle. Thus $\theta = 180° + 75.85° = 255.85°$.

**ROUND:** Round to three significant figures: $v = 25.6$ m/s, $\theta = 256°$, or $75.8°$ south of west.

**DOUBLE-CHECK:** Since $v_1$ is much larger than $v_2$, $v_3$ is roughly the same speed as $v_1$ but in the opposite direction.

**7.45.** **THINK:** A soccer ball and a basketball have masses $m_1 = 0.400$ kg and $m_2 = 0.600$ kg respectively. The soccer ball has an initial energy of 100. J and the basketball 112 J. After collision, the second ball flew off at an angle of $32.0°$ with 95.0 J of energy. I need to calculate the speed and angle of the first ball. Let subscript 1 denote the soccer ball, and subscript 2 denote the basketball.

**SKETCH:**



**RESEARCH:** I need to calculate the speed of the balls using $1/2mv^2 = K$, or $v = \sqrt{2K/m}$, and then apply the conservation of momentum to get $p_{xi} = p_{xf}$ and $p_{yi} = p_{yf}$. I also use $p_{yi} = 0$.

**SIMPLIFY:** $p_{xi} = p_{xf} \Rightarrow m_1v_{1i} - m_2v_{2i} = m_1v_{1f}\cos\theta_1 - m_2v_{2f}\cos\theta_2$,

$p_{yi} = p_{yf} = 0 \Rightarrow m_1v_{1f}\sin\theta_1 - m_2v_{2f}\sin\theta_2 = 0$,

$v_{1x} = v_{1f}\cos\theta_1$ and $v_{1y} = v_{1f}\sin\theta_1 \Rightarrow v_{1x} = \dfrac{m_1v_{1i} - m_2v_{2i} + m_2v_{2f}\cos\theta_2}{m_1}$ and $v_{1y} = \dfrac{m_2v_{2f}\sin\theta_2}{m_1}$,

$v_{1i} = \sqrt{\dfrac{2K_{1i}}{m_1}}$, $v_{2i} = \sqrt{\dfrac{2K_{2i}}{m_2}}$, $v_{2f} = \sqrt{\dfrac{2K_{2f}}{m_2}}$, and $v_{1f} = \sqrt{v_{1x}^2 + v_{1y}^2}$.

**CALCULATE:** $v_{1i} = \sqrt{\dfrac{2(100.\text{ J})}{0.400\text{ kg}}} = 22.36\text{ m/s}$, $v_{2i} = \sqrt{\dfrac{2(112\text{ J})}{0.600\text{ kg}}} = 19.32\text{ m/s}$, $v_{2f} = \sqrt{\dfrac{2(95.0\text{ J})}{0.600\text{ kg}}} = 17.80\text{ m/s}$,

$$v_{1x} = \frac{(0.400\text{ kg})(22.36\text{ m/s}) - (0.600\text{ kg})(19.32\text{ m/s}) + (0.600\text{ kg})(17.80\text{ m/s})\cos 32.0°}{0.400\text{ kg}} = 16.02\text{ m/s},$$

$v_{1y} = \dfrac{(0.600\text{ kg})(17.80\text{ m/s})\sin 32.0°}{0.400\text{ kg}} = 14.15\text{ m/s}$, $v_{1f} = \sqrt{(16.02\text{ m/s})^2 + (14.15\text{ m/s})^2} = 21.37\text{ m/s}$, and

$\theta_1 = \tan^{-1}\left(\dfrac{14.15\text{ m/s}}{16.02\text{ m/s}}\right) = 41.5°$.

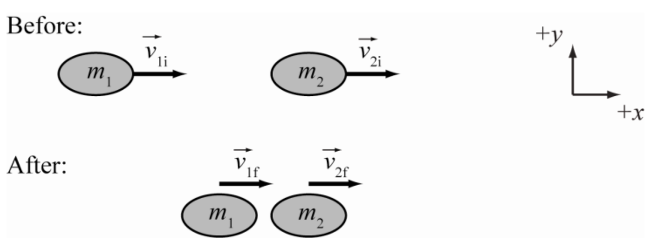
**ROUND:** $v_{1f} = 21.4\text{ m/s}$ and $\theta_1 = 41.5°$.

**DOUBLE-CHECK:** The results for speed and angle are comparable to $v_2$ and $\theta_2$, which is expected. From energy conservation (assuming elastic collision), the energy is $E_{1f} = E_{1i} + E_{2i} - E_{2f}$ = 100. J + 112 J – 95.0 J = 117 J, which corresponds to a speed of 24.2 m/s for $v_{1f}$. The result $v_{1f} = 21.4$ m/s is less than this because the energy is not conserved in this case.



**7.46.** **THINK:** Two bumper cars have masses $m_1 = 188$ kg and $m_2 = 143$ kg and speeds $v_1 = 20.4$ m/s and $v_2 = 9.00$ m/s respectively. I want to calculate $v_1$ after the elastic collision.

**SKETCH:**




**RESEARCH:** Use the conservation of momentum and the conservation of energy. $p_i = p_f$ and $E_i = E_f$.

**SIMPLIFY:** $p_i = p_f \Rightarrow m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \Rightarrow m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$ (1)

$$E_i = E_f$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_iv_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$

$$m_1(v_{1i} + v_{1f})(v_{1i} - v_{1f}) = m_2(v_{2f} + v_{2i})(v_{2f} - v_{2i})$$

$$v_{1i} + v_{1f} = \frac{m_2(v_{2f} - v_{2i})}{m_1(v_{1i} - v_{1f})}(v_{2f} + v_{2i})$$

Using (1) above, $v_{1i} + v_{1f} = v_{2f} + v_{2i} \Rightarrow v_{2f} = v_{1i} + v_{1f} - v_{2i}$. Substituting back into the equation of conservation of momentum,

$$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2\left(v_{1i}+v_{1f}-v_{2i}\right)$$
$$m_1v_{1i}+m_2v_{2i}=\left(m_1+m_2\right)v_{1f}+m_2v_{1i}-m_2v_{2i}$$
$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_{1i}+\frac{2m_2}{m_1+m_2}v_{2i}$$

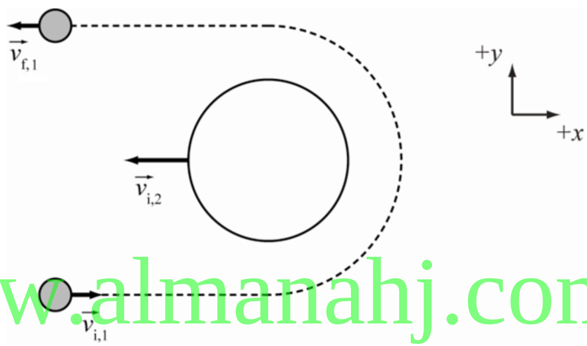
**CALCULATE:** $v_{1f}=\left(\frac{188\text{ kg}-143\text{ kg}}{188\text{ kg}+143\text{ kg}}\right)(20.4\text{ m/s})+\left(\frac{2\cdot 143\text{ kg}}{188\text{ kg}+143\text{ kg}}\right)(9.00\text{ m/s})=10.55\text{ m/s}$

**ROUND:** Rounding to three significant figures: $v_{1f}=10.6\text{ m/s}$

**DOUBLE-CHECK:** It is expected that some of the kinetic energy of $m_1$ is transferred to $m_2$. As a result, $v_{1f}$ is smaller than $v_{2i}$. Since $m_1>m_2$, $v_{1f}$ should be smaller than $v_{2i}$.

**7.47.** **THINK:** The mass of the satellite is $m_1=274\text{ kg}$ and its initial speed is $v_{i,1}=13.5\text{ km/s}$ The initial speed of the planet is $v_{i,2}=-10.5\text{ km/s}$. I want to calculate the speed of the satellite after collision. It is assumed that the mass of the planet is much larger than the mass of the satellite, i.e. $m_2\gg m_1$.

**SKETCH:**






**RESEARCH:** Use the conservation of energy and the conservation of momentum; $E_i=E_f$ and $p_i=p_f$.

**SIMPLIFY:** $p_i=p_f \;\Rightarrow\; m_1v_{i,1}+m_2v_{i,2}=m_1v_{f,1}+m_2v_{f,2} \;\Rightarrow\; m_1\left(v_{i,1}-v_{f,1}\right)=m_2\left(v_{f,2}-v_{i,2}\right)$ (1)

$$E_i=E_f$$
$$\frac{1}{2}m_1v_{i,1}^2+\frac{1}{2}m_2v_{i,2}^2=\frac{1}{2}m_1v_{f,1}^2+\frac{1}{2}m_2v_{f,2}^2$$
$$m_1\left(v_{i,1}^2-v_{f,1}^2\right)=m_2\left(v_{f,2}^2-v_{i,2}^2\right)$$
$$m_1\left(v_{i,1}+v_{f,1}\right)\left(v_{i,1}-v_{f,1}\right)=m_2\left(v_{f,2}+v_{i,2}\right)\left(v_{f,2}-v_{i,2}\right)$$
$$v_{i,1}+v_{f,1}=\frac{m_2\left(v_{f,2}-v_{i,2}\right)}{m_1\left(v_{i,1}-v_{f,1}\right)}\left(v_{f,2}+v_{i,2}\right)$$

Using (1), $v_{i,1}+v_{f,1}=v_{f,2}+v_{i,2} \;\Rightarrow\; v_{f,2}=v_{i,1}+v_{f,1}-v_{i,2}$. Substituting back into the conservation of momentum equation above,

$$m_1v_{i,1}+m_2v_{i,2}=m_1v_{f,1}+m_2\left(v_{i,1}+v_{f,1}-v_{i,2}\right)$$
$$m_1v_{i,1}+m_2v_{i,2}=\left(m_1+m_2\right)v_{f,1}+m_2v_{i,1}-m_2v_{i,2}$$
$$v_{f,1}=\frac{m_1-m_2}{m_1+m_2}v_{i,1}+\frac{2m_2}{m_1+m_2}v_{i,2}$$

Using the fact that $m_2\gg m_1$, $\left(m_1-m_2\right)/\left(m_1+m_2\right)\approx -1$ and $2m_2/\left(m_1+m_2\right)\approx 2$. Therefore, $v_{f,1}\approx -v_{i,1}+2v_{i,2}$.

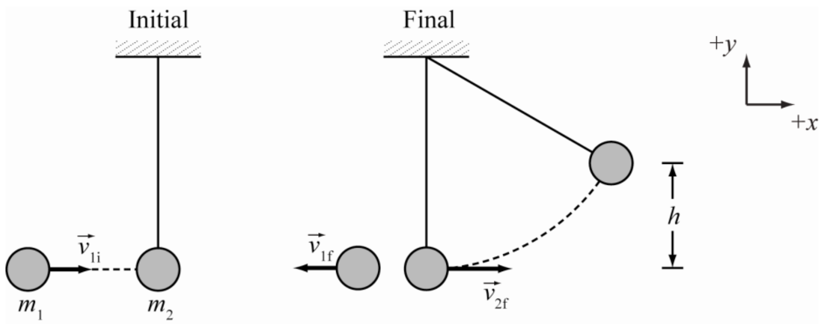
**CALCULATE:** $v_{i,1}=13.5\text{ km/s}$, $v_{i,2}=-10.5\text{ km/s}$, $v_{f,1}=-13.5\text{ km/s}+2\left(-10.5\text{ km/s}\right)=-34.5\text{ km/s}$

**ROUND:** $v_{\text{f},1} = -34.5 \text{ km/s}$

**DOUBLE-CHECK:** The result makes sense. $v_{\text{f},1}$ should be negative since it is in the opposite direction.

**7.48.** **THINK:** A stone has mass of $m_1 = 0.250$ kg. The mass of one of the shoes is $m_2 = 0.370$ kg. I need to calculate the speed of the shoe after collision, and then the height of the shoe.

**SKETCH:**




**RESEARCH:** Use the conservation of momentum and energy, $p_\text{i} = p_\text{f}$ and $E_\text{i} = E_\text{f}$, as well as $K_2 = mgh$.

**SIMPLIFY:** $p_\text{f} = p_\text{i} \;\Rightarrow\; m_1 v_{1\text{i}} + 0 = m_1 v_{1\text{f}} + m_2 v_{2\text{f}} \;\Rightarrow\; m_1\left(v_{1\text{i}} - v_{1\text{f}}\right) = m_2 v_{2\text{f}}$, and

$m_1\left(v_{1\text{i}} - v_{1\text{f}}\right)\left(v_{1\text{i}} + v_{1\text{f}}\right) = m_2 v_{2\text{f}}^2 \;\Rightarrow\; m_2 v_{2\text{f}}\left(v_{1\text{i}} + v_{1\text{f}}\right) = m_2 v_{2\text{f}}^2 \;\Rightarrow\; v_{1\text{i}} + v_{1\text{f}} = v_{2\text{f}} \;\Rightarrow\; v_{1\text{f}} = v_{2\text{f}} - v_{1\text{i}}$.

Substituting back into the conservation of momentum equation,

$$m_1 v_{1\text{i}} = m_1\left(v_{2\text{f}} - v_{1\text{i}}\right) + m_2 v_{2\text{f}} \;\Rightarrow\; \left(m_1 + m_1\right)v_{1\text{i}} = \left(m_1 + m_2\right)v_{2\text{f}} \;\Rightarrow\; v_{2\text{f}} = \frac{2m_1}{m_1 + m_2}v_{1\text{i}}$$

Using $K = \dfrac{1}{2}mv_{2\text{f}}^2 = mgh,\; h = \dfrac{v_{2\text{f}}^2}{2g} \Rightarrow \left(\dfrac{2m_1 v_{1\text{i}}}{m_1 + m_2}\right)^2 \dfrac{1}{2g}$.



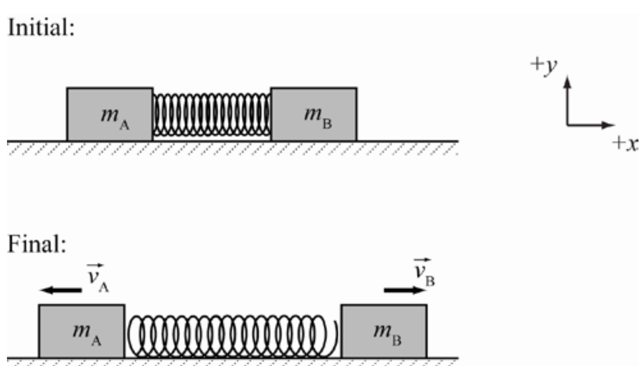
**CALCULATE:** $h = \left(\dfrac{2(0.250 \text{ kg})(2.30 \text{ m/s})}{0.250 \text{ kg} + 0.370 \text{ kg}}\right)^2 \left(\dfrac{1}{2\left(9.81 \text{ m/s}^2\right)}\right) = 0.1754 \text{ m}$

**ROUND:** Rounding to three significant figures, $h = 0.175$ m.

**DOUBLE-CHECK:** This is a reasonable height. As a comparison, the length of a shoelace is between about 0.5 m and 1.8 m.

**7.49.** **THINK:** Two blocks with a spring between them sit on an essentially frictionless surface. The spring constant is $k$ = 2500. N/m. The spring is compressed such that $\Delta x = 3.00 \text{ cm} = 3.00 \cdot 10^{-2}$ m. I need to calculate the speeds of the two blocks. $m_\text{A} = 1.00$ kg, and $m_\text{B} = 3.00$ kg.

**SKETCH:**




**RESEARCH:** I use the conservation of momentum and the conservation of energy. Thus $p_\text{i} = p_\text{f}$, and $E_\text{i} = E_\text{f}$. I also know that $p_i = 0$ and $E_\text{i} = E_\text{s} = (1/2)k\Delta x^2$.

**SIMPLIFY:** $p_{\text{i}} = p_{\text{f}} = 0 \Rightarrow m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} = 0 \Rightarrow m_{\text{A}}v_{\text{A}} = -m_{\text{B}}v_{\text{B}}$

$$E_{\text{i}} = E_{\text{f}} \Rightarrow \frac{1}{2}k\Delta x^2 = \frac{1}{2}m_{\text{A}}v_{\text{A}}^2 + \frac{1}{2}m_{\text{B}}v_{\text{B}}^2 \Rightarrow k\Delta x^2 = m_{\text{A}}\left(-\frac{m_{\text{B}}}{m_{\text{A}}}v_{\text{B}}\right)^2 + m_{\text{B}}v_{\text{B}}^2 \Rightarrow k\Delta x^2 = \frac{m_{\text{B}}^2}{m_{\text{A}}}v_{\text{B}}^2 + m_{\text{B}}v_{\text{B}}^2$$

Simplifying further gives:

$$\left(\frac{m_{\text{B}}^2}{m_{\text{A}}} + m_{\text{B}}\right)v_{\text{B}}^2 = k\Delta x^2 \Rightarrow v_{\text{B}} = \sqrt{\frac{k\Delta x^2}{\frac{m_{\text{B}}^2}{m_{\text{A}}} + m_{\text{B}}}} = \sqrt{\frac{k\Delta x^2}{m_{\text{B}}\left(1 + \frac{m_{\text{B}}}{m_{\text{A}}}\right)}} \quad \text{and} \quad v_{\text{A}} = -\frac{m_{\text{B}}}{m_{\text{A}}}v_{\text{B}}.$$

**CALCULATE:** $v_{\text{B}} = \sqrt{\dfrac{(2500.\text{ N/m})(3.00\cdot 10^{-2}\text{ m})^2}{(3.00\text{ kg})\left(1 + \dfrac{3.00\text{ kg}}{1.00\text{ kg}}\right)}} = 0.4330\text{ m/s}$

$$v_{\text{A}} = -\frac{3.00\text{ kg}}{1.00\text{ kg}}(0.4330\text{ m/s}) = -1.299\text{ m/s}$$

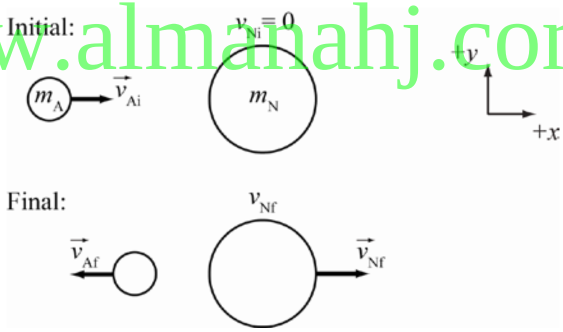
**ROUND:** $v_{\text{B}} = 0.433\text{ m/s}$ and $v_{\text{A}} = -1.30\text{ m/s}$.

**DOUBLE-CHECK:** The speed of block A should be larger than the speed of block B since $m_{\text{A}}$ is less than $m_{\text{B}}$.

**7.50.** **THINK:** An alpha particle has mass $m_{\text{A}} = 4.00\text{ u}$ and speed $v_{\text{Ai}}$, and a nucleus has mass $m_{\text{N}} = 166\text{ u}$ and is at rest. Conservation of momentum and energy can be used to calculate the kinetic energy of the nucleus after the elastic collision.

**SKETCH:**






**RESEARCH:** Conservation of momentum and energy are: $p_{\text{i}} = p_{\text{f}}$ and $E_{\text{i}} = E_{\text{f}}$.

**SIMPLIFY:** Conservation of momentum gives

$$p_{\text{i}} = p_{\text{f}} \Rightarrow m_{\text{A}}v_{\text{Ai}} + 0 = m_{\text{A}}v_{\text{Af}} + m_{\text{N}}v_{\text{Nf}} \Rightarrow m_{\text{A}}(v_{\text{Ai}} - v_{\text{Af}}) = m_{\text{N}}v_{\text{Nf}}$$

Conservation of energy gives:

$$E_{\text{i}} = E_{\text{f}}$$

$$\frac{1}{2}m_{\text{A}}v_{\text{Ai}}^2 + 0 = \frac{1}{2}m_{\text{A}}v_{\text{Af}}^2 + \frac{1}{2}m_{\text{N}}v_{\text{Nf}}^2$$

$$m_{\text{A}}(v_{\text{Ai}}^2 - v_{\text{Af}}^2) = m_{\text{N}}v_{\text{Nf}}^2$$

$$m_{\text{A}}(v_{\text{Ai}} - v_{\text{Af}})(v_{\text{Ai}} + v_{\text{Af}}) = m_{\text{N}}v_{\text{Nf}}^2$$

$$m_{\text{N}}v_{\text{Nf}}(v_{\text{Ai}} + v_{\text{Af}}) = m_{\text{N}}v_{\text{Nf}}^2$$

$$v_{\text{Ai}} + v_{\text{Af}} = v_{\text{Nf}}$$

$$v_{\text{Af}} = v_{\text{Nf}} - v_{\text{Ai}}$$

Substituting this back into the equation of conservation of momentum gives:

$$m_A\left(v_{Ai}-\left(v_{Nf}-v_{Ai}\right)\right)=m_N v_{Nf}$$
$$-m_A v_{Nf}+2m_A v_{Ai}=m_N v_{Nf}$$
$$m_N v_{Nf}+m_A v_{Nf}=2m_A v_{Ai}$$
$$v_{Nf}=\frac{2m_A}{m_A+m_N}v_{Ai}$$

The kinetic energy of the nucleus is:

$$K_N=\frac{1}{2}m_N v_{Nf}^2=\frac{1}{2}m_N\left(\frac{2m_A}{m_A+m_N}\right)^2 v_{Ai}^2=\frac{4m_A m_N}{\left(m_A+m_N\right)^2}\left(\frac{1}{2}m_A v_{Ai}^2\right)=\frac{4m_A m_N}{\left(m_A+m_N\right)^2}K_A,$$

which gives

$$\frac{K_N}{K_A}=\frac{4m_A m_N}{\left(m_A+m_N\right)^2}.$$

**CALCULATE:** $\frac{K_N}{K_A}=\frac{4(4.00\text{ u})(166\text{ u})}{(4.00\text{ u}+166\text{ u})^2}=0.09190=9.190\%$

**ROUND:** To three significant figures, $\frac{K_N}{K_A}=9.19\ \%$.

**DOUBLE-CHECK:** To check the equation, set the masses equal:

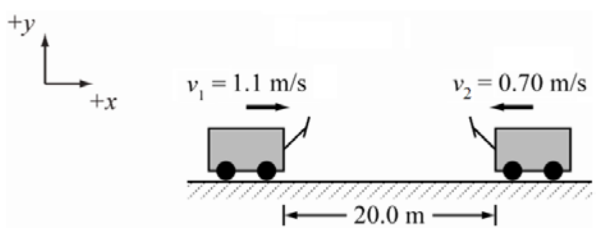
$$\frac{K_2}{K_1}=\frac{4m_1m_2}{\left(m_1+m_2\right)^2}=\frac{4m^2}{(2m)^2}=\frac{4m^2}{4m^2}=1.$$

This means that all of energy is transferred, which is expected for two equal masses (i.e. billiard balls). This confirms that the derived equation is correct. Here, since the mass of the nucleus is much larger than the mass of the alpha particle, it is reasonable that the ratio is small.



**7.51.** **THINK:** Two carts, separated by a distance $x_0=20.0$ m, are travelling towards each other with speeds $v_1=1.10$ m/s and $v_2=0.700$ m/s. They collide for $\Delta t=0.200$ s. This is an elastic collision. I need to plot *x* vs. *t*, *v* vs. *t* and *F* vs. *t*.

**SKETCH:**




**RESEARCH:** Use the conservation of momentum and energy to get the speeds after collision. Then use the impulse $\vec{J}=\vec{F}\Delta t=\Delta\vec{p}$ to get the force.

**SIMPLIFY:** First, need the position of the collision. Using $x=x_0+v_0t \Rightarrow x_1=0+v_1t$ and $x_2=x_0-v_2t$, $x_1=x_2=v_1t=x_0-v_2t \Rightarrow t=x_0/\left(v_1+v_2\right)$. Conservation of momentum:

$$p_i=p_f \Rightarrow m_1v_{i1}+m_2v_{i2}=m_1v_{f1}+m_2v_{f2}$$

$m_1=m_2=m$ because they are both the same type of cart. Then

$$v_{i1}-v_{f1}=v_{f2}-v_{i2}. \qquad (1)$$

$$K_i = K_f$$
$$\frac{1}{2}m_1v_{i1}^2 + \frac{1}{2}m_2v_{i2}^2 = \frac{1}{2}m_1v_{f1}^2 + \frac{1}{2}m_2v_{f2}^2$$
$$v_{i1}^2 - v_{f1}^2 = v_{f2}^2 - v_{i2}^2$$
$$(v_{i1} - v_{f1})(v_{i1} + v_{f1}) = (v_{f2} - v_{i2})(v_{f2} + v_{i2})$$
$$v_{i1} + v_{f1} = v_{f2} + v_{i2}$$
$$v_{f2} = v_{i1} + v_{f1} - v_{i2}$$

Substituting back into (1):

$$v_{i1} - v_{f1} = v_{i1} + v_{f1} - v_{i2} - v_{i2} \Rightarrow 2v_{f1} = 2v_{i2} \Rightarrow v_{f1} = v_{i2} \text{ and } v_{f2} = v_{i1}.$$

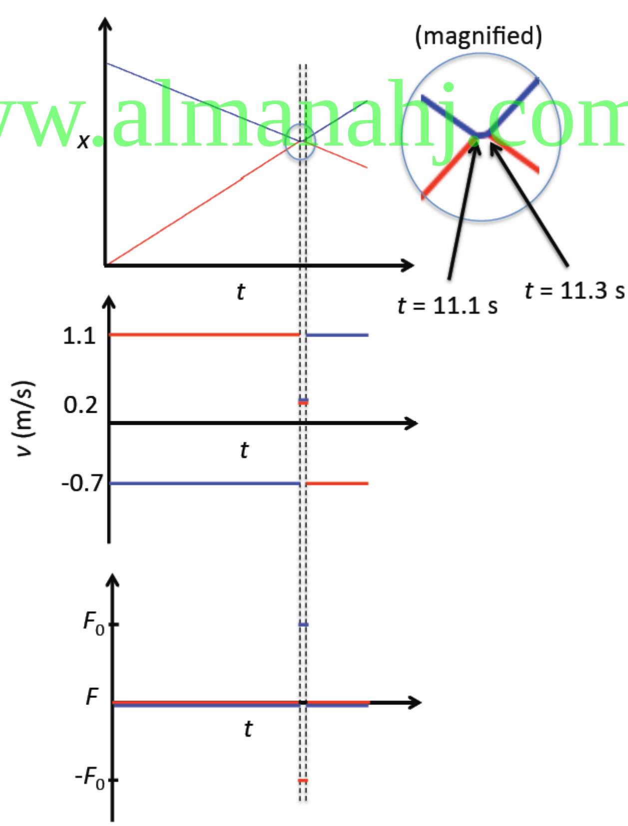
The change of momentum is $\Delta p_2 = m(v_{f2} - v_{i2}) = m(v_{i1} - v_{i2})$. The force on the other cart is

$$F_2\Delta t = \Delta p_2 \Rightarrow F_2 = \frac{\Delta p_2}{\Delta t} = \frac{m(v_{i1} - v_{i2})}{\Delta t}.$$

The force on your car is equal and opposite.

**CALCULATE:** The time for the collision to occur is $t = \frac{20.0 \text{ m}}{0.700 \text{ m/s} + 1.10 \text{ m/s}} = 11.11 \text{ s}$ and during this time the other cart has moved $x = (0.700 \text{ m/s})(11.11 \text{ s}) = 7.78 \text{ m}$.

**ROUND:** For the two calculations shown above three significant figures are required: $t = 11.1$ s and $x = 7.78$ m.

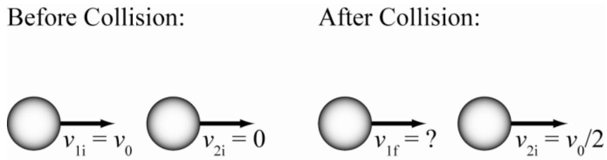





**DOUBLE-CHECK:** Since $m_1 = m_2$ it makes sense that $v_{f1} = v_{i2}$ and $v_{f2} = v_{i1}$. This means that energy is transferred completely from one to the other.

**7.52.** **THINK:** There are two balls with masses $m_1 = 0.280$ kg and $m_2$. The initial speeds are $v_{1\text{i}} = v_0$ and $v_{2i} = 0$. After the collision, the speeds are $v_{1\text{f}}$ and $v_{2\text{f}} = (1/2)v_0$. I want to calculate the mass of the second ball. This is an elastic collision.

**SKETCH:**




**RESEARCH:** I use the conservation of momentum and energy. $p_\text{i} = p_\text{f}$ and $E_\text{i} = E_\text{f}$.

**SIMPLIFY:**

(a) $p_\text{i} = p_\text{f} \Rightarrow m_1 v_{1\text{i}} + m_2 v_{2\text{i}} = m_1 v_{1\text{f}} + m_2 v_{2\text{f}} \Rightarrow m_1 v_{1\text{i}} + 0 = m_1 v_{1\text{f}} + m_2 v_{2\text{f}} \Rightarrow m_2 v_{2\text{f}} = m_1\left(v_{1\text{i}} - v_{1\text{f}}\right)$ (1)

$$E_\text{i} = E_\text{f}$$
$$\frac{1}{2}m_1 v_{1\text{i}}^2 + 0 = \frac{1}{2}m_1 v_{1\text{f}}^2 + \frac{1}{2}m_2 v_{2\text{f}}^2$$
$$m_1\left(v_{1\text{i}}^2 - v_{1\text{f}}^2\right) = m_2 v_{2\text{f}}^2$$
$$m_1\left(v_{1\text{i}} - v_{1\text{f}}\right)\left(v_{1\text{i}} + v_{1\text{f}}\right) = m_2 v_{2\text{f}}^2$$
$$v_{1\text{i}} + v_{1\text{f}} = v_{2\text{f}}$$
$$v_{1\text{f}} = v_{2\text{f}} - v_{1\text{i}}$$

Substituting back into (1), I have:

$$m_2 v_{2\text{f}} = m_1\left(v_{1\text{i}} - \left(v_{2\text{f}} - v_{1\text{i}}\right)\right) = m_1\left(2v_{1i} - v_{2f}\right) \Rightarrow m_2 = m_1\frac{2v_{1i} - v_{2f}}{v_{2f}} = m_1\left(2\frac{v_{1i}}{v_{2f}} - 1\right)$$



(b) The fraction of kinetic energy is $f = \dfrac{\Delta K}{K} = \dfrac{(1/2)m_2 v_{2\text{f}}^2}{(1/2)m_1 v_{1\text{i}}^2}$.

**CALCULATE:**

(a) $m_2 = (0.280\text{ kg})\left(2(2) - 1\right) = 3(0.280\text{ kg}) = 0.840$ kg

(b) Using $m_2 = 3m_1$ and $v_{2\text{f}} = v_{1\text{i}}/2$, $f = \dfrac{3m_1\left(v_{1\text{i}}/2\right)^2}{m_1 v_{1\text{i}}^2} = \dfrac{3}{4}$.

**ROUND:**

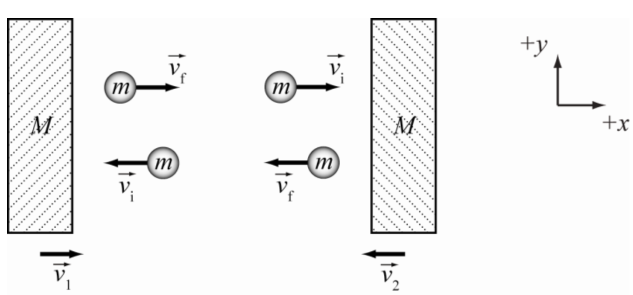
(a) $m_2 = 0.840$ kg

(b) $f = 3/4$

**DOUBLE-CHECK:** Because $v_{2\text{f}}$ is less than $v_{1\text{i}}$ it is expected that $m_2 > m_1$.

**7.53.** **THINK:** A particle has an initial velocity $v_\text{i} = -2.21\cdot 10^3$ m/s. I want to calculate the speed after 6 collisions with the left wall (which has a speed of $v_1 = 1.01\cdot 10^3$ m/s) and 5 collisions with the right wall (which has a speed of $v_2 = -2.51\cdot 10^3$ m/s). The magnetic walls can be treated as walls of mass M.

**SKETCH:**



**RESEARCH:** Consider one wall with speed $v_W$. Using the conservation of momentum and energy, $p_i = p_f$ and $E_i = E_f$.

**SIMPLIFY:** $p_i = p_f \Rightarrow mv_i + Mv_{Wi} = mv_f + Mv_{Wf}$

$$m(v_i - v_f) = M(v_{Wf} - v_{Wi}) \quad (1)$$

$$E_i = E_f$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}Mv_{Wi}^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}Mv_{Wf}^2$$

$$m(v_i^2 - v_f^2) = M(v_{Wf}^2 - v_{Wi}^2)$$

$$m(v_i - v_f)(v_i + v_f) = M(v_{Wf} - v_{Wi})(v_{Wf} + v_{Wi})$$

$$v_i + v_f = v_{Wf} + v_{Wi}$$

$$v_{Wf} = v_i + v_f - v_{Wi}$$

Substituting back into (1):

$$mv_i + Mv_{Wi} = mv_f + M(v_i + v_f - v_{Wi}) \Rightarrow v_f = \frac{m-M}{m+M}v_i + \frac{2M}{m+M}v_{Wi}$$

If $m \ll M$ then $K_{Sf} = 121$ J.. This means that every collision results in an additional speed of $2v_W$. So after 6 collisions with the left wall and 5 collisions with the right wall, I get $v_f = -v_i + 6(2v_1) - 5(2v_2)$.

**CALCULATE:** $v_i = -2.21 \cdot 10^3$ m/s, $v_1 = 1.01 \cdot 10^3$ m/s, and $v_2 = -2.51 \cdot 10^3$ m/s.

$v_f = 2.21 \cdot 10^3 \text{ m/s} + 12(1.01 \cdot 10^3 \text{ m/s}) - 10(-2.51 \cdot 10^3 \text{ m/s}) = 3.943 \cdot 10^4$ m/s

Since the last collision is with the left wall, the particle is moving to the right and the velocity is positive.

**ROUND:** $v_f = 3.94 \cdot 10^4$ m/s

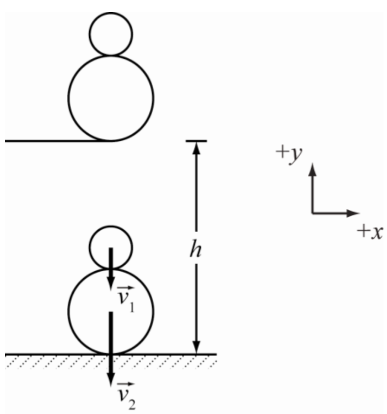
**DOUBLE-CHECK:** Since there have been 11 collisions, it is expected that the resulting speed is about 10 times the original speed.



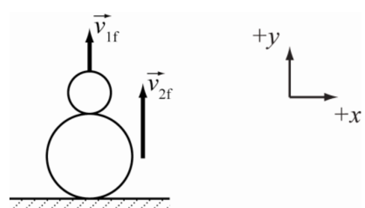
**7.54.** **THINK:** We have a golf ball with mass $m_1 = 0.0459$ kg and a basketball with mass $m_2 = 0.619$ kg. The balls are dropped from a height of 0.701 m.

**SKETCH:**

(a, b)




(c)




**RESEARCH:** Use the conservation of momentum and energy as well as $v^2 = 2gh \Rightarrow v = \sqrt{2gh}$.

**SIMPLIFY:**

(a) The momentum of the basketball is $p_2 = m_2v_2 = m_2\sqrt{2gh}$.

(b) The momentum of the golf ball is $p_1 = m_1v_1 = m_1\sqrt{2gh}$.

(c) The basketball collides with the floor first then collides with the golf ball. After the collision with the floor, the basketball's velocity is opposite the initial velocity. (See diagram (ii) above.) Using conservation of momentum, and conservation of energy: $p_i = p_f \Rightarrow m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ and therefore

$$m_1\left(v_{1i}-v_{1f}\right)=m_2\left(v_{2f}-v_{2i}\right) \quad (1)$$
$$E_i=E_f$$
$$\frac{1}{2}m_1v_{1i}^2+\frac{1}{2}m_2v_{2i}^2=\frac{1}{2}m_1v_{1f}^2+\frac{1}{2}m_2v_{2f}^2$$
$$m_1\left(v_{1i}^2-v_{1f}^2\right)=m_2\left(v_{2f}^2-v_{2i}^2\right)$$
$$m_1\left(v_{1i}-v_{1f}\right)\left(v_{1i}+v_{1f}\right)=m_2\left(v_{2f}-v_{2i}\right)\left(v_{2i}+v_{2f}\right)$$
$$v_{1i}+v_{1f}=v_{2f}+v_{2i}$$
$$v_{2f}=v_{1i}+v_{1f}-v_{2i}$$

Substituting back into (1), we have:

$$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2\left(v_{1i}+v_{1f}-v_{2i}\right) \Rightarrow v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_{1i}+\frac{2m_2}{m_1+m_2}v_{2i}$$

The speed of the golf ball is calculated using $v_{1i}=-v_1=-\sqrt{2gh}$ and $v_{2i}=v_2=\sqrt{2gh}$.

$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}\left(-\sqrt{2gh}\right)+\frac{2m_2}{m_1+m_2}\left(\sqrt{2gh}\right)=\frac{-m_1+m_2+2m_2}{m_1+m_2}\sqrt{2gh}=\frac{-m_1+3m_2}{m_1+m_2}\sqrt{2gh}$$

(d) The height is calculated using $v^2=2gh \Rightarrow h=v^2/(2g)$.

**CALCULATE:**

(a) $p_2=(0.619\text{ kg})\sqrt{2\left(9.81\text{ m/s}^2\right)(0.701\text{ m})}=2.296\text{ kg m/s}$

(b) $p_1=(0.459\text{ kg})\sqrt{2\left(9.81\text{ m/s}^2\right)(0.701\text{ m})}=0.1702\text{ kg m/s}$

(c) $v_{1f}=\dfrac{-0.0459\text{ kg}+3(0.619\text{ kg})}{0.0459\text{ kg}+0.619\text{ kg}}\sqrt{2\left(9.81\text{ m/s}^2\right)(0.701\text{ m})}=10.102\text{ m/s}$



(d) $h=\dfrac{(10.102\text{ m/s})^2}{2\left(9.81\text{ m/s}^2\right)}=5.201\text{ m}$

**ROUND:** Rounding to three significant figures:

(a) $p_2=2.30\text{ kg m/s}$

(b) $p_1=0.170\text{ kg m/s}$

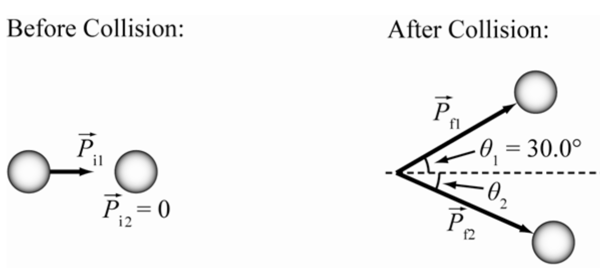
(c) $v_{1f}=10.1\text{ m/s}$

(d) $h=5.20\text{ m}$

**DOUBLE-CHECK:** We can see that $v_{1f}\approx 3v_{1i}$, so $h\approx(3v_{1i})^2/(2g)=9\left(v_{1i}^2\right)/(2g)=9h_0$. Our result in (d) should be about 9 times the original height.

**7.55.** **THINK:** There are two hockey pucks with equal mass $m_1=m_2=0.170$ kg. The first puck has an initial speed of 1.50 m/s and a final speed after collision of 0.750 m/s at an angle of 30.0°. We want to calculate the speed and direction of the second puck.

**SKETCH:**




**RESEARCH:** We need to use the conservation of momentum, i.e. $\vec{p}_i=\vec{p}_f$, or, in component form, $p_{ix}=p_{fx}$ and $p_{iy}=p_{fy}$.

**SIMPLIFY:** $p_{ix} = p_{fx} \Rightarrow m_1 v_{1i} + 0 = m_1 v_{1f} \cos\theta_1 + m_2 v_{2x}$. Since $m_1 = m_2 = m$, $v_{2x} = v_{1i} - v_{1f}\cos\theta_1$.
$p_{iy} = p_{fy} = 0 \Rightarrow 0 = m_1 v_{1f} \sin\theta_1 + m_2 v_{2y}$. Since $m_1 = m_2 = m$, $v_{2y} = -v_{1f}\sin\theta_1$.

The speed of the second puck is $v_2 = \sqrt{v_{2x}^2 + v_{2y}^2}$ and the angle is: $\theta = \tan^{-1}\left(\dfrac{-v_{1f}\sin\theta_1}{v_{1i} - v_{1f}\cos\theta_1}\right)$.

**CALCULATE:** $v_2 = \sqrt{\left(1.50 \text{ m/s} - (0.750 \text{ m/s})\cos 30.0°\right)^2 + \left(-(0.750 \text{ m/s})\sin 30.0°\right)^2} = 0.92949 \text{ m/s}$, and

$\theta = \tan^{-1}\left(\dfrac{-(0.750 \text{ m/s})\sin 30.0°}{1.50 \text{ m/s} - (0.750 \text{ m/s})\cos 30.0°}\right) = -23.79°$. Since $m_1 = m_2 = m$, the energy is conserved if $v_{1i}^2 = v_{1f}^2 + v_2^2$ we calculate these values to determine if the collision was elastic.

$$v_{1i}^2 = (1.50 \text{ m/s})^2 = 2.25 \text{ (m/s)}^2 \quad \text{and} \quad v_{1f}^2 + v_2^2 = (0.750 \text{ m/s})^2 + (0.930 \text{ m/s})^2 = 1.43 \text{ (m/s)}^2.$$

These values are not equal, thus $K_{1i} \neq K_{1f} + K_2$, and the collision is not elastic.

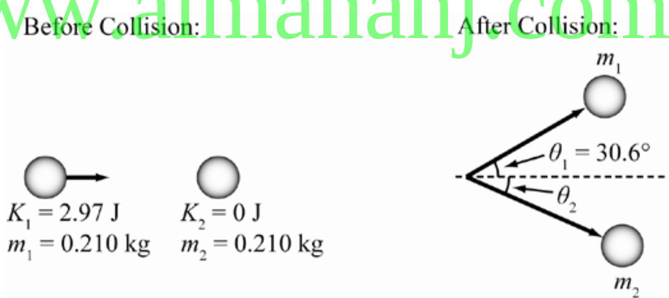
**ROUND:** $v_2 = 0.929$ m/s, and $\theta = -23.8°$.

**DOUBLE-CHECK:** The angle and speed for the second puck are reasonable since they are comparable to the angle and speed of the first puck.

**7.56.** **THINK:** We want to find the kinetic energy of a ball after it collides elastically with a ball at rest. We know the energy and can easily calculate the momentum of the balls before collision. The kinetic energy of the two balls respectively are $K_1 = 2.97$ J and $K_2 = 0$ J. The masses of both balls are the same, $m = 0.210$ kg. After the collision we know only the angle the first ball makes with its own path, $\theta_1 = 30.6°$. This means we have three unknowns: the velocities of the balls after collision and the angle of the second ball. Having three unknowns means we should have three equations.

**SKETCH:**






**RESEARCH:** The three equations we will use are the conservation of energy and one for each of the *x* and *y* components of the conservation of momentum: $K_{1i} + K_{2i} = K_{1f} + K_{2f}$, $p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$, and $p_{1iy} + p_{2iy} = p_{1fy} + p_{2fy}$. Here, the kinetic energy is given by $(1/2)mv^2$ and the momentum by $mv$.

**SIMPLIFY:** $K_{1i} + K_{2i} = K_{1f} + K_{2f} \Rightarrow \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 \Rightarrow v^2 = v_{1f}^2 + v_{2f}^2$ (1)

$$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx} \Rightarrow mv + 0 = mv_{1fx} + mv_{2fx} \Rightarrow v = v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2 \quad (2)$$

$$p_{1iy} + p_{2iy} = p_{1fy} + p_{2fy} \Rightarrow 0 + 0 = mv_{1fy} + mv_{2fy} \Rightarrow 0 = v_{1f}\sin\theta_1 + v_{2f}\sin\theta_2 \quad (3)$$

Our goal is to solve for $v_{1f}$ in equations (1), (2) and (3) so that we can calculate the kinetic energy. With this in mind, first rewrite equation (2) and (3) and then square them:

$$v = v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2 \Rightarrow v_{2f}\cos\theta_2 = v - v_{1f}\cos\theta_1 \Rightarrow v^2{}_{2f}\cos^2\theta_2 = (v - v_{1f}\cos\theta_1)^2 \quad (4)$$

$$0 = v_{1f}\sin\theta_1 + v_{2f}\sin\theta_2 \Rightarrow v_{2f}\sin\theta_2 = -v_{1f}\sin\theta_1 \Rightarrow v^2{}_{2f}\sin^2\theta_2 = (-v_{1f}\sin\theta_1)^2 \quad (5)$$

Next we add equations (4) and (5) so that $\theta_2$ can be removed from the equation

$$v^2{}_{2f}\cos^2\theta_2 + v^2{}_{2f}\sin^2\theta_2 = (-v_{1f}\sin\theta_1)^2 + (v - v_{1f}\cos\theta_1)^2$$

$$\Rightarrow v^2{}_{2f} = (-v_{1f}\sin\theta_1)^2 + (v - v_{1f}\cos\theta_1)^2 = v^2 - 2vv_{1f}\cos\theta_1 + v^2{}_{1f}$$

Substituting $v^2{}_{2f}$ into equations (1), we obtain

$$v^2 = v_{1f}^2 + v_{2f}^2 = v_{1f}^2 + v^2 - 2vv_{1f}\cos\theta_1 + v^2{}_{1f} = v^2 - 2vv_{1f}\cos\theta_1 + 2v^2{}_{1f}$$
$$\Rightarrow\; 2v^2{}_{1f} = 2vv_{1f}\cos\theta_1$$
$$\Rightarrow\; v_{1f} = v\cos\theta_1$$

Note the kinetic energy of ball 1 after collision can now be represented in term of $K_1$

$$K_{1f} = \frac{1}{2}m_1 v^2{}_{1f} = \frac{1}{2}m_1 v^2\cos^2\theta_1 = K_1\cos^2\theta_1$$

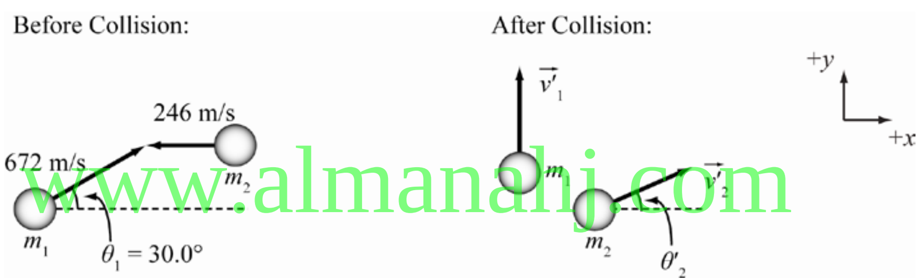
**CALCULATE:** $K_{1f} = 2.97\text{ J}\cdot\cos^2(30.6°) = 2.200404\text{ J}$

**ROUND:** The kinetic energy will be given to three significant digits since both $K_1$ and $\theta_1$ are given to three significant figures. $K_{1f} = 2.20$ J.

**DOUBLE-CHECK:** The kinetic energy after the collision is less than the original kinetic energy, which makes sense.

**7.57.** **THINK:** I want to find the final velocity of the molecules after they collide elastically. The first molecule has a speed of $v_1 = 672$ m/s at an angle of $30.0°$ along the positive horizontal. The second has a speed of 246 m/s in the negative horizontal direction. After the collision, the first particle travels vertically. For simplicity, we ignore rotational effects and treat the molecules as simple spherical masses.

**SKETCH:**





**RESEARCH:** Since this is an elastic collision, there is conservation of momentum in the $x$ and $y$ components and conservation of energy. $p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$, $p_{1iy} + p_{2iy} = p_{1fy} + p_{2fy}$ and $K_i = K_f$.

**SIMPLIFY:** In the $x$-direction, the momentum equation gives:

$$mv_{1i}\cos\theta_1 - mv_{2i} = mv_{2f}\cos\theta_2 \;\Rightarrow\; v_{1i}\cos\theta_1 - v_{2i} = v_{2fx} \qquad (1)$$

The $y$-component of the momentum gives:

$$mv_{1i}\sin\theta_1 = mv_{1f} + mv_2\sin\theta_2 \;\Rightarrow\; v_{1i}\sin\theta_1 = v_{1f} + v_{2fy} \;\Rightarrow\; v_{1i}\sin\theta_1 - v_{1f} = v_{2fy} \qquad (2)$$

The kinetic energy gives:

$$\frac{1}{2}mv_{1i}^2 + \frac{1}{2}mv_{2i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 \;\Rightarrow\; v_{1i}^2 + v_{2i}^2 = v_{1f}^2 + v_{2fx}^2 + v_{2fy}^2 \qquad (3)$$

Squaring and adding equations (1) and (2),

$$v^2{}_{2fx} + v^2{}_{2fy} = \left(v_{1i}\cos\theta_1 - v_{2i}\right)^2 + \left(v_{1i}\sin\theta_1 - v_{1f}\right)^2 \qquad (4)$$

Substituting (4) into equation (3),

$$v_{1i}^2 + v_{2i}^2 = v_{1f}^2 + \left(v_{1i}\cos\theta_1 - v_{2i}\right)^2 + \left(v_{1i}\sin\theta_1 - v_{1f}\right)^2$$
$$\Rightarrow\; v_{1i}^2 + v_{2i}^2 = v_{1f}^2 + \left(v^2{}_{1i}\cos^2\theta_1 - 2v_{1i}v_{2i}\cos\theta_1 + v^2{}_{2i}\right) + \left(v^2{}_{1i}\sin^2\theta_1 - 2v_{1i}v_{1f}\sin\theta_1 + v^2{}_{1f}\right)$$
$$\Rightarrow\; v_{1i}^2 + v_{2i}^2 = 2v_{1f}^2 + v^2{}_{1i} + v^2{}_{2i} - 2v_{1i}v_{2i}\cos\theta_1 - 2v_{1i}v_{1f}\sin\theta_1$$
$$\Rightarrow\; v_{1f}^2 - v_{1i}v_{1f}\sin\theta_1 - v_{1i}v_{2i}\cos\theta_1 = 0$$

There is only one unknown in this equation, so I can solve for $v_{1f}$.

$$v_{1f} = \frac{v_{1i}\sin\theta_1 \pm \sqrt{\left(-v_{1i}\sin\theta_1\right)^2 + 4v_{1i}v_{2i}\cos\theta_1}}{2}$$

I can solve for $v_{2fy}$ in terms of $v_{1f}$ using $v_{2fy} = v_{1i}\sin\theta_1 - v_{1f}$. The angle $\theta_2' = \arctan\left(\frac{v_{2fy}}{v_{2fx}}\right)$.

**CALCULATE:** $v_{1f} = \dfrac{672 \text{ m/s}\cdot\sin 30.0° \pm \sqrt{\left(-672 \text{ m/s}\cdot\sin 30.0°\right)^2 + 4\cdot 672 \text{ m/s}\cdot 246 \text{ m/s}\cdot\cos 30.0°}}{2}$

$= 581.9908 \text{ m/s or } -245.9908 \text{ m/s}$

Since I know that the molecule travels in the positive $y$ direction, $v_{1f} = 581.9908$ m/s .

$$v_{2fy} = v_{1i}\sin\theta_1 - v_{1f} = 672 \text{ m/s}\cdot\sin 30° - 581.9908 \text{ m/s} = -245.9908 \text{ m/s}$$

$$v_{2fx} = v_{1i}\cos\theta_1 - v_{2i} = 672 \text{ m/s}\cdot\cos 30° - 246 \text{ m/s} = 335.9691 \text{ m/s}$$

Therefore, $v_{2f} = \sqrt{\left(335.9691 \text{ m/s}\right)^2 + \left(-245.9908 \text{ m/s}\right)^2} = 416.3973$ m/s at an angle of $\theta_2' = \arctan\left(\frac{-245.9908 \text{ m/s}}{335.9691 \text{ m/s}}\right) = -36.211°$.

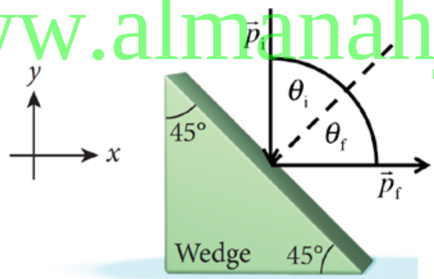
**ROUND:** $v_{1f} = 582$ m/s in the positive $y$-direction and $v_{2f} = 416$ m/s at an angle of 36.2° below the positive $x$-axis.

**DOUBLE-CHECK:** The results show $v_{1f} < v_1$ and $v_{2f} > v_{2i}$ as expected, so the answers look reasonable.

**7.58.** **THINK:** Since the wedge is solidly attached to the ground, it will not move during the collision, because the Earth has, for all practical purposes, infinite mass. This means that we can consider the surface of the wedge as a rigid wall and the angle of deflection relative to the normal will be equal to the angle of incidence relative to the normal.

**SKETCH:** We can simply use the figure supplied in the problem as our sketch, where we indicate the surface normal to the wedge (dashed line), as well as the angle of incidence and the angle of reflection.






**RESEARCH:** In equation 7.19 we found that $\theta_f = \theta_i$. Since the normal to the wedge surface makes a 45°-angle with the $x$-axis, this implies that the final momentum of the ball after the collision points horizontally. Since the collision is totally elastic the kinetic energy is conserved, which means that the length of the ball's momentum vector does not change. Consequently, $\vec{p}_i = (0, -mv)$; $\vec{p}_f = (mv, 0)$ . The momentum change of the ball in the collision is $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$ . Since the total momentum is conserved in all collisions, the recoil momentum that the Earth receives in this collision is then $\vec{p}_{recoil} = -\Delta\vec{p}$ .

**SIMPLIFY:** $\vec{p}_{recoil} = -\Delta\vec{p} = -(\vec{p}_f - \vec{p}_i) = \vec{p}_i - \vec{p}_f = (0, -mv) - (mv, 0) = (-mv, -mv)$

The absolute value of the recoil momentum is $\left|\vec{p}_{recoil}\right| = \sqrt{(-mv)^2 + (-mv)^2} = \sqrt{2}mv$

**CALCULATE:** $\vec{p}_{recoil} = (-1,-1)(3.00 \text{ kg})(4.50 \text{ m/s}) = (-1,-1)(13.5 \text{ kg m/s})$

$\left|\vec{p}_{recoil}\right| = \sqrt{2}(13.5 \text{ kg m/s}) = 19.0919 \text{ kg m/s}$

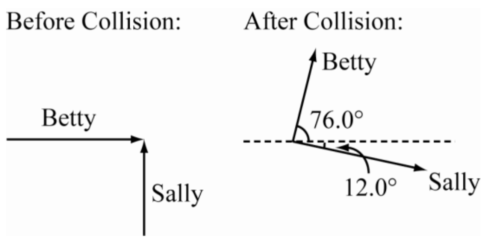
**ROUND:** We round the absolute value of the recoil momentum to three significant figures:

$\left|\vec{p}_{recoil}\right| = 19.1 \text{ kg m/s}$

**DOUBLE-CHECK:** We have assumed that the wedge, which is attached to the Earth, does not move in the collision process. Is it reasonable then to find that the wedge+Earth system receives a finite recoil momentum in the collision process? The answer is yes, but only because we can assume that the mass of the Earth is practically infinitely large compared to the mass of the ball.

**7.59.** **THINK:** I want to find the kinetic energy of Betty and Sally after they collide together. Also, I would like to know if the collision is elastic. Betty and Sally have masses and velocities of $m_{\text{B}} = 55.0$ kg, $v_{\text{B}} = v_{\text{B},x} = 22.0 \text{ km/h} = 6.111 \text{ m/s}$, $m_{\text{S}} = 45.0$ kg and $v_{\text{S}} = v_{\text{S},y} = 28.0 \text{ km/h} = 7.778 \text{ m/s}$ respectively. After the collision, Betty is travelling $\theta_{\text{B}} = 76.0°$ from the horizontal and Sally is moving $\theta_{\text{S}} = 12.0°$ below the horizontal.

**SKETCH:**




**RESEARCH:** Use the conservation of momentum $\vec{p}_{\text{Bi}} + \vec{p}_{\text{Si}} = \vec{p}_{\text{Bf}} + \vec{p}_{\text{Sf}}$ to get the velocities after the collision. This information will allow calculation of the kinetic energy $mv^2/2$ for the skaters.

**SIMPLIFY:** The momentum gives the two following equations:

$$m_{\text{B}}v_{\text{Bi}} = m_{\text{S}}v_{\text{Sf}}\cos\theta_{\text{S}} + m_{\text{B}}v_{\text{Bf}}\cos\theta_{\text{B}} \qquad (1)$$

$$m_{\text{S}}v_{\text{Si}} = -m_{\text{S}}v_{\text{Sf}}\sin\theta_{\text{S}} + m_{\text{B}}v_{\text{Bf}}\sin\theta_{\text{B}} \qquad (2)$$

Solving equation (1) for $v_{\text{Sf}}$,



$$v_{\text{Sf}} = \frac{m_{\text{B}}v_{\text{Bi}} - m_{\text{B}}v_{\text{Bf}}\cos\theta_{\text{B}}}{m_{\text{S}}\cos\theta_{\text{S}}}$$

Substituting into equation (2),

$$m_{\text{S}}v_{\text{Si}} = -m_{\text{S}}\left(\frac{m_{\text{B}}v_{\text{Bi}} - m_{\text{B}}v_{\text{Bf}}\cos\theta_{\text{B}}}{m_{\text{S}}\cos\theta_{\text{S}}}\right)\sin\theta_{\text{S}} + m_{\text{B}}v_{\text{Bf}}\sin\theta_{\text{B}}$$

$$m_{\text{S}}v_{\text{Si}} = -m_{\text{B}}v_{\text{Bi}}\tan\theta_{\text{S}} + m_{\text{B}}v_{\text{Bf}}\cos\theta_{\text{B}}\tan\theta_{\text{S}} + m_{\text{B}}v_{\text{Bf}}\sin\theta_{\text{B}}$$

$$v_{\text{Bf}} = \frac{m_{\text{S}}v_{\text{Si}} + m_{\text{B}}v_{\text{Bi}}\tan\theta_{\text{S}}}{m_{\text{B}}\left(\cos\theta_{\text{B}}\tan\theta_{\text{S}} + \sin\theta_{\text{B}}\right)}$$

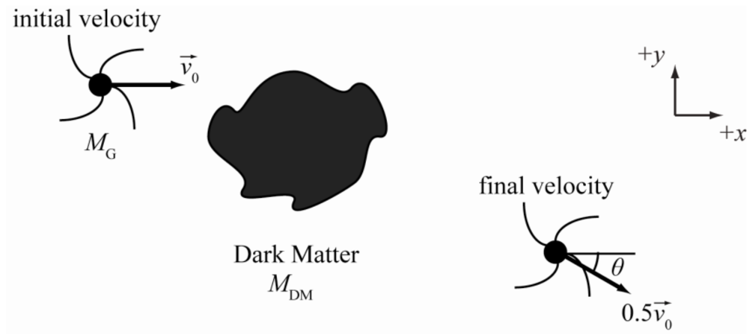
Similarly, $v_{\text{Sf}} = \dfrac{m_{\text{B}}v_{\text{Bi}}\tan\theta_{\text{B}} - m_{\text{S}}v_{\text{Si}}}{m_{\text{S}}\left(\sin\theta_{\text{S}} + \cos\theta_{\text{S}}\tan\theta_{\text{B}}\right)}$. To get the kinetic energy, we simply plug the result into the equation $K = \dfrac{1}{2}mv^2$.

**CALCULATE:** $v_{\text{Bf}} = \dfrac{(45.0 \text{ kg})(7.778 \text{ m/s}) + (55.0 \text{ kg})(6.111 \text{ m/s})\tan 12.0°}{(55.0 \text{ kg})\left(\cos 76.0° \tan 12.0° + \sin 76°\right)} = 7.49987 \text{ m/s}$ and

$v_{\text{Sf}} = \dfrac{(55.0 \text{ kg})(6.111 \text{ m/s})\tan 76.0° - (45.0 \text{ kg})(7.778 \text{ m/s})}{(45.0 \text{ kg})\left(\sin 12.0° + \cos 12.0° \tan 76.0°\right)} = 5.36874 \text{ m/s}.$ Betty's final kinetic energy is then $\dfrac{1}{2}m_{\text{B}}v_{\text{Bf}}^2 = 1546.82 \text{ J}$. Sally's final kinetic energy is then $\dfrac{1}{2}m_{\text{S}}v_{\text{Sf}}^2 = 648.526 \text{ J}$. The ratio of the final and initial kinetic energy is $\dfrac{K_{\text{f}}}{K_{\text{i}}} = \dfrac{m_{\text{B}}v_{\text{Bf}}^2 + m_{\text{S}}v_{\text{Sf}}^2}{m_{\text{B}}v_{\text{Bi}}^2 + m_{\text{S}}v_{\text{Si}}^2} = 0.9193$.

**ROUND:** Our results will be reported to 3 significant figures, the same accuracy as the given values. $K_{\text{Bf}} = 1.55$ kJ and $K_{\text{Sf}} = 649$ J. The ratio $K_{\text{f}}/K_{\text{i}}$ is not equal to one, so the collision is inelastic.
**DOUBLE-CHECK:** These are reasonable results.

**7.60.** **THINK:** I want to find the mass of dark matter in terms of $M_{\text{G}}$, $v_0$ and $\theta$. The initial velocity of the galaxy is in the *x*-direction. After it interacts with the dark matter it travels at 50% of its original speed in the direction of $\theta$ below the *x*-axis.
**SKETCH:**




**RESEARCH:** Use the conservation of momentum in the *x* and *y* directions. Also use the conservation of energy. $p_{\text{Gi}} + p_{\text{DMi}} = p_{\text{Gf}} + p_{\text{DMf}}$, $K_{\text{i}} = K_{\text{f}}$.
**SIMPLIFY:** The momentum in the *x* and *y* direction gives:

$$M_{\text{G}}v_0 + 0 = M_{\text{G}}\left(0.50v_0\right)\cos\theta + p_{\text{DMf}x};\quad 0 = M_{\text{G}}\left(0.50v_0\right)\sin\theta + p_{\text{DMf}y}.$$

The conservation of energy gives:

$$\frac{1}{2}M_{\text{G}}v_0^2 = \frac{1}{2}M_{\text{G}}\left(0.50v_0\right)^2 + \frac{p_{\text{DMf}x}^2 + p_{\text{DMf}y}^2}{2M_{\text{DM}}} \Rightarrow M_{\text{G}}v_0^2 = M_{\text{G}}\left(0.50v_0\right)^2 + \frac{p_{\text{DMf}x}^2 + p_{\text{DMf}y}^2}{M_{\text{DM}}}.$$



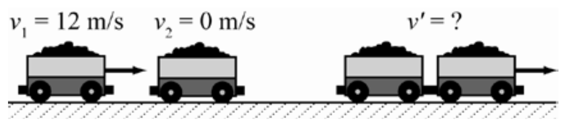
Use the conservation of energy to solve for $M_{\text{DM}}$.

$$\begin{aligned}
M_{\text{DM}} &= \frac{p_{\text{DMf}x}^2 + p_{\text{DMf}y}^2}{M_{\text{G}}v_0^2\left(1-0.50^2\right)} \\
&= \frac{\left(M_{\text{G}}v_0\left(1-0.50\cos\theta\right)\right)^2 + \left(-M_{\text{G}}v_0\left(0.50\sin\theta\right)\right)^2}{M_{\text{G}}v_0^2\left(1-0.50^2\right)} \\
&= \frac{M_{\text{G}}^2v_0^2}{M_{\text{G}}v_0^2}\,\frac{\left(1-0.50\cos\theta\right)^2 + \left(-0.50\sin\theta\right)^2}{1-0.50^2} \\
&= M_{\text{G}}\frac{1-\cos\theta + 0.25\cos^2\theta + 0.25\sin^2\theta}{0.75} \\
&= M_{\text{G}}\frac{1.25-\cos\theta}{3/4} \\
&= \frac{4}{3}M_{\text{G}}\left(1.25-\cos\theta\right)
\end{aligned}$$

**CALCULATE:** There are no values to calculate.
**ROUND:** There is no rounding to do.
**DOUBLE-CHECK:** This result is reasonable.

**7.61.** **THINK:** I want to know what the speed of the railroad car is after a perfectly inelastic collision occurs. Knowing $m_1 = m_2 = 1439$ kg, $v_1 = 12.0$ m/s and $v_2 = 0$ m/s.

**SKETCH:**




**RESEARCH:** The equation for a perfectly inelastic collision with identical masses is given by $v_{1i} + v_{2i} = 2v_f$.

**SIMPLIFY:** $v_f = (v_{1i} + v_{2i})/2$

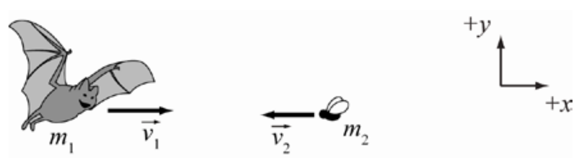
**CALCULATE:** $v_f = (12.0 \text{ m/s} + 0 \text{ m/s})/2 = 6.00 \text{ m/s}$

**ROUND:** Because the velocity before the collision is given to three significant figures, keep the result to three significant figures. The velocity of the cars after the collision is 6.00 m/s.

**DOUBLE-CHECK:** This is equivalent to a speed of 22 km/h, which is reasonable for railroad cars.

**7.62.** **THINK:** I want to know the speed of a 50.0 g bat after it catches a 5.00 g insect if they travel at 8.00 m/s and 6.00 m/s in opposite directions.

**SKETCH:**




**RESEARCH:** Since the bat catches the insect this is an elastic collision. Use the equation $m_{1i}v_{1i} + m_{2i}v_{2i} = (m_1 + m_2)v_f$.



**SIMPLIFY:** $v_f = \dfrac{m_{1i}v_{1i} + m_{2i}v_{2i}}{m_1 + m_2}$

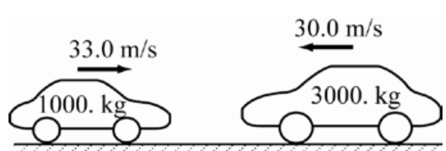
**CALCULATE:** $v_f = \dfrac{(50.0 \text{ g})(8.00 \text{ m/s}) + (5.00 \text{ g})(-6.00 \text{ m/s})}{50.0 \text{ g} + 5.00 \text{ g}} = 6.727 \text{ m/s}$

**ROUND:** To three significant figures, the speed of the bat after a tasty treat is 6.73 m/s.

**DOUBLE-CHECK:** I would expect a small loss in the speed of the bat since the insect is small compared to it.

**7.63.** **THINK:** I want to know the acceleration of the occupants of each car after a perfectly inelastic collision. The first car has mass $m_1 = 1000.$ kg and velocity $v_1 = 33.0$ m/s while the second has mass $m_2 = 3000.$ kg and velocity $v_2 = -30.0$ m/s. The collision lasts for 100. ms, or 0.100 s.

**SKETCH:**




**RESEARCH:** First to find the change of moment each car experiences using the equation of perfectly inelastic collision, $m_1v_1 + m_2v_2 = (m_1 + m_2)v_f$. Using this find the force experienced with the help of the equation $F\Delta t = \Delta p$.

**SIMPLIFY:** $v_f = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}$. Then $\Delta p_1 = (m_1 + m_2)v_f - m_1v_1$ and $\Delta p_2 = (m_1 + m_2)v_f - m_2v_2$.

So $a_1 = \dfrac{m_2(v_2 - v_1)}{(m_1 + m_2)\Delta t}$ and $a_2 = \dfrac{m_1(v_1 - v_2)}{(m_1 + m_2)\Delta t}$.

**CALCULATE:** $a_1 = \dfrac{(3000.\text{ kg})(-30.0 \text{ m/s} - 33.0 \text{ m/s})}{(3000.\text{ kg} + 1000.\text{ kg})(0.100 \text{ s})} = -472.5 \text{ m/s}^2$, and

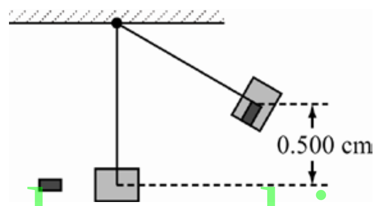
$$a_2 = \frac{(1000.\text{ kg})(33.0 \text{ m/s} - (-30.0 \text{ m/s}))}{(3000.\text{ kg} + 1000.\text{ kg})(0.100 \text{ s})} = 157.5 \text{ m/s}^2.$$

**ROUND:** The acceleration the occupants of the smaller car feel is $a_1 = -473 \text{ m/s}^2$, or $-48.2g$. The acceleration the occupants of the larger car feel is $a_2 = 158 \text{ m/s}^2$, or $16.1g$.

**DOUBLE-CHECK:** This makes sense since we often hear how the drivers of smaller cars fair worse than those in larger cars.

**7.64.** **THINK:** I am looking for the speed of the bullet of mass $m_{\text{bu}} = 2.00 \text{ g}$ that moves the 2.00 kg block on a string. The kinetic energy of the block and bullet is converted to potential energy and attains a height of 0.500 cm. First start by converting the mass of the bullet to kilograms, $m_{\text{bu}} = 0.00200 \text{ kg}$, and the height to meters; $h$ = 0.00500 m.

**SKETCH:**






**RESEARCH:** First use the relation between the kinetic energy $T = \frac{1}{2}mv^2$ and the potential energy $U = mgh$. From these find the final velocity of the block and bullet. Then using the conservation of momentum for perfectly inelastic collisions, find the initial speed of the bullet. $m_1v_1 + m_2v_2 = (m_1 + m_2)v_{\text{f}}$

**SIMPLIFY:** Set the kinetic energy equal to the potential energy and solve for the velocity. Use this in the conservation of momentum equation: $\frac{1}{2}mv_{\text{f}}^2 = mgh \;\Rightarrow\; v_{\text{f}} = \sqrt{2gh}$. Note that $v_2 = v_{\text{bl}} = 0$.

$$m_1v_1 = (m_1 + m_2)v_{\text{f}} = (m_1 + m_2)\sqrt{2gh} \;\Rightarrow\; v_1 = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$

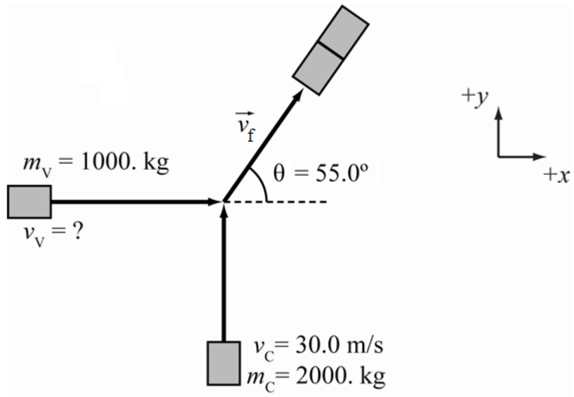
**CALCULATE:** $v_1 = \dfrac{2.00 \text{ kg} + 0.00200 \text{ kg}}{0.00200 \text{ kg}}\sqrt{2(9.81 \text{ m/s}^2)(0.00500 \text{ m})} = 313.522 \text{ m/s}$

**ROUND:** The height attained by the block and bullet was only given to three significant figures, thus the velocity of the bullet will be reported as 314 m/s.

**DOUBLE-CHECK:** The speed of a typical bullet is 1000 m/s, thus our answer is reasonable.

**7.65.** **THINK:** The Volkswagen of mass $m_{\text{V}} = 1000.\text{ kg}$ was going eastward before the collision and the Cadillac had mass $m_{\text{C}} = 2000.\text{ kg}$ and velocity $v_{\text{C}} = 30.0 \text{ m/s}$ northward, and after the collision both cars stuck together travelling $\theta = 55.0°$ north of east.

**SKETCH:**




**RESEARCH:** The collision was perfectly inelastic so use the equation $m_C v_C + m_V v_V = (m_C + m_V)v_f$ for each component of the motion.

**SIMPLIFY:** In the east-west direction:

$$m_V v_V = (m_C + m_V)v_f \cos\theta \Rightarrow v_f = \frac{m_V v_V}{m_C + m_V}\frac{1}{\cos\theta},$$

and in the north-south direction:

$$m_C v_C = (m_C + m_V)v_f \sin\theta \Rightarrow v_f = \frac{m_C v_C}{m_C + m_V}\frac{1}{\sin\theta}$$

Equating these two expressions for the final velocity gives:

$$\frac{m_V v_V}{m_C + m_V}\frac{1}{\cos\theta} = \frac{m_C v_C}{m_C + m_V}\frac{1}{\sin\theta} \Rightarrow v_V = \frac{m_C}{m_V}v_C\frac{\cos\theta}{\sin\theta} = \frac{m_C}{m_V}v_C \cot\theta.$$



**CALCULATE:** $v_V = \frac{2000. \text{ kg}}{1000. \text{ kg}}(30.0 \text{ m/s})\cot 55.0° = 42.01245 \text{ m/s}$

**ROUND:** The Volkswagen's velocity is 42.0 m/s.

**DOUBLE-CHECK:** This is a reasonable result. It's in the same order as the Cadillac.

**7.66.** **THINK:** There are three things to calculate:

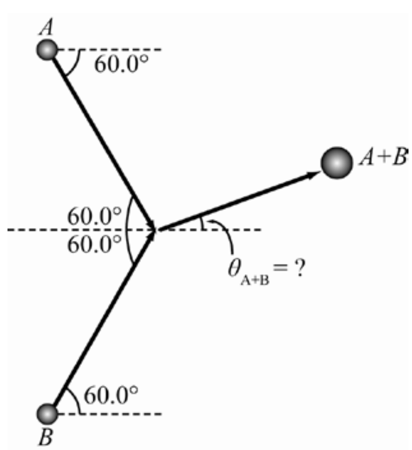
(a) the angle above the horizontal the mass *A*+*B* makes;

(b) the ratio $v_f / v_A$;

(c) the ratio $E_f / E_i$.

It is known that $m_A = m_B = m$ and that $v_B = 2v_A = 2v$. By inspection, $\theta_A = 60.0°$ and $\theta_B = 60.0°$.

**SKETCH:**



**RESEARCH:** The relevant equations are those for conservation of momentum for a perfectly inelastic collision for the $x$ and $y$ components, and for the kinetic energy.

$$p_{Ax} + p_{Bx} = p_{(A+B)x}, \quad p_{Ay} + p_{By} = p_{(A+B)y}, \quad \text{and} \quad K = \frac{1}{2}mv^2.$$

Also, $m_A = m_B = m;\ m_A + m_B = 2m;\ v_B = 2v_A$

**SIMPLIFY:** In the $x$ -direction:

$$m_A v_A \cos\theta_A + m_B v_B \cos\theta_B = (m_A + m_B) v_{AB} \cos\theta_{A+B}$$
$$mv_A \cos(60.0°) + m(2v_A)\cos 60.0° = 2mv_{AB}\cos\theta_{A+B}$$
$$v_A \cos(60.0°) + (2v_A)\cos 60.0° = 2v_{AB}\cos\theta_{A+B}$$
$$\frac{v_A}{2} + \frac{2v_A}{2} = 2v_{AB}\cos\theta_{A+B}$$
$$\frac{3}{4}v_A = v_{AB}\cos\theta_{A+B}$$

In the $y$-direction:

$$-m_A v_A \sin\theta_A + m_B v_B \sin\theta_B = (m_A + m_B) v_{AB} \sin\theta_{A+B}$$
$$-mv_A \sin(60.0°) + m(2v_A)\sin 60.0° = 2mv_{AB}\sin\theta_{A+B}$$
$$-v_A \sin(60.0°) + (2v_A)\sin 60.0° = 2v_{AB}\sin\theta_{A+B}$$
$$\frac{-\sqrt{3}v_A}{2} + \frac{2\sqrt{3}v_A}{2} = 2v_{AB}\sin\theta_{A+B}$$
$$\frac{\sqrt{3}}{2}v_A = 2v_{AB}\sin\theta_{A+B}$$
$$\frac{\sqrt{3}v_A}{4} = v_{AB}\sin\theta_{A+B}$$



To find the angle $\theta_{A+B}$, divide the equation found for the $y$-component by the one for the $x$-component.

$$\frac{\frac{\sqrt{3}v_A}{4}}{\frac{3}{4}v_A} = \frac{v_{AB}\sin\theta_{A+B}}{v_{AB}\cos\theta_{A+B}} \Rightarrow \frac{1}{\sqrt{3}} = \tan\theta_{AB} \Rightarrow \theta_{AB} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

To find the ratio $v_{AB}/v_A$ rearrange the $y$-component equation.

$$\frac{\sqrt{3}v_A}{4} = v_{AB}\sin\theta_{A+B} \Rightarrow \frac{v_{AB}}{v_A} = \frac{\sqrt{3}}{4\sin\theta_{A+B}}$$

The ratio $K_f/K_i$ is:

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(m_A + m_B)v_{AB}^2}{\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2} = \frac{(m+m)v_{AB}^2}{mv^2 + m(2v_A)^2} = \frac{2v_{AB}^2}{v^2 + 4v_A^{\ 2}} = \frac{2}{5}\left(\frac{v_{AB}}{v_A}\right)^2 = \frac{2}{5}\left[\frac{\sqrt{3}}{4\sin\theta_{A+B}}\right]^2$$

**CALCULATE:**

(a) $\theta_{A+B} = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = 30.0°$

(b) $\dfrac{v_{AB}}{v_A} = \dfrac{\sqrt{3}}{4\sin 30.0°} = \dfrac{\sqrt{3}}{2}$

(c) $\dfrac{K_f}{K_i} = \dfrac{2}{5}\left[\dfrac{\sqrt{3}}{4\sin 30.0°}\right]^2 = \left(\dfrac{2}{5}\right)\left(\dfrac{3}{4}\right) = \dfrac{3}{10}$

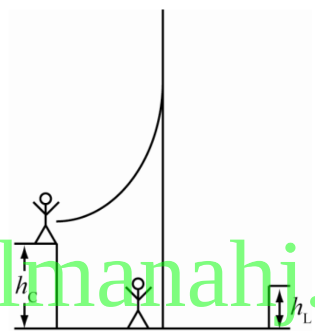
**ROUND:**
(a) $\theta_{A+B} = 30.0°$

(b) $\dfrac{v_f}{v_A} = \dfrac{\sqrt{3}}{2} = 0.866$

(c) $\dfrac{K_f}{K_i} = \dfrac{3}{10} = 0.300$

**DOUBLE-CHECK:** These results are reasonable. When the objects collide and stick together is known as perfectly inelastic so we would expect the ratio $K_f/K_i$ to be less than one.

**7.67.** **THINK:** This is essentially an inelastic collision. Since Jane is standing still, she has no initial momentum. Tarzan must have initial kinetic energy such that when his and Jane's mass are combined, their momentum is sufficient to make it to the tree. Because this is an inelastic collision, energy is not conserved when Tarzan catches Jane, but momentum is conserved. Tarzan's mass is 80.0 kg, and Jane's mass is 40.0 kg. The vine Tarzan swings from is 30.0 m long. The cliff from which Tarzan jumps is 20.0 meters high, and the tree limb Tarzan and Jane must reach is 10.0 m high.
**SKETCH:**






**RESEARCH:** The problem is most easily solved by working backwards. The potential energy of Tarzan and Jane when they reach the tree is $U_{TJ} = (m_T + m_J)gh_{tree}$. This potential energy must be equal to the kinetic energy just after the "collision": $K_{TJ} = \frac{1}{2}(m_T + m_J)v_{TJ}^2$. The combined momentum of Tarzan and Jane after the collision, $P_{TJ} = (m_T + m_J)v_{TJ}$, must be equal to the sum of their momenta before the collision, $m_T v_T + m_J v_J = m_T v_T$ (since Jane's initial momentum is zero). Tarzan's kinetic energy just before he catches Jane is $K_T = \frac{1}{2}m_T v_T^2$, which must be equal to his initial total energy, $U_T + K_{T,0} = m_T g h_{cliff} + \frac{1}{2}m_T v_{T,0}^2$. Tarzan's initial velocity, $v_{T,0}$, is the desired quantity.

**SIMPLIFY:**

$U_{TJ} = (m_T + m_J)gh_{tree} = \frac{1}{2}(m_T + m_J)v_{TJ}^2$. Solving for $v_{TJ}$: $v_{TJ} = \sqrt{2gh_{tree}}$.

$P_{TJ} = (m_T + m_J)v_{TJ} = (m_T + m_J)\sqrt{2gh_{tree}} = m_T v_T$. Solving for $v_T$: $v_T = \dfrac{(m_T + m_J)\sqrt{2gh_{tree}}}{m_T}$.

$$K_T = \frac{1}{2}m_T v_T^2 = \frac{1}{2}m_T\left(\frac{(m_T + m_J)\sqrt{2gh_{tree}}}{m_T}\right)^2 = \frac{(m_T + m_J)^2 gh_{tree}}{m_T}$$

$U_T + K_{T,0} = m_T g h_{cliff} + \frac{1}{2} m_T v_{T,0}^2 = K_T = \frac{(m_T + m_J)^2 g h_{tree}}{m_T}$. Solving for $v_{T,0}$ :

$$v_{T,0} = \sqrt{\frac{2}{m_T}\left(\frac{(m_T + m_J)^2 g h_{tree}}{m_T} - m_T g h_{cliff}\right)}.$$

**CALCULATE:**

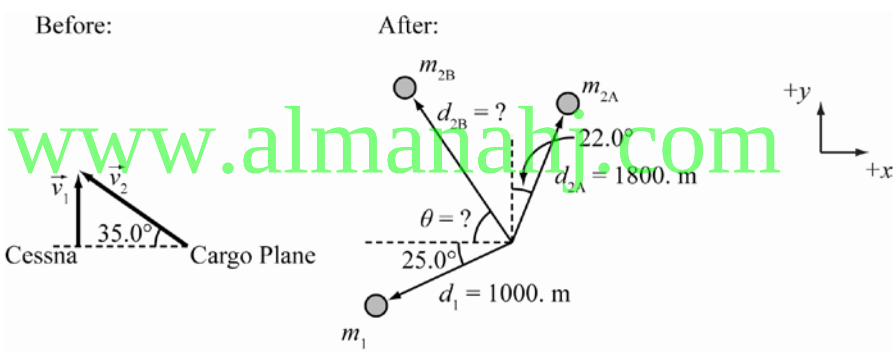
$$v_{T,0} = \sqrt{\frac{2}{80.0 \text{ kg}}\left(\frac{(80.0 \text{ kg} + 40.0 \text{ kg})^2 (9.81 \text{ m/s}^2)(10.0 \text{ m})}{80.0 \text{ kg}} - (80.0 \text{ kg})(9.81 \text{ m/s}^2)(20.0 \text{ m})\right)} = 7.00 \text{ m/s}.$$

**ROUND:** Tarzan must jump from the ledge at a speed of 7.00 m/s.

**DOUBLE-CHECK:** 7.00 m/s is a fast but reasonable speed for a fit person to achieve with a running jump.

**7.68.** **THINK:** I hope to find the region the second part of the cargo plane lands after the collision. Knowing the initial speed and mass of the Cessna to be $m_1 = 3000.0$ kg and $v_1 = 75.0$ m/s northward and the initial speed and mass of the cargo plane to be $m_2 = 7000.$ kg and $v_2 = 100.$ m/s $35.0°$ north of west. After the collision the plane drops $z = 1600.$ m to the ground. The Cessna is $d_1 = 1000.$ m at $25.0°$ south of west and one piece of the cargo plane of mass $m_{2A} = 4000.$ kg is 1800. m $22.0°$ east of north.

**SKETCH:**






**RESEARCH:** In order to calculate the position of the second piece of the cargo plane I need the conservation of momentum in the *x* (east-west) component and *y* (north-south) component.

$p_{1x} + p_{2x} = p'_{1x} + p_{2Ax} + p_{2Bx}$ .

To find the speed of the planes after impact, use $z = gt^2/2$ and $d = vt$ . The time it takes the pieces of the planes to fall to the ground is $t = \sqrt{2z/g}$ , and the velocity of each piece is $v = d/t = d\sqrt{g/(2z)}$ , where *d* is the horizontal distance traveled by a given piece. Therefore the distance traveled by each piece of debris is $d = vt = v\sqrt{(2z)/g}$ .

**SIMPLIFY:** Now solve for the *x* and *y* components of the missing piece of debris.

$$p_{2Bx} = p_{1x} + p_{2x} - p'_{1x} - p_{2Ax} \quad \text{and} \quad p_{2By} = p_{1y} + p_{2y} - p'_{1y} - p_{2Ay}$$

Being careful with directions, these become:

$$p_{2Bx} = -m_2 v_2 \cos\theta_2 + m_1 v_1' \cos\theta_1' - m_{2A} v_{2A} \sin\theta_{2A}$$

$$m_{2B} v_{2Bx} = -m_2 v_2 \cos\theta_2 + m_1 d_1 \sqrt{\frac{g}{2z}} \cos\theta_1' - m_{2A} d_{2A} \sqrt{\frac{g}{2z}} \sin\theta_{2A}$$

$$m_{2B} d_{2Bx} \sqrt{\frac{g}{2z}} = -m_2 v_2 \cos\theta_2 + m_1 d_1 \sqrt{\frac{g}{2z}} \cos\theta_1' - m_{2A} d_{2A} \sqrt{\frac{g}{2z}} \sin\theta_{2A}$$

$$d_{2Bx} = -\frac{m_2 v_2 \cos\theta_2}{m_{2B}\sqrt{g/(2z)}} + \frac{m_1}{m_{2B}} \cos\theta_1' - \frac{m_{2A}}{m_{2B}} d_{2A} \sin\theta_{2A}$$

The $y$-component is:

$$p_{2By} = m_1 v_1 + m_2 v_2 \sin\theta_2 + m_1 v_1' \sin\theta_1' - m_{2A} v_{2A} \cos\theta_{2A}$$

$$m_{2B} d_{2By} \sqrt{g/(2z)} = m_1 v_1 + m_2 v_2 \sin\theta_2 + m_1 d_1 \sqrt{g/(2z)} \sin\theta_1' - m_{2A} d_{2A} \sqrt{g/(2z)} \cos\theta_{2A}$$

$$d_{2By} = \frac{m_1 v_1 + m_2 v_2 \sin\theta_2}{m_{2B}\sqrt{g/(2z)}} + \frac{m_1}{m_{2B}} d_1 \sin\theta_1' - \frac{m_{2A}}{m_{2B}} d_{2A} \cos\theta_{2A}$$

The total distance is: $d_{2B} = \sqrt{(d_{2Bx})^2 + (d_{2By})^2}$. The direction of the missing piece of wreckage is: $\theta = \tan^{-1}(d_{2By}/d_{2Bx})$.

**CALCULATE:**

$$d_{2bx} = \frac{-(7000.\text{ kg})(100.\text{ m/s})\cos 35.0^\circ}{(3000.\text{ kg})\sqrt{(9.81\text{ m/s}^2)/(2(1600.\text{ m}))}} + \left(\frac{3000.\text{ kg}}{3000.\text{ kg}}\right)(1000.\text{ m})\cos 25.0^\circ$$

$$-\left(\frac{4000.\text{ kg}}{3000.\text{ kg}}\right)(1800.\text{ m})\sin 22.0^\circ = -3444.84\text{ m}$$



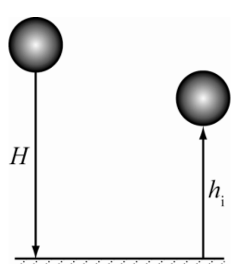
$$d_{2by} = \frac{(3000.\text{ kg})(75.0\text{ m/s}) + (7000.\text{ kg})(100.\text{ m/s})\sin 35.0^\circ}{(3000.\text{ kg})\sqrt{(9.81\text{ m/s}^2)/(2(1600.\text{ m}))}} + \left(\frac{3000.\text{ kg}}{3000.\text{ kg}}\right)(1000.\text{ m})\sin 25.0^\circ$$

$$-\left(\frac{4000.\text{ kg}}{3000.\text{ kg}}\right)(1800.\text{ m})\cos 22.0^\circ = 1969.126\text{ m}$$

$$d_{2B} = \sqrt{(-3444.84\text{ m}) + (1969.126\text{ m})} = 3967.92\text{ m},\ \theta = \tan^{-1}\left(\frac{1969.126\text{ m}}{-3444.84\text{ m}}\right) = 29.75^\circ$$

**ROUND:** 3970 m from the point of the collision, at an angle of $29.8^\circ$ clockwise from the negative $x$-axis.
**DOUBLE-CHECK:** This is a reasonable answer. The distance is of the same order as the other crash sites.

**7.69.** **THINK:** I want to find the coefficient of restitution for a variety of balls.
**SKETCH:**




**RESEARCH:** Using the equation for the coefficient of restitution for heights. $\varepsilon = \sqrt{H/h_1}$.
**SIMPLIFY:** There is no need to simplify.

**CALCULATE:** An example calculation: A range golf ball has an initial height $H = 85.0$ cm and a final height $h_1 = 62.6$ cm.

$$\varepsilon = \sqrt{\frac{62.6 \text{ cm}}{85.0 \text{ cm}}} = 0.85818$$

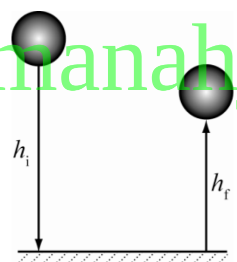
**ROUND:** All of the coefficients of restitution will be given to 3 significant figures because all the heights are given to 3 significant figures.

| Object | $H$ [cm] | $h_1$ [cm] | ε |
|---|---|---|---|
| Range golf ball | 85.0 | 62.6 | 0.858 |
| Tennis ball | 85.0 | 43.1 | 0.712 |
| Billiard ball | 85.0 | 54.9 | 0.804 |
| Hand ball | 85.0 | 48.1 | 0.752 |
| Wooden ball | 85.0 | 30.9 | 0.603 |
| Steel ball bearing | 85.0 | 30.3 | 0.597 |
| Glass marble | 85.0 | 36.8 | 0.658 |
| Ball of rubber bands | 85.0 | 58.3 | 0.828 |
| Hollow, hard plastic balls | 85.0 | 40.2 | 0.688 |

**DOUBLE-CHECK:** All these values are less than one, which is reasonable.

**7.70.** **THINK:** I want to find the maximum height a ball reaches if it is started at 0.811 m and has a coefficient of restitution of 0.601.

**SKETCH:**






**RESEARCH:** Using the equation $\varepsilon = \sqrt{h_f / h_i}$.

**SIMPLIFY:** $\varepsilon = \sqrt{h_f / h_i} \Rightarrow \varepsilon^2 = h_f / h_i \Rightarrow h_f = h_i \varepsilon^2$

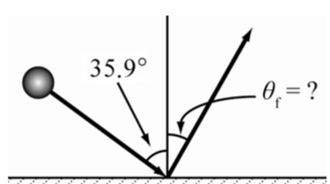
**CALCULATE:** $h_f = (0.811 \text{ m})(0.601)^2 = 0.292934011$ m

**ROUND:** The values are given to 3 significant figures so the final height is $h_f = 0.293$ m.

**DOUBLE-CHECK:** This is a reasonable answer since $h_f < h_i$.

**7.71.** **THINK:** I want to know the angle relative to the wall after the ball hits the wall. The ball has mass $m$ = 0.162 kg, a speed of $v$ = 1.91 m/s and collides at an angle $\theta_i = 35.9^\circ$ with a coefficient of restitution $\varepsilon = 0.841$.

**SKETCH:**



**RESEARCH:** We will use $\theta_{\text{f}} = \cot^{-1}\left(\dfrac{\varepsilon p_{\text{i}\perp}}{p_{\text{i}\parallel}}\right)$.

**SIMPLIFY:** $\theta_{\text{f}} = \cot^{-1}\left(\dfrac{\varepsilon mv\cos\theta_{\text{i}}}{mv\sin\theta_{\text{i}}}\right) = \cot^{-1}\left(\dfrac{\varepsilon\cos\theta_{\text{i}}}{\sin\theta_{\text{i}}}\right)$

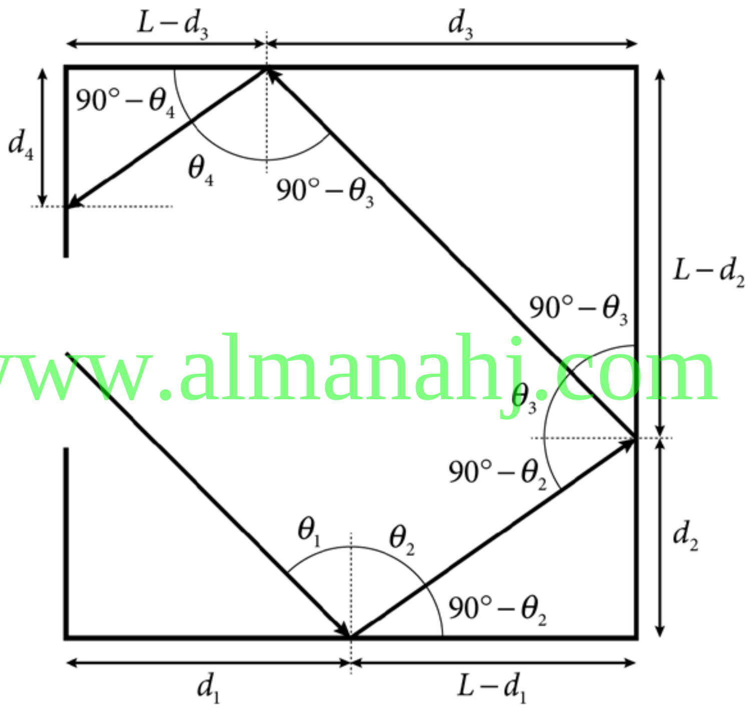
**CALCULATE:** $\theta_{\text{f}} = \cot^{-1}\left(\dfrac{(0.841)\cos 35.9^\circ}{\sin 35.9^\circ}\right) = 40.719775^\circ$

**ROUND:** All values are given to 3 significant figures. The final answer is $40.7^\circ$.

**DOUBLE-CHECK:** This is reasonable since $\theta_{\text{f}} > \theta_{\text{i}}$.

**7.72.** **THINK:** We want to find out if the ball will escape the room. The room is $L$ = 6.00 m by 6.00 m with a 2.00 m wide doorway located in the center of the wall. The coefficient of restitution for the ball is 0.850.

**SKETCH:**






**RESEARCH:** The angle will be given by $\theta_{\text{f}} = \cot^{-1}\left(\varepsilon p_{\text{i}\perp}/p_{\text{i}\parallel}\right) = \cot^{-1}\left(\varepsilon\cot\theta_{\text{i}}\right)$ and this will be used through trigonometry to find the distances.

**SIMPLIFY:** The angles are:

$\theta_2 = \cot^{-1}\left(\varepsilon\cot\theta_1\right)$,

$\theta_3 = \cot^{-1}\left(\varepsilon\cot\left(90^\circ - \theta_2\right)\right)$, and

$\theta_4 = \cot^{-1}\left(\varepsilon\cot\left(90.0^\circ - \theta_3\right)\right)$.

The distances are:

$d_1 = L/2$,

$d_2 = \left(L - d_1\right)\tan\left(90^\circ - \theta_2\right)$,

$d_3 = \left(L - d_2\right)\tan\left(90^\circ - \theta_3\right)$, and

$d_4 = \left(L - d_3\right)\tan\left(90^\circ - \theta_4\right)$.

**CALCULATE:** First calculate the angles

$\theta_2 = \cot^{-1}\left(0.850\cot 45.0°\right) = 49.64°,$

$\theta_3 = \cot^{-1}\left(0.850\cot\left(90.00° - 49.64°\right)\right) = 45.00°,$ and

$\theta_4 = \cot^{-1}\left(0.850\cot^{-1}\left(90.00° - 45.00°\right)\right) = 49.64°.$

Now calculate the distances

$d_1 = 6.00 \text{ m}/2 = 3.00 \text{ m},$

$d_2 = \left(6.00 \text{ m} - 3.00 \text{ m}\right)\tan\left(90.00° - 49.64°\right) = 2.550 \text{ m},$

$d_3 = \left(6.00 \text{ m} - 2.550 \text{ m}\right)\tan\left(90.00° - 45.00°\right) = 3.450 \text{ m},$ and

$d_4 = \left(6.00 \text{ m} - 3.450 \text{ m}\right)\tan\left(90.00° - 49.64°\right) = 2.168 \text{ m}.$

**ROUND:** The last distance $d_4$ is 2.17 m, which is more than 2.00 m from the wall where the door begins. Thus, the soccer ball does bounce back out of the room on the first trip around the room.

**DOUBLE-CHECK:** What would we expect if the coefficient of restitution were 1? We would have $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 45°$ and $d_1 = d_2 = d_3 = d_4 = L/2.$ The soccer ball would return to the same place it entered the room and would exit the room. By calculating $d_4$ for a given $\varepsilon,$ you can show that for $\varepsilon > 0.817,$ the soccer ball will exit the room on its first trip around the room.

**7.73.** **THINK:** I want to know if Jerry will make it over the second fence. Each yard begins and ends with a 2.00 m fence. The range and maximum height of Jerry's initial trajectory are 15.0 m and 5.00 m respectively. Jerry is 7.50 m away from the next fence and he has a coefficient of restitution of 0.80.

**SKETCH:**




**RESEARCH:** From the range and maximum height the initial velocity can be found, along with the angle of Jerry's trajectory. $R = \left(v_0^2/g\right)\sin\left(2\theta\right)$ and $H = \left(v_0^2/\left(2g\right)\right)\sin^2\theta$. With this I can find the *x*- and *y*-components of the velocity. Since the coefficient of restitution only acts on the momentum perpendicular to the ground, $v_{yf} = v_{yi}$ and $v_x$ remains constant. With this information the height Jerry attains after travelling another 7.50 m can be found by using $x = v_x t$ and $y = v_y t - \frac{1}{2}gt^2$.

**SIMPLIFY:** $v_0^2 = \frac{Rg}{\sin\left(2\theta\right)} = \frac{2Hg}{\sin^2\theta} \Rightarrow \frac{R}{2\cos\theta} = \frac{2H}{\sin\theta} \Rightarrow \tan\theta = \frac{4H}{R},$ $\qquad v_0 = \sqrt{\frac{Rg}{\sin 2\theta}} = \sqrt{\frac{2Hg}{\sin^2\theta}}$

$v_{xi} = v_{xf} = v_0\cos\theta = \sqrt{\frac{2Hg}{\sin^2\theta}}\cos\theta = \frac{\sqrt{2Hg}}{\tan\theta} = \frac{\sqrt{2Hg}}{4H/R} = \frac{R\sqrt{2Hg}}{4H},$ and $v_{yf} = \varepsilon v_{yi} = \varepsilon v_0\sin\theta = \varepsilon\sqrt{2Hg}$. The time it takes to reach the fence is given by $x = v_x t$, or $t = x/v_x = \left(4Hx\right)/\left(R\sqrt{2Hg}\right)$, where *x* = 7.5 m. The height it attains in this time is:

$$y = v_y t - \frac{1}{2}gt^2 = \varepsilon\sqrt{2Hg}\frac{4Hx}{R\sqrt{2Hg}} - \frac{1}{2}g\left(\frac{4Hx}{R\sqrt{2Hg}}\right)^2 = \frac{4Hx\varepsilon}{R} - \frac{4Hx^2}{R^2}$$

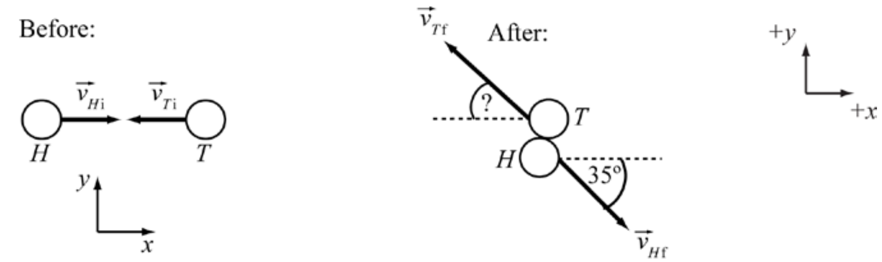
**CALCULATE:** $y = \frac{4\left(5 \text{ m}\right)\left(7.50 \text{ m}\right)\left(0.800\right)}{15.0 \text{ m}} - \frac{4\left(5 \text{ m}\right)\left(7.50 \text{ m}\right)^2}{\left(15.0 \text{ m}\right)^2} = 3 \text{ m}$

**ROUND:** Jerry is at a height of 3 m when he reaches the fence, which means that he does make it over the next fence, with exactly 1 meter to spare.

**DOUBLE-CHECK:** This is a reasonable answer for the world of cartoon characters.

**7.74.** **THINK:** I want to find the angle $\theta_{\text{T}}$ at which Toyohibiki moves after collision. Hakurazan and Toyohibiki have masses and speeds of $m_{\text{H}} = 135 \text{ kg}$, $v_{\text{Hi}} = 3.5 \text{ m/s}$, $m_{\text{T}} = 173 \text{ kg}$, and $v_{\text{Ti}} = 3.0 \text{ m/s}$. After collision, there is a loss of 10% of the kinetic energy, and $\theta_{\text{H}} = 35°$.

**SKETCH:**




**RESEARCH:** I can use the conservation of momentum along the *x* and *y* axes. $\vec{p}_{\text{Hi}} + \vec{p}_{\text{Ti}} = \vec{p}_{\text{Hf}} + \vec{p}_{\text{Tf}}$. Since the relation between the initial and final kinetic energy is known, I can also use the equation $K_{\text{f}} = 0.90 K_{\text{i}}$.

**SIMPLIFY:** First set up the three equations starting with momentum along the *x* axis:

$$m_{\text{H}} v_{\text{Hi}} - m_{\text{T}} v_{\text{Ti}} = m_{\text{H}} v_{\text{Hf}} \cos\theta_{\text{H}} - m_{\text{T}} v_{\text{Tf}} \cos\theta_{\text{T}} \quad (1)$$

Along the *y*-axis:

$$0 = m_{\text{T}} v_{\text{Tf}} \sin\theta_{\text{T}} - m_{\text{H}} v_{\text{Hf}} \sin\theta_{\text{H}} \quad (2)$$

The energy gives:



$$0.90\left(m_{\text{H}} v_{\text{Hi}}^2 + m_{\text{T}} v_{\text{Ti}}^2\right) = m_{\text{H}} v_{\text{Hf}}^2 + m_{\text{T}} v_{\text{Tf}}^2 \quad (3)$$

Use the first two equations to find $\left(m_{\text{T}} v_{\text{Tf}} \sin\theta_{\text{T}}\right)^2$ and $\left(m_{\text{T}} v_{\text{Tf}} \cos\theta_{\text{T}}\right)^2$.

$$\left(m_{\text{T}} v_{\text{Tf}} \cos\theta_{\text{T}}\right)^2 = \left(m_{\text{H}} v_{\text{Hf}} \cos\theta_{\text{H}} + m_{\text{T}} v_{\text{Ti}} - m_{\text{H}} v_{\text{Hi}}\right)^2 \quad \text{and} \quad \left(m_{\text{T}} v_{\text{Tf}} \sin\theta_{\text{T}}\right)^2 = m_{\text{H}}^2 v_{\text{Hf}}^2 \sin^2\theta_{\text{H}}$$

$$\left(m_{\text{T}} v_{\text{Tf}} \cos\theta_{\text{T}}\right)^2 + \left(m_{\text{T}} v_{\text{Tf}} \sin\theta_{\text{T}}\right)^2 = \left(m_{\text{H}} v_{\text{Hf}} \cos\theta_{\text{H}} + m_{\text{T}} v_{\text{Ti}} - m_{\text{H}} v_{\text{Hi}}\right)^2 + m_{\text{H}}^2 v_{\text{Hf}}^2 \sin^2\theta_{\text{H}}$$

$$\left(m_{\text{T}} v_{\text{Tf}}\right)^2 \left(\sin^2\theta_{\text{T}} + \cos^2\theta_{\text{T}}\right) = \left(m_{\text{H}} v_{\text{Hf}} \cos\theta_{\text{H}} + m_{\text{T}} v_{\text{Ti}} - m_{\text{H}} v_{\text{Hi}}\right)^2 + m_{\text{H}}^2 v_{\text{Hf}}^2 \sin^2\theta_{\text{H}}$$

$$m_{\text{T}}^2 v_{\text{Tf}}^2 = \left(m_{\text{H}} v_{\text{Hf}} \cos\theta_{\text{H}} + m_{\text{T}} v_{\text{Ti}} - m_{\text{H}} v_{\text{Hi}}\right)^2 + m_{\text{H}}^2 v_{\text{Hf}}^2 \sin^2\theta_{\text{H}} \quad (4)$$

Substituting this into the third equation gives:

$$0.90\left(m_{\text{H}} v_{\text{Hi}}^2 + m_{\text{T}} v_{\text{Ti}}^2\right) = m_{\text{H}} v_{\text{Hf}}^2 + \frac{\left(m_{\text{H}} v_{\text{Hf}} \cos\theta_{\text{H}} + m_{\text{T}} v_{\text{Ti}} - m_{\text{H}} v_{\text{Hi}}\right)^2}{m_{\text{T}}} + \frac{m_{\text{H}}^2 v_{\text{Hf}}^2}{m_{\text{T}}} \sin^2\theta_{\text{H}}.$$

This quadratic equation in $v_{\text{Hf}}$ which simplifies to: $0.90\left((135 \text{ kg})(3.5 \text{ m/s})^2 + (173 \text{ kg})(3 \text{ m/s})^2\right)$

$$= (135 \text{ kg}) v_{\text{Hf}}^2 + \frac{\left((135 \text{ kg}) v_{\text{Hf}} \cos 35 + (173 \text{ kg})(3 \text{ m/s}) - (135 \text{ kg})(3.5 \text{ m/s})\right)^2}{(173 \text{ kg})} + \frac{(135 \text{ kg})^2 v_{\text{Hf}}^2}{(173 \text{ kg})} \sin^2 35°$$

$$\Rightarrow 240.3468 v_{\text{Hf}}^2 + 59.4477 v_{\text{Hf}} - 2877.176 = 0 \quad (5)$$

Solving equation (4) for $v_{\text{Tf}}$ gives the equation: $v_{\text{Tf}} = \sqrt{\dfrac{\left(m_{\text{H}}v_{\text{Hf}}\cos\theta_{\text{H}} + m_{\text{T}}v_{\text{Ti}} - m_{\text{H}}v_{\text{Hi}}\right)^2 + m_{\text{H}}^2 v_{\text{Hf}}^2 \sin^2\theta_{\text{H}}}{m_{\text{T}}^2}}$.

**CALCULATE:** Solving equation (5), gives $v_{\text{Hf}} = 3.3384$ m/s. Using this in equation (4), gives:

$$v_{\text{Tf}} = \sqrt{\frac{\left[(135)(3.3384)\cos 35° + (173)(3.0) - (135)(3.5)\right]^2 + (135)^2(3.3384)^2\sin^2 35°}{(173)^2}}$$

$= 2.829$, with units of:

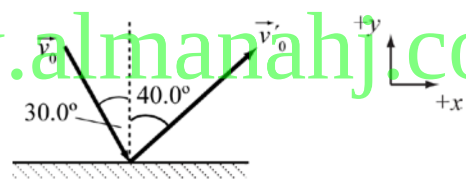
$\sqrt{\dfrac{\left[(\text{kg})(\text{m/s}) + (\text{kg})(\text{m/s}) - (\text{kg})(\text{m/s})\right]^2 + (\text{kg})^2(\text{m/s})^2}{(\text{kg})^2}} = \text{m/s}$. Therefore, $v_{\text{Tf}} = 2.829$ m/s. Use equation (2) to find $\theta_{\text{T}}$: $\theta_{\text{T}} = \sin^{-1}\left(\dfrac{(135\text{ kg})(3.3384\text{ m/s})}{(173\text{ kg})(2.829\text{ m/s})}\sin 35°\right) = 31.88°$.

**ROUND:** The angle $\theta_{\text{H}}$ is given to two significant figures and limits our answer to two significant figures. $\theta_{\text{T}} = 32°$.

**DOUBLE-CHECK:** The sumo wrestlers' masses and initial speeds and directions are similar, so in is reasonable that their final speeds and directions would be similar as well.

**7.75.** **THINK:** I want to find the coefficient of restitution and the ratio of the final and initial kinetic energies. The puck initially has a mass, velocity and angle of $m$ = 170. g, $v_0 = 2.00$ m/s, and $\theta_i = 30.0°$ respectively. The puck bounces off the board with an angle of $\theta_{\text{f}} = 40.0°$.

**SKETCH:**






**RESEARCH:** To find the coefficient of restitution we will use $\theta_{\text{f}} = \cot^{-1}\left(\varepsilon p_{\text{i}\wedge}/p_{\text{iP}}\right)$. To find the ratio for the initial kinetic energy we will use $p_{\text{f}\wedge} = p_{\text{i}\wedge}$ and $p_{\text{fP}} = p_{\text{iP}}$.

**SIMPLIFY:** The coefficient of restitution is given by:

$$\cot\theta_{\text{f}} = \frac{\varepsilon p_{\text{i}\perp}}{p_{\text{i}\parallel}} = \frac{\varepsilon v_0\cos\theta_{\text{i}}}{v_0\sin\theta_{\text{i}}} = \varepsilon\cot\theta_{\text{i}} \Rightarrow \varepsilon = \frac{\cot\theta_{\text{f}}}{\cot\theta_{\text{i}}}$$

Now we use $p_{\text{f}\parallel} = p_{\text{i}\parallel}$ to find $v_0'$.

$$p_{\text{i}\parallel} = p_{\text{f}\parallel} \Rightarrow mv_{x\text{i}} = mv_{x\text{f}} \Rightarrow v_0\sin\theta_{\text{i}} = v_0'\sin\theta_{\text{f}} \Rightarrow v_0' = v_0\frac{\sin\theta_{\text{i}}}{\sin\theta_{\text{f}}}$$

The ratio $K_{\text{f}}/K_{\text{i}} = \dfrac{(1/2)mv_0'^2}{(1/2)mv_0^2} = \dfrac{\sin^2\theta_{\text{i}}}{\sin^2\theta_{\text{f}}}$.

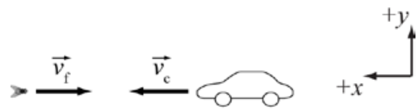
**CALCULATE:** $\varepsilon = \dfrac{\cot 40.0°}{\cot 30.0°} = 0.688059$, and $\dfrac{K_{\text{f}}}{K_{\text{i}}} = \dfrac{\sin^2 30.0°}{\sin^2 40.0°} = 0.6051$.

**ROUND:** The coefficient of restitution is $\varepsilon = 0.688$ and the kinetic energy ratio $K_{\text{f}}/K_{\text{i}} = 0.605$.

**DOUBLE-CHECK:** These numbers seem reasonable.

**7.76.** **THINK:** I want to know the speed a 5.00 g fly must have to slow a 1900. kg car by 5.00 mph. The car is travelling at an initial speed of 55.0 mph.

**SKETCH:**




**RESEARCH:** Using the conservation of momentum: $m_F\vec{v}_F + m_C\vec{v}_C = (m_F + m_C)\vec{v}'$.

**SIMPLIFY:** $-m_Fv_F + m_Cv_C = (m_F + m_C)(v_C - \Delta v)$

$$m_Fv_F = m_Cv_C - (m_F + m_C)(v_C - \Delta v)$$

$$v_F = \frac{m_Cv_C - (m_F + m_C)(v_C - \Delta v)}{m_F}$$

**CALCULATE:** $v_F = \frac{(1900.\text{ kg})(55.0\text{ mph}) - (1900.\text{ kg} + 0.00500\text{ kg})(50.0\text{ mph})}{0.00500\text{ kg}} = 1899950\text{ mph}$

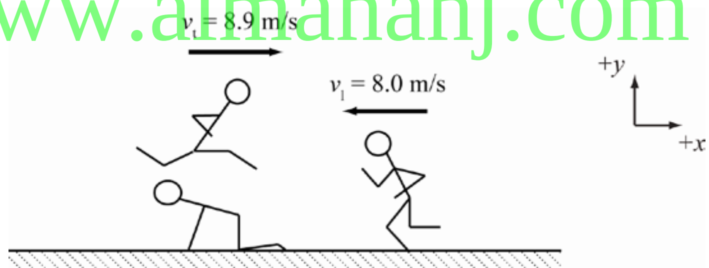
**ROUND:** The fly must travel $1.90\cdot 10^6$ mph to change the speed of the car by 5.00 mph.

**DOUBLE-CHECK:** This is a crazy speed for a fly to attain. It is about 800000 m/s. This value is extreme, but the notion of a fly being able to slow a car from 55 mph to 50 mph is absurd, and this is verified by the very high speed required of the fly.

**7.77.** **THINK:** I want to find the speed of the tailback and linebacker and if the tailback will score a touchdown. The tailback has mass and velocity $m_t = 85.0$ kg and $v_t = 8.90$ m/s, and the linebacker has mass and velocity $m_l = 110.$ kg and $v_l = -8.00$ m/s.

**SKETCH:**






**RESEARCH:** The conservation of momentum for this perfectly inelastic collision is

$$m_tv_t + m_lv_l = (m_t + m_l)v.$$

**SIMPLIFY:** Rearranging the equation to solve for the final velocity, $v = \frac{m_tv_t + m_lv_l}{m_t + m_l}$.

**CALCULATE:** $v = \frac{(85.0\text{ kg})(8.90\text{ m/s}) - (110.\text{ kg})(8.00\text{ m/s})}{85.0\text{ kg} + 110.\text{ kg}} = -0.633\text{ m/s}$

**ROUND:**

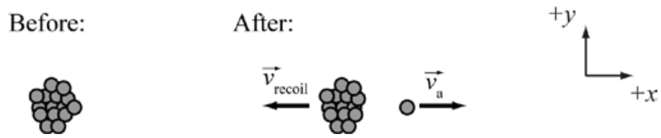
(a) The values are given to 3 significant figures so the final speed is 0.633 m/s.

(b) Since the velocity is negative, the two go in the direction of the linebacker and the tailback does not score a touchdown.

**DOUBLE-CHECK:** The speed is quite small, as would be expected of two people opposing each other's motion. Since the initial momentum of the linebacker is greater than the initial momentum of the tailback, the tailback should not be able to score. This is consistent with the calculated result.

**7.78.** **THINK:** I want to know the recoil speed of the remaining nucleus. The thorium-228 nucleus starts at rest with a mass of $m_t = 3.8 \cdot 10^{-25}$ kg and the emitted alpha particle has mass $m_a = 6.64 \cdot 10^{-27}$ kg and velocity $v_a = 1.8 \cdot 10^7$ m/s.

**SKETCH:**




**RESEARCH:** Using the conservation of momentum: $0 = (m_t - m_a)v_{recoil} - m_a v_a$.

**SIMPLIFY:** $(m_t - m_a)v_{recoil} = m_a v_a \Rightarrow v_{recoil} = \dfrac{m_a v_a}{m_t - m_a}$

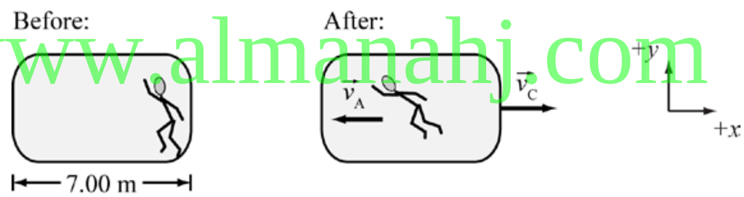
**CALCULATE:** $v_{recoil} = \dfrac{(6.64 \cdot 10^{-27} \text{ kg})(1.8 \cdot 10^7 \text{ m/s})}{(3.8 \cdot 10^{-25} \text{ kg}) - (6.64 \cdot 10^{-27} \text{ kg})} = 320{,}120 \text{ m/s}$

**ROUND:** The given values have two significant figures. Therefore, the recoil velocity is $3.2 \cdot 10^5$ m/s.

**DOUBLE-CHECK:** This value seems like a reasonable speed because it is less than the speed of the alpha particle.

**7.79.** **THINK:** I want to know the time it takes the astronaut to reach the other side of the 7.00 m long capsule. The astronaut and the capsule both start at rest. The astronaut and capsule have masses of 60.0 kg and 500. kg respectively. After the astronaut's kick, he reaches a velocity is 3.50 m/s.

**SKETCH:**






**RESEARCH:** I can find the speed of the capsule by using the conservation of momentum. The time it takes is the distance divided by the sum of the velocities.

**SIMPLIFY:** $m_A v_A = m_C v_C \Rightarrow v_C = \dfrac{m_A}{m_C} v_A$. Using this in our distance equation:

$$d = (v_A + v_C)t = \left(v_A + \frac{m_A}{m_C} v_A\right)t = v_A\left(1 + \frac{m_A}{m_C}\right)t \Rightarrow t = \frac{d}{v_A\left(1 + \dfrac{m_A}{m_C}\right)}$$

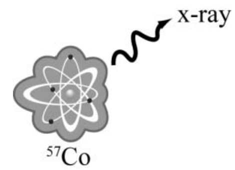
**CALCULATE:** $t = \dfrac{7.00 \text{ m}}{(3.50 \text{ m/s})(1 + (60.0 \text{ kg})/(500. \text{ kg}))} = 1.7857 \text{ s}$

**ROUND:** The time to cross the capsule is reported as 1.79 s.

**DOUBLE-CHECK:** This is a reasonable time.

**7.80.** **THINK:** The conservation of momentum and the definition of kinetic energy and momentum can be used to find the momentum and kinetic energy of a $^{57}$Co nucleus that emits an x-ray. The nucleus has a mass of $m_{Co} = 9.52 \cdot 10^{-26}$ kg and the x-ray has a momentum and kinetic energy of 14 keV/c and 14 keV, respectively.

**SKETCH:**




**RESEARCH:** $p_{\text{Co}} = -p_{\text{x-ray}}$, $K = \frac{1}{2}mv^2$ and $p = mv$.

**SIMPLIFY:** $K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = \frac{p^2}{2m}$

**CALCULATE:**

$$p_{\text{Co}} = -p_{\text{x-ray}} = -14 \text{ keV/c} = \frac{(-14\cdot 10^3 \text{ eV/c})(1.602\cdot 10^{-19} \text{ J/eV})}{(2.998\cdot 10^8 \text{ (m/s)/c})} = -7.4810\cdot 10^{-24} \text{ kg m/s}$$

$$K = \frac{(-7.4810\cdot 10^{-24} \text{ kg m/s})^2}{2(9.52\cdot 10^{-26} \text{ kg})} = 2.939\cdot 10^{-22} \text{ J} = 1.83\cdot 10^{-3} \text{ eV}$$

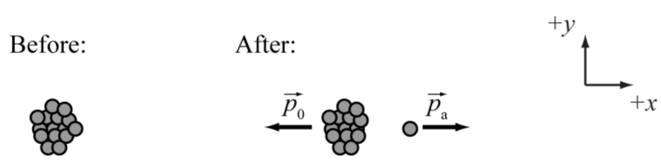
**ROUND:** The momentum is given to two significant figures so the answer can be reported to two significant figures: $p_{\text{Co}} = -14$ keV/c and $K = 1.8\cdot 10^{-3}$ eV. The negative sign means that the $^{57}$Co nucleus is in the opposite direction of the x-ray.

**DOUBLE-CHECK:** These are reasonable values.

**7.81.** **THINK:** I am looking for the velocity of the nucleus after the decay. The atom starts at rest, i.e. $v_0 = 0$ m/s, and its nucleus has mass $m_0 = 3.68\cdot 10^{-25}$ kg. The alpha particle has mass $m_\text{a} = 6.64\cdot 10^{-27}$ kg and energy $8.79\cdot 10^{-13}$ J.



**SKETCH:**




**RESEARCH:** I can find the velocity of the alpha particle with the equation $K = \frac{1}{2}m_\text{a}v_\text{a}^2$. The conservation of momentum gives $p_\text{a} = p_0$.

**SIMPLIFY:** $v_\text{a} = \sqrt{\frac{2K}{m_\text{a}}}$, $p_\text{a} = p_0 \Rightarrow m_\text{a}v_\text{a} = (m_0 - m_\text{a})v_0 \Rightarrow v_0 = \frac{m_\text{a}}{(m_0 - m_\text{a})}v_\text{a} = \frac{m_\text{a}}{(m_0 - m_\text{a})}\sqrt{\frac{2K}{m_\text{a}}}$

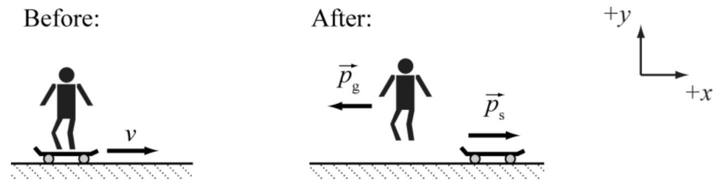
**CALCULATE:** $v_0 = \frac{6.64\cdot 10^{-27} \text{ kg}}{(3.68\cdot 10^{-25} \text{ kg} - 6.64\cdot 10^{-27} \text{ kg})}\sqrt{\frac{2(8.79\cdot 10^{-13} \text{ J})}{6.64\cdot 10^{-27} \text{ kg}}} = 298988$ m/s

**ROUND:** The values are given to three significant figures, so $v_0 = 2.99\cdot 10^5$ m/s.

**DOUBLE-CHECK:** Such a high speed is reasonable for such small masses.

**7.82.** **THINK:** I am looking for the speed of the skateboarder after she jumps off her skateboard. She has a mass of $m_\text{g} = 35.0$ kg and the skateboard has mass $m_\text{s} = 3.50$ kg. They initially travel at $v = 5.00$ m/s in the same direction.

**SKETCH:**




**RESEARCH:** To find the speed we can use the conservation of momentum.

**SIMPLIFY:** $(m_g + m_s)v = m_s v_s - m_g v_g \Rightarrow m_g v_g = m_s v_s - (m_g + m_s)v \Rightarrow v_g = \left|\dfrac{m_s v_s - (m_g + m_s)v}{m_g}\right|$

**CALCULATE:** $v_g = \left|\dfrac{(3.50 \text{ kg})(8.50 \text{ m/s}) - (35.0 \text{ kg} + 3.50 \text{ kg})(5.00 \text{ m/s})}{35.0 \text{ kg}}\right| = 4.65 \text{ m/s}$

**ROUND:** The speed is accurate to three significant figures since all of our values are given to three significant figures. The speed of the girl is $v_g = 4.65$ m/s.

**DOUBLE-CHECK:** This is a reasonable speed.

**7.83.** **THINK:** I am looking for the recoil the archer experiences. The mass of the archer and the arrow are $m_A = 50.0$ kg and $m_B = 0.100$ kg respectively. The initial velocity is 0 and the arrow has a velocity $v_B = 95.0$ m/s.

**SKETCH:**




**RESEARCH:** I use the conservation of momentum to find the recoil velocity: $p_A = -p_B$.

**SIMPLIFY:** $m_A v_A = -m_B v_B \Rightarrow v_A = -\dfrac{m_B}{m_A} v_B$

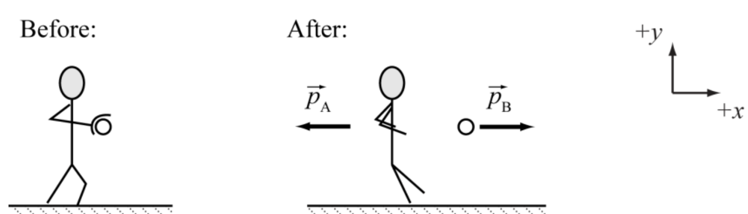
**CALCULATE:** $v_A = -\dfrac{0.100 \text{ kg}}{50.0 \text{ kg}}(95.0 \text{ m/s}) = -0.190 \text{ m/s}$

**ROUND:** The three significant figures of the values limit the answers to three significant figures. The recoil speed of the archer is 0.190 m/s.

**DOUBLE-CHECK:** This is reasonable recoil.

**7.84.** **THINK:** I want to find the recoil of an astronaut starting at rest after he throws a baseball. The astronaut and baseball have masses $m_A = 55.0$ kg and $m_B = 0.145$ kg respectively. The ball is thrown with a speed of 31.3 m/s.

**SKETCH:**




**RESEARCH:** I can find the recoil speed of the astronaut with the conservation of momentum: $p_A = -p_B$.

**SIMPLIFY:** $m_A v_A = -m_B v_B \Rightarrow v_A = -\dfrac{m_B}{m_A} v_B$

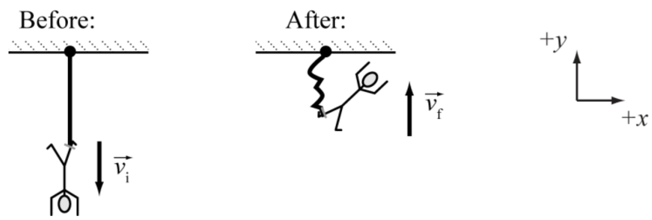
**CALCULATE:** $v_A = -\dfrac{0.145 \text{ kg}}{55.0 \text{ kg}}(31.3 \text{ m/s}) = -0.082518 \text{ m/s}$

**ROUND:** The values are given to three significant figures so the recoil speed will be reported to three significant figures. The recoil speed is $v_A = 0.0825$ m/s.

**DOUBLE-CHECK:** This is a reasonable speed.

**7.85.** **THINK:** I want to find the average force exerted on the jumper and the number of $g$'s she experiences. She has a mass of $m_j = 55.0$ kg and reaches a speed of $v_i = 13.3$ m/s downwards then goes $v_f = 10.5$ m/s upwards after the cord pulls her back up in $\Delta t = 1.25$ s.

**SKETCH:**




**RESEARCH:** I use the impulse equation, $F\Delta t = \Delta p$, to find the net force acting on the jumper. I can then use $F = ma$ to find the net force (cord pulling up plus gravity pulling down) and then the number of $g$'s experienced. Number of $g$'s is determined by the action of forces *other* than gravity, so in this case the cord tension. (A person standing motionless on the ground experiences 1 $g$ from the upward normal force.)

**SIMPLIFY:** $F = \dfrac{\Delta p}{\Delta t} = \dfrac{m_j(v_f - v_i)}{\Delta t}$ and $a = \dfrac{F}{m_j}$.



**CALCULATE:** $F_{net} = \dfrac{(55.0 \text{ kg})(10.5 \text{ m/s} - (-13.3 \text{ m/s}))}{1.25 \text{ s}} = 1047.2 \text{ N}$

$F_{net} = F_{cord} - mg \Rightarrow F_{cord} = F_{net} + mg = 1047.2 \text{ N} + (55.0 \text{ kg})(9.81 \text{ m/s}^2) = 1586.75 \text{ N}$

Acceleration due to cord: $a = \dfrac{F_{cord}}{m} = \dfrac{1586.75 \text{ N}}{55.0 \text{ kg}} = 28.85 \text{ m/s}^2$.

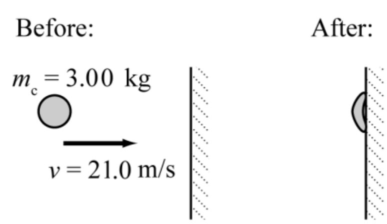
Dividing 28.85 by 9.81, the cord subjects the number to 2.9408 $g$'s.

**ROUND:** The values are given to three significant figures, so the average force is 1590 N and the jumper experiences 2.94 $g$'s.

**DOUBLE-CHECK:** These numbers are within reasonable levels. A person can experience a few $g$'s without harm and without losing consciousness.

**7.86.** **THINK:** I want to find the impulse exerted on the ball of clay when it sticks to a wall. The ball has a mass of $m_c = 3.00$ kg and speed $v = 21.0$ m/s.

**SKETCH:**




**RESEARCH:** I use the impulse equation. $J = \Delta p = F\Delta t$.

**SIMPLIFY:** $J = m_c(v_f - v_i)$

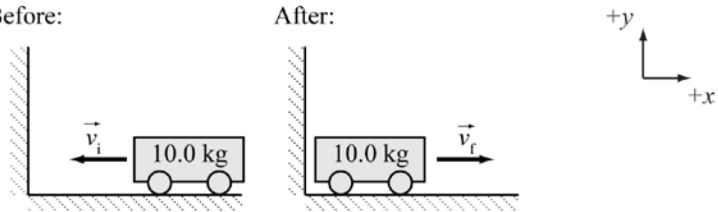
**CALCULATE:** $|J| = \left|(3.00\text{ kg})(0\text{ m/s} - 21.0\text{ m/s})\right| = 63.0\text{ kg m/s}$

**ROUND:** Our result will have threesignificant figures since our values are accurate to three significant figures. The impulse exerted on the ball is 63.0 kg m/s.

**DOUBLE-CHECK:** This is a reasonable value.

**7.87.** **THINK:** I want to find the change in the momentum of the cart. (This is the same as the impulse.) The cart has a mass of 10.0 kg and initially travels at $v_\text{i} = 2.00\text{ m/s}$ to the left then travels at $v_\text{f} = 1.00\text{ m/s}$ after it hits the wall.

**SKETCH:**




**RESEARCH:** All I need to do is find the momentum in each case then subtract them to find the change in momentum. $\Delta p = p_\text{f} - p_\text{i}$.

**SIMPLIFY:** $\Delta p = p_\text{f} - p_\text{i} = mv_\text{f} - mv_\text{i} = m(v_\text{f} - v_\text{i})$

**CALCULATE:** $\Delta p = (10.0\text{ kg})(1.00\text{ m/s} - (-2.00\text{ m/s})) = 30.0\text{ kg m/s}$

**ROUND:** The change in momentum is 30.0 kg m/s.

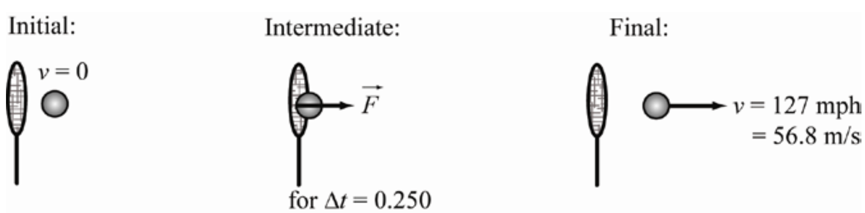
**DOUBLE-CHECK:** This is a reasonable value.



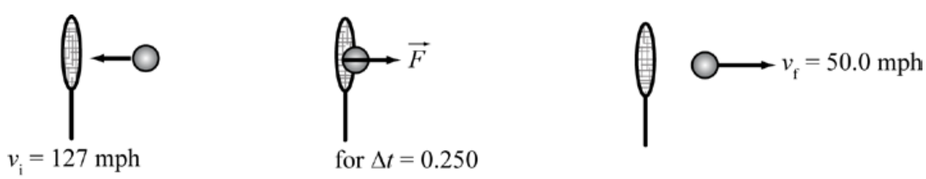
**7.88.** **THINK:** I have a tennis ball with mass $57.0\text{ g} = 5.70 \cdot 10^{-2}\text{ kg}$ and speed 127 mph $= (127\text{ mph})(0.447\text{ (m/s)/mph}) = 56.8\text{ m/s}$. I want to calculate impulse. I am given that $\Delta t = 0.250\text{ s}$.

**SKETCH:**

(a)




(b)




**RESEARCH:** I use the definition of impulse. $J = F_{ave}\Delta t = \Delta p$.

**SIMPLIFY:**

(a) The tennis ball is initially at rest before the serve. $v_\text{i} = 0\text{ m/s}$.

$$J = F\Delta t = \Delta p = m(v_\text{f} - v_\text{i}) = mv_\text{f} \quad \Rightarrow \quad F = \frac{mv_\text{f}}{\Delta t}$$

(b) The tennis ball has an initial speed $v_i = -127 \text{ mph} = -56.8 \text{ m/s}$ and a final speed $v_f = 50.0 \text{ mph} = 22.4 \text{ m/s}$.

$$J = F\Delta t = \Delta p = m(v_f - v_i) \Rightarrow F = \frac{m(v_f - v_i)}{\Delta t}$$

**CALCULATE:**

(a) $F = \dfrac{(5.70 \cdot 10^{-2} \text{ kg})(56.8 \text{ m/s})}{0.250 \text{ s}} = 12.95 \text{ N}$

(b) $F = \dfrac{(5.70 \cdot 10^{-2} \text{ kg})(22.4 \text{ m/s} - (-56.8 \text{ m/s}))}{0.250 \text{ s}} = 18.06 \text{ N}$

**ROUND:**

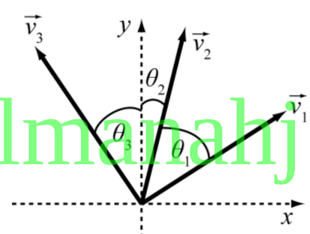
(a) $F = 13.0$ N

(b) $F = 18.1$ N

**DOUBLE-CHECK:** I expect the answer for (a) to be less than that of (b) because $v_f - v_i$ in (a) is less than in (b).

**7.89.** **THINK:** I have three birds with masses $m_1 = 0.100 \text{ kg}$, $m_2 = 0.123 \text{ kg}$, and $m_3 = 0.112 \text{ kg}$ and speeds $v_1 = 8.00 \text{ m/s}$, $v_2 = 11.0 \text{ m/s}$, and $v_3 = 10.0 \text{ m/s}$. They are flying in directions $\theta_1 = 35.0°$ east of north, $\theta_2 = 2.00°$ east of north, and $\theta_3 = 22.0°$ west of north, respectively. I want to calculate the net momentum.

**SKETCH:**






**RESEARCH:** $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$, or in component form, $p_x = p_{1x} + p_{2x} + p_{3x}$ and $p_y = p_{1y} + p_{2y} + p_{3y}$.

**SIMPLIFY:** $p_x = m_1v_1\sin\theta_1 + m_2v_2\sin\theta_2 - m_3v_3\sin\theta_3$, $p_y = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2 + m_3v_3\cos\theta_3$,

$\vec{p} = p_x\hat{x} + p_y\hat{y}$, and $\vec{v} = \dfrac{\vec{p}}{m}$.

**CALCULATE:**

$$p_x = (0.100 \text{ kg})(8.00 \text{ m/s})\sin 35.0° + (0.123 \text{ kg})(11.0 \text{ m/s})\sin 2.00° - (0.112 \text{ kg})(10.0 \text{ m/s})\sin 22.0°$$
$$= 0.0865 \text{ kg m/s}$$

$$p_y = (0.100 \text{ kg})(8.00 \text{ m/s})\cos 35.0° + (0.123 \text{ kg})(11.0 \text{ m/s})\cos 2.00° + (0.112 \text{ kg})(10.0 \text{ m/s})\cos 22.0°$$
$$= 3.0459 \text{ kg m/s}$$

The speed of a 0.115 kg bird is: $\vec{v} = \dfrac{0.0865 \text{ kg m/s } \hat{x} + 3.0459 \text{ kg m/s } \hat{y}}{0.115 \text{ kg}} = 0.752 \text{ m/s } \hat{x} + 26.486 \text{ m/s } \hat{y}$.

$|\vec{v}| = \sqrt{(0.752 \text{ m/s})^2 + (26.486 \text{ m/s})^2} = 26.497 \text{ m/s}$,

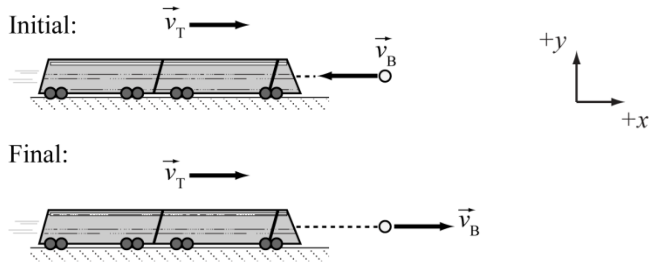
$\tan\theta = \dfrac{0.752 \text{ m/s}}{26.486 \text{ m/s}} = 0.02839 \Rightarrow \theta = \tan^{-1}(0.02839) = 1.626°$ east of north

**ROUND:** $p_x = 0.0865 \text{ kg m/s}$, $p_y = 3.05 \text{ kg m/s}$, $\vec{p} = 0.0865 \text{ kg m/s } \hat{x} + 3.05 \text{ kg m/s } \hat{y}$, $|\vec{v}| = 26.5 \text{ m/s}$, $\theta = 1.63°$ east of north

**DOUBLE-CHECK:** The speed of the fourth bird must be less than the sum of the speeds of the three birds. $v = v_1 + v_2 + v_3 = 8.00 \text{ m/s} + 11.0 \text{ m/s} + 10.0 \text{ m/s} = 29.0 \text{ m/s}$.

**7.90.** **THINK:** I have a golf ball with mass $m_B = 45.0\text{ g} = 0.0450\text{ kg}$ and speed $v_B = 120.\text{ km/h} = 33.3\text{ m/s}$. A train has mass $m_T = 3.80 \cdot 10^5\text{ kg}$ with speed $v_T = 300.\text{ km/h} = 83.3\text{ m/s}$. I want to calculate the speed of the golf ball after collision.

**SKETCH:**




**RESEARCH:** I use the conservation of momentum and energy. $p_i = p_f$ and $K_i = K_f$.

**SIMPLIFY:** $p_i = p_f \Rightarrow m_B v_{Bi} + m_T v_{Ti} = m_B v_{Bf} + m_T v_{Tf} \Rightarrow m_B\left(v_{Bi} - v_{Bf}\right) = m_T\left(v_{Tf} - v_{Ti}\right)$ (1)

$$K_i = K_f$$

$$\frac{1}{2}m_B v_{Bi}^2 + \frac{1}{2}m_T v_{Ti}^2 = \frac{1}{2}m_B v_{Bf}^2 + \frac{1}{2}m_T v_{Tf}^2$$

$$m_B\left(v_{Bi}^2 - v_{Bf}^2\right) = m_T\left(v_{Ti}^2 - v_{Tf}^2\right)$$

$$m_B\left(v_{Bi} - v_{Bf}\right)\left(v_{Bi} + v_{Bf}\right) = m_T\left(v_{Ti} - v_{Tf}\right)\left(v_{Ti} + v_{Tf}\right)$$

Using equation (1) above, I have $v_{Bi} + v_{Bf} = v_{Tf} + v_{Ti}$, or $v_{Tf} = v_{Bi} + v_{Bf} - v_{Ti}$. Therefore,

$$m_B v_{Bi} + m_T v_{Ti} = m_B v_{Bf} + m_T\left(v_{Bi} + v_{Bf} - v_{Ti}\right)$$

$$\left(m_B - m_T\right)v_{Bi} + 2m_T v_{Ti} = \left(m_B + m_T\right)v_{Bf}$$

$$v_{Bf} = \left(\frac{m_B - m_T}{m_B + m_T}\right)v_{Bi} + \left(\frac{2m_T}{m_B + m_T}\right)v_{Ti}$$



Since $m_B$ is much smaller than $m_T$, i.e. $m_B \ll m_T$, I can approximate: $\frac{m_B - m_T}{m_B + m_T} \approx -1$, , $\frac{2m_T}{m_B + m_T} \approx 2$, and $v_{Bf} \approx -v_{Bi} + 2v_{Ti}$

**CALCULATE:** $v_{Bi} = -33.3\text{ m/s}$, $v_{Ti} = 83.3\text{ m/s}$, and $v_{Bf} = -(-33.3\text{ m/s}) + 2(83.3\text{ m/s}) = 199.9\text{ m/s}$.

**ROUND:** Rounding to three significant figures: $v_{Bf} = 200.\text{ m/s}$

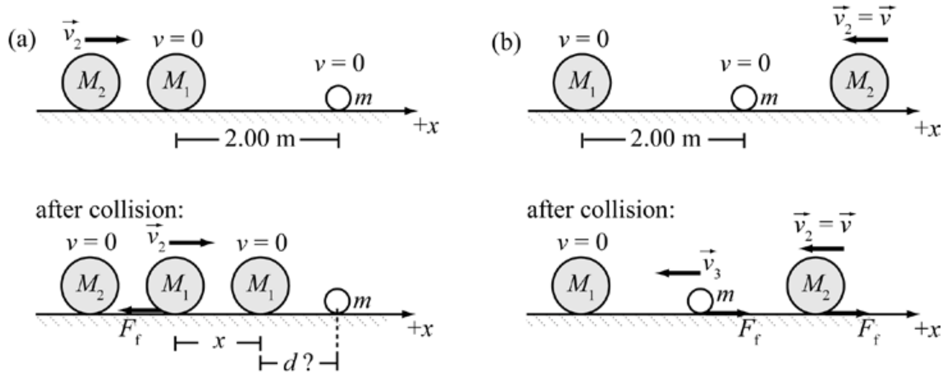
**DOUBLE-CHECK:** Let us compute $\left(m_B - m_T\right)/\left(m_B + m_T\right)$.

$$\frac{m_B - m_T}{m_B + m_T} = \frac{0.0450\text{ kg} - 3.80 \cdot 10^5\text{ kg}}{0.0450\text{ kg} + 3.80 \cdot 10^5\text{ kg}} = -0.9999997...$$

The approximation is correct.

**7.91.** **THINK:** I have two balls with masses $M_1 = 1.00\text{ kg}$ and $m = 0.0450\text{ kg}$ which are a distance of $d_0 = 2.00\text{ m}$ apart. I threw a third ball with mass $M_2$ at a speed $v = 1.00\text{ m/s}$. Calculate the distance between the balls after the collision. $M_1 = M_2 = M$.

**SKETCH:**




**RESEARCH:**

(a) Since $M_1 = M_2 = M$ and there is an elastic collision, the final speeds after collision are $v_2 = 0$ m/s and $v_1 = v = 1.00$ m/s.

(b) Since $m \ll M$ and there is an elastic collision, the final speeds after the collision are $v_2 = v$ and $v_3 = 2v_2 = 2v = 2.00$ m/s.

**SIMPLIFY:** Using Newton's second law, $F_f = \mu_k N = \mu_k mg = ma$. Therefore the acceleration is $a = \mu_k g$. The distance travelled by a ball is $x = v_0^2 / (2a) \Rightarrow x = v_0^2 / (2\mu_k g)$. Therefore the distance between the two balls is $d = d_0 - x = d_0 - v_0^2 / (2\mu_k g)$.

**CALCULATE:**

(a) $v_0 = 1.00$ m/s. The distance between the first ball and the pallina is:



$$d = 2.00 \text{ m} - \frac{(1.00 \text{ m/s})^2}{2(0.200)(9.81 \text{ m/s}^2)} = 1.745 \text{ m}.$$

The distance between the second ball and the pallina is 2.00 m because it stops after the collision.

(b) (i) $2v_0 = v_3 = 2.00$ m/s. The distance between the first ball and the pallina is:

$$d_1 = d_0 - \frac{v_0^2}{2\mu_k g} = 2.00 \text{ m} - \frac{(2.00 \text{ m/s})^2}{2(0.200)(9.81 \text{ m/s}^2)} = 0.9806 \text{ m}.$$

(ii) $v_0 = v_2 = 1.00$ m/s. The distance between the first ball and second ball is:

$$d_2 = d_0 - \frac{v_0^2}{2\mu_k g} = 2.00 \text{ m} - \frac{(1.00 \text{ m/s})^2}{2(0.200)(9.81 \text{ m/s}^2)} = 1.745 \text{ m}$$

The distance between the second ball and the pallina is $d = d_2 - d_1 = 1.745 \text{ m} - 0.9806 \text{ m} = 0.764 \text{ m}$.

**ROUND:**

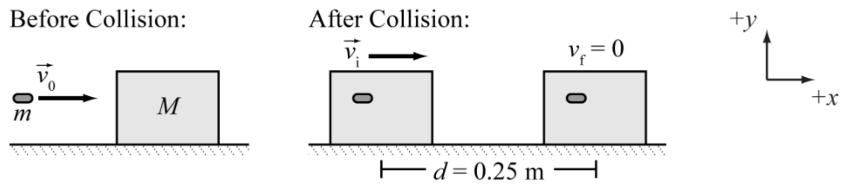
(a) Three significant figures: the distance between the first ball and the pallina is 1.75 m and the distance between the second ball and the pallina is 2.00 m.

(b) Two significant figures, because of subtraction: The distance between the first ball and the pallina is 0.98 m and the distance between the second ball and the pallina is 0.76 m.

**DOUBLE-CHECK:** Only the first ball is in motion after the collision in part (a) and in part (b) the second ball and the pallina are in motion. It makes sense that the distances in part (b) are shorter than the distances in part (a).

**7.92.** **THINK:** I have a soft pellet with mass $m = 1.2 \text{ g} = 1.2 \cdot 10^{-3}$ kg and an initial speed $v_0 = 65$ m/s. The pellet gets stuck in a piece of cheese with mass $M = 0.25$ kg. The cheese slides 25 cm before coming to a stop. I want to calculate the coefficient of friction between the cheese and the surface of the ice.

**SKETCH:**



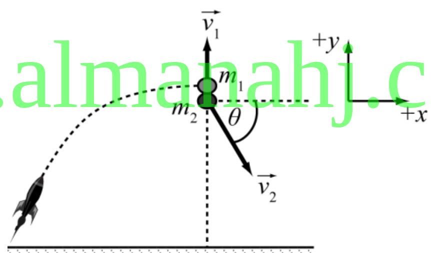

**RESEARCH:** I apply the conservation of momentum to calculate the speed of the cheese and the pellet after collision and then use $v_f^2 = v_i^2 - 2ad$ and $F_f = \mu_k N$ to obtain the coefficient of friction.

**SIMPLIFY:** $p_i = p_f \Rightarrow mv_0 + 0 = (m+M)v_i \Rightarrow v_i = \dfrac{mv_0}{m+M}$. Since $v_f = 0$, we have $a = v_i^2/(2d)$. Using Newton's second law, we get: $N = (m+M)g$ and $F_f = (m+M)a = \mu_k N = \mu_k (m+M)g$. The coefficient of friction is $\mu_k g = a$, or $\mu_k = a/g = v_i^2/(2gd)$.

**CALCULATE:** $v_i = \dfrac{(1.2\cdot 10^{-3}\text{ kg})(65\text{ m/s})}{0.25\text{ kg} + 1.2\cdot 10^{-3}\text{ kg}} = 0.311\text{ m/s}$ and $\mu_k = \dfrac{(0.311\text{ m/s})^2}{2(9.81\text{ m/s}^2)(0.25\text{ m})} = 0.01966.$

**ROUND:** To three significant figures, $\mu_k = 0.0200$.
**DOUBLE-CHECK:** This is reasonable since the initial speed is small.

**7.93.** **THINK:** I have a rocket which at the top of the trajectory breaks into two equal pieces. One piece has half the speed of the rocket travelling upward. I want to calculate the speed and angle of the second piece.
**SKETCH:**

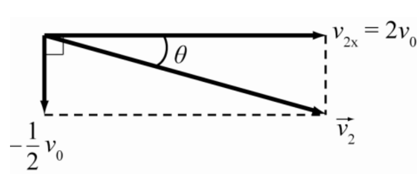





**RESEARCH:** Use the conservation of momentum. $\vec{p}_i = \vec{p}_f$, or in component form, $p_{xi} = p_{xf}$ and $p_{yi} = p_{yf}$. I also know that $p_{yi} = 0$. Let us assume that the speed of the rocket before it breaks is $v_0$ and mass $m_0$.

**SIMPLIFY:** $p_{xi} = p_{xf} \Rightarrow m_0 v_0 = m_1 v_{1x} + m_2 v_{2x}$. Since $v_{1x} = 0$ and $m_2 = \frac{1}{2}m_0$, $m_0 v_0 = \frac{1}{2}m_0 v_{2x}$ $\Rightarrow v_{2x} = 2v_0$. $p_{yi} = p_{yf} = 0 \Rightarrow 0 = m_1 v_{1y} + m_2 v_{2y}$. Since $m_1 = m_2 = \frac{1}{2}m_0$ and $v_{1y} = \frac{1}{2}v_0$, $v_{2y} = -v_{1y} \Rightarrow v_{2y} = -\frac{1}{2}v_0$. $v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{2^2 v_0^2 + (-1/2)^2 v_0^2}$ ; $\theta = \tan^{-1}\left(\dfrac{(-1/2)v_0}{2v_0}\right)$.

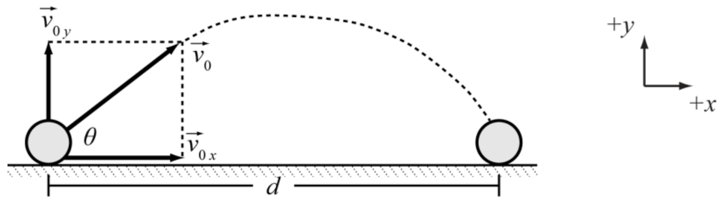
Drawing the vector $\vec{v}_2$:




**CALCULATE:** $v_2 = v_0\sqrt{4 + 1/4} = \dfrac{\sqrt{17}}{2}v_0$, $\theta = \tan^{-1}\left(-\dfrac{1}{4}\right) = -14.04°$

**ROUND:** Rounding is not needed here.
**DOUBLE-CHECK:** It makes sense that $\theta$ is negative since the first piece is travelling upwards. The $y$ component of $v_2$ must be in the negative $y$-direction.

**7.94.** **THINK:** A soccer ball has mass 0.265 kg. The ball is kicked at an angle of $20.8^\circ$ with respect to the horizontal. It travels a distance of 52.8 m. Calculation of the impulse received by the ball is needed.
**SKETCH:**




**RESEARCH:** I use the definition of impulse. $J = \Delta p = p_f - p_i$, and $p_i = 0$ since the ball is initially at rest. Thus $J = mv_0$. I need to determine $v_0$.
**SIMPLIFY:** I can determine the time to reach the maximum height by: $v = v_{0y} - gt = 0 \Rightarrow t = v_{0y} / g$. The time to reach a distance $d$ is twice the time taken to reach the maximum height. So, $t_d = 2v_{0y} / g = 2v_0 \sin\theta / g$. I can also use:

$$d = v_{0x}t_d = v_0 \cos\theta \frac{2v_0 \sin\theta}{g} = \frac{v_0^2 \sin 2\theta}{g} \Rightarrow v_0 = \sqrt{\frac{dg}{\sin 2\theta}}$$

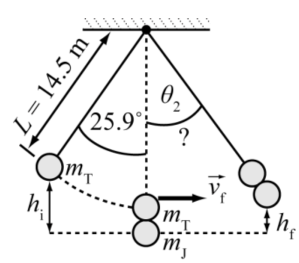
The impulse is $J = m\sqrt{\frac{dg}{\sin 2\theta}}$.

**CALCULATE:** $J = (0.265 \text{ kg})\sqrt{\frac{(52.8 \text{ m})(9.81 \text{ m/s}^2)}{\sin(2 \cdot 20.8^\circ)}} = 7.402 \text{ kg m/s}$



**ROUND:** $J = 7.40$ kg m/s
**DOUBLE-CHECK:** This is a reasonable value.

**7.95.** **THINK:** Tarzan swinging on a vine 14.5 m long picks Jane up at the bottom of his trajectory. At the beginning of his swing, the vine was at an angle of $25.9^\circ$ to the vertical. What will be the maximum angle relative to the vertical Tarzan and Jane will reach? Tarzan and Jane have masses $m_T = 70.4$ kg and $m_J = 43.4$ kg.
**SKETCH:**




**RESEARCH:** Use conservation of energy to calculate the speed of Tarzan just before he picks up Jane. Use conservation of momentum to find the speed just after Tarzan picks up Jane. Then use conservation of energy again to find the final height. Relate the initial and final heights to the angles and $L$.
**SIMPLIFY:**
By conservation of energy, noting that Tarzan starts with $v = 0$ at $h_i$,

$$m_T g h_i = \frac{1}{2} m_T v_i^2 \Rightarrow v_i = \sqrt{2gh_i}$$

By the conservation of momentum,

$$p_i = p_f \Rightarrow m_T v_i = (m_T + m_J)v_f \Rightarrow v_f = \left(\frac{m_T}{m_T + m_J}\right)v_i = \left(\frac{m_T}{m_T + m_J}\right)\sqrt{2gh_i}$$

The final height is determined using conservation of energy, noting that $v = 0$ at the maximum height, $\sqrt{\quad}$

$$\frac{1}{2}(m_T + m_J)v_f^2 = (m_T + m_J)gh_f \Rightarrow h_f = \frac{v_f^2}{2g} = \left(\frac{m_T}{m_T + m_J}\right)^2 \frac{2gh_i}{2g} = \left(\frac{m_T}{m_T + m_J}\right)^2 h_i.$$

Knowing that $h_f = L - L\cos\theta_2$ and $h_i = L - L\cos\theta_1$.

$$L - L\cos\theta_2 = \left(\frac{m_T}{m_T + m_J}\right)^2 (L - L\cos\theta_1)$$

$$1 - \cos\theta_2 = \left(\frac{m_T}{m_T + m_J}\right)^2 (1 - \cos\theta_1)$$

$$\cos\theta_2 = 1 - \left(\frac{m_T}{m_T + m_J}\right)^2 (1 - \cos\theta_1)$$

**CALCULATE:**

$$\cos\theta_2 = 1 - \left(\frac{70.4 \text{ kg}}{70.4 \text{ kg} + 43.4 \text{ kg}}\right)^2 (1 - \cos 25.9°) \Rightarrow \cos\theta_2 = 0.9616 \Rightarrow \theta_2 = \cos^{-1}(0.9616) = 15.929°$$

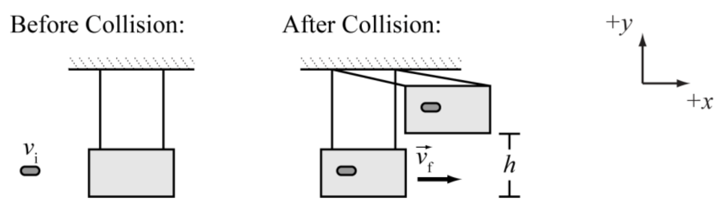
**ROUND:** $\theta_2 = 15.9°$

**DOUBLE-CHECK:** $\theta_2$ must be less than $\theta_1$ because $v_f$ is less than $v_i$. This is the case.

**7.96.** **THINK:** Since the bullet has a mass $m = 35.5 \text{ g} = 0.0355 \text{ kg}$ and a block of wood with mass $M = 5.90$ kg. The height is $h = 12.85 \text{ cm} = 0.1285 \text{ m}$. Determine the speed of the bullet.



**SKETCH:**




**RESEARCH:** First determine $v_f$ using $v = \sqrt{2gh}$ and then determine the speed of the bullet using the conservation of momentum.

**SIMPLIFY:** $\frac{1}{2}mv_f^2 = mgh \Rightarrow v_f = \sqrt{2gh}$,

$$p_i = p_f \Rightarrow mv_i = (m + M)v_f \Rightarrow v_i = \left(\frac{m+M}{m}\right)v_f = \left(\frac{m+M}{m}\right)\sqrt{2gh}$$

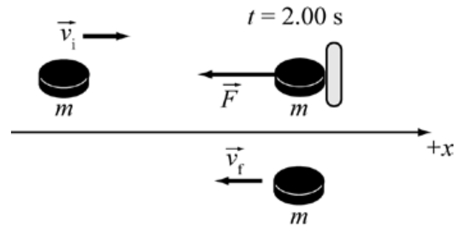
**CALCULATE:** $v_i = \left(\frac{0.0355 \text{ kg} + 5.90 \text{ kg}}{0.0355 \text{ kg}}\right)\sqrt{2(9.81 \text{ m/s}^2)(0.1285 \text{ m})} = 265.48 \text{ m/s}$

**ROUND:** $v_i = 265$ m/s

**DOUBLE-CHECK:** The result is reasonable for a bullet.

**7.97.** **THINK:** I have a 170. g hockey puck with initial velocity $v_{\text{i}} = 30.0 \text{ m/s}$ and final velocity $v_{\text{f}} = -25.0 \text{ m/s}$, changing over a time interval of $\Delta t = 0.200 \text{ s}$.

**SKETCH:**




**RESEARCH:** The initial and final momentums are calculated by $p_{\text{i}} = mv_{\text{i}}$ and $p_{\text{f}} = mv_{\text{f}}$. The force is calculated using $J = F\Delta t = \Delta p = m(v_{\text{f}} - v_{\text{i}})$.

**SIMPLIFY:** Simplification is not necessary.

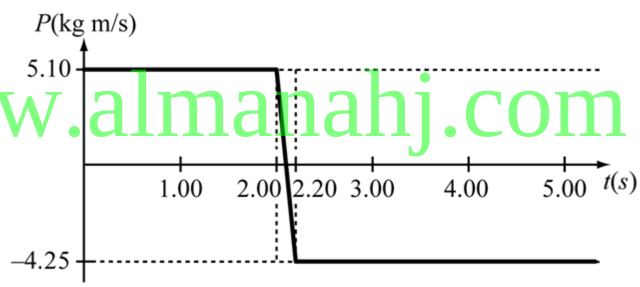
**CALCULATE:** $p_{\text{i}} = (0.170 \text{ kg})(30.0 \text{ m/s}) = 5.10 \text{ kg m/s}$, $p_{\text{f}} = (0.170 \text{ kg})(-25.0 \text{ m/s}) = -4.25 \text{ kg m/s}$, and

$F = \dfrac{p_{\text{f}} - p_{\text{i}}}{\Delta t} = \dfrac{(-4.25 \text{ kg m/s} - 5.10 \text{ kg m/s})}{0.200 \text{ s}} = -46.75 \text{ N}$. The position of the puck at $t = 2.00$ s is:

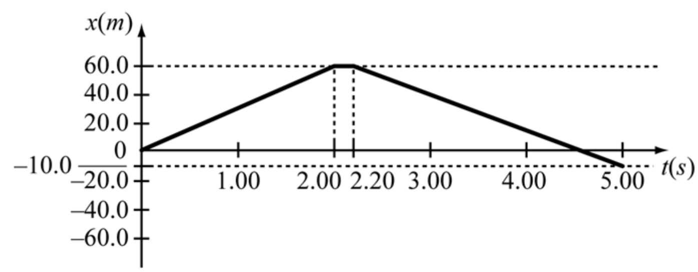
$x_2 = v_i t = (30.0 \text{ m/s})(2.00 \text{ s}) = 60.0 \text{ m}$. The position of the puck at $t = 5.00$ s is:

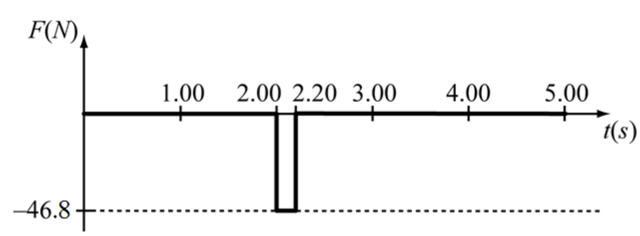
$$x_5 = x_2 + v_{\text{f}}(5.00 \text{ s} - 2.20 \text{ s}) = 60.0 \text{ m} + (-25.0 \text{ m/s})(2.80 \text{ s}) = -10.0 \text{ m}.$$

With all this information I can plot *p* vs. *t*, *x* vs. *t* and *F* vs. *t*.





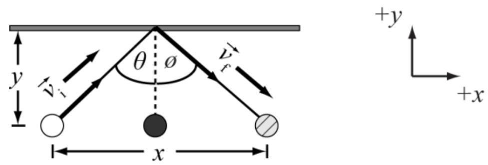




**ROUND:** $p_{\text{i}} = 5.10 \text{ kg m/s}$, $p_{\text{f}} = -4.25 \text{ kg m/s}$, $F = -46.8 \text{ N}$, $x_2 = 60.0 \text{ m}$, and $x_5 = -10.0 \text{ m}$.

**DOUBLE-CHECK:** The force *F* is applied only during the interval of 0.200 s. At other times *F* = 0, or $a = 0$.

**7.98.** **THINK:** I know the distance between the cue ball and the stripe ball is $x$ = 30.0 cm, and the distance between the cue ball and the bumper is $y$ = 15.0 cm. I want (a) the angle of incidence $\theta_1$ for the cue ball given an elastic collision between the ball and the bumper and (b) the angle $\theta_2$ given a coefficient of restitution of $c_{\text{r}} = 0.600$.

**SKETCH:**




**RESEARCH:**

(a) To conserve momentum in a purely elastic collision, the incidence and reflection angles are equal. I can use basic trigonometry to find $\theta$.

(b) When $c_{\text{r}} = 0.600$ I know how the speed of the ball changes after colliding with the bumper. Since there are no horizontal ($x$-direction) forces, only the vertical ($y$-direction) speed changes, and does so by a factor of $c_{\text{r}}$. That is, $v_{\text{f}x} = v_{\text{i}x}$ while $v_{\text{f}y} = c_{\text{r}} v_{\text{i}y}$

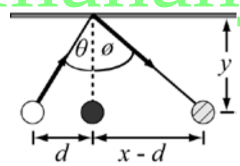
**SIMPLIFY:**

(a) From the sketch we see that $\tan\theta = \dfrac{(1/2)x}{y}$. Then $\theta = \tan^{-1}\left(\dfrac{x}{2y}\right)$.

(b) I know $v_{\text{f}x} = v_{\text{i}x} \Rightarrow v_{\text{f}} \sin\phi = v_{\text{i}} \sin\theta$, and $v_{\text{f}y} = c_{\text{r}} v_{\text{i}y} \Rightarrow v_{\text{f}} \cos\phi = c_{\text{r}} v_{\text{i}} \cos\theta$. Dividing these two equations gives:

$$\frac{v_{\text{f}} \sin\phi}{v_{\text{f}} \cos\phi} = \frac{v_{\text{i}} \sin\theta}{c_{\text{r}} v_{\text{i}} \cos\theta} \Rightarrow \tan\theta = c_{\text{r}} \tan\phi.$$






I know that $\tan\theta = d/y$ and that $\tan\phi = (x-d)/y$. Then $\tan\phi = x/y - d/y = x/y - \tan\theta$.

Now $\tan\theta = c_{\text{r}} \tan\phi$ becomes:

$$\tan\theta = c_{\text{r}}\left(\frac{x}{y} - \tan\theta\right) \Rightarrow \tan\theta(1+c_{\text{r}}) = c_{\text{r}}\frac{x}{y} \Rightarrow \theta = \tan^{-1}\left(\frac{c_{\text{r}} x}{y(1+c_{\text{r}})}\right)$$

**CALCULATE:**

(a) $\theta = \tan^{-1}\left(\dfrac{30.0 \text{ cm}}{2(15.0 \text{ cm})}\right) = 45.0°$

(b) $\theta = \tan^{-1}\left(\dfrac{0.600(30.0 \text{ cm})}{(15.0 \text{ cm})(0.600+1)}\right) = 36.87°$

**ROUND:**

Rounding to three significant figures:

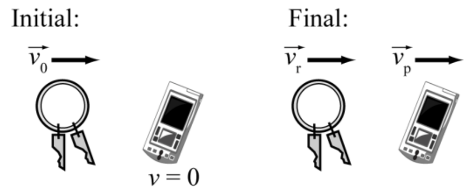
(a) $\theta = 45.0°$

(b) $\theta = 36.9°$

**DOUBLE-CHECK:** When $c_{\text{r}}$ decreases from 1 (a perfectly inelastic collision), $\theta$ should become smaller (steeper).

**7.99.** **THINK:** I know that the phone's mass $m_\text{p} = 0.111\text{ kg}$, the key ring's mass $m_\text{r} = 0.020\text{ kg}$ and the mass per key $m_\text{k} = 0.023\text{ kg}$. I want to find the minimum number of keys, *n*, to make the keys and the phone come out on the same side of the bookcase, and the final velocities of the phone, $v_{2\text{f}}$ and the key ring, $v_{1\text{f}}$, if the key ring has five keys and an initial velocity of $v_{1\text{i}} = 1.21\text{ m/s}$.

**SKETCH:**




**RESEARCH:** Note that this is an elastic collision, therefore kinetic energy is conserved. Also, the phone is initially stationary and the collision is one dimensional. I can use the following equations:

$$v_{1\text{f}} = \frac{m_1 - m_2}{m_1 + m_2} v_{1\text{i}} \quad \text{and} \quad v_{2\text{f}} = \frac{2m_1}{m_1 + m_2} v_{1\text{i}}$$

When determining the minimum number of keys, keep in mind that the final key velocity must be positive, as negative would imply that the keys and the phone come out on opposite sides.

**SIMPLIFY:** Find *n* given the condition that $v_{1\text{f}}$ is positive. Note the condition that ensures $v_{1\text{f}} > 0$ is $m_1 > m_2$. Then

$$(m_\text{r} + nm_\text{k}) > m_\text{p} \;\Rightarrow\; n > \frac{1}{m_\text{k}}(m_\text{p} - m_\text{r})$$

$$v_{1\text{f}} = \frac{(m_\text{r} + nm_\text{k}) - m_\text{p}}{(m_\text{r} + nm_\text{k}) + m_\text{p}} v_{1\text{i}} \quad \text{and} \quad v_{2\text{f}} = \frac{2(m_\text{r} + nm_\text{k})}{(m_\text{r} + nm_\text{k}) + m_\text{p}} v_{1\text{i}}.$$



**CALCULATE:** $n > \dfrac{1}{0.023\text{ kg}}(0.111\text{ kg} - 0.020\text{ kg}) = 3.96$

$$v_{1\text{f}} = \frac{(0.020\text{ kg} + 5(0.023\text{ kg})) - 0.111\text{ kg}}{0.020\text{ kg} + 5(0.023\text{ kg}) + 0.111\text{ kg}}(1.21\text{ m/s}) = 0.118\text{ m/s}$$

$$v_{2\text{f}} = \frac{2(0.020\text{ kg} + 5(0.023\text{ kg}))}{0.020\text{ kg} + 5(0.023\text{ kg}) + 0.111\text{ kg}}(1.21\text{ m/s}) = 1.3280\text{ m/s}$$

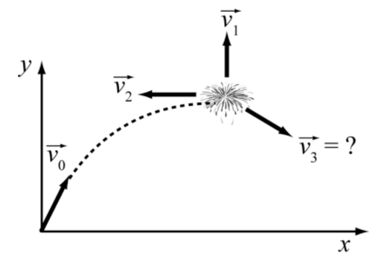
**ROUND:** $m_\text{r}$ and $m_\text{k}$ have two significant figures. As *n* is an integer, the minimum *n* is 4, $v_{1\text{f}} = 0.12\text{ m/s}$ and $v_{2\text{f}} = 1.33\text{ m/s}$.

**DOUBLE-CHECK:** The combined mass of four keys and the key ring is just slightly more than the mass of the cell phone. The result is reasonable.

**7.100.** **THINK:** I know the ball's initial mass *M* = 7.00 kg, initial speed $v_0 = 10.0\text{ m/s}$, launch angle $\theta_0 = 40.0^\circ$ and that it explodes at the peak of its trajectory. By choosing "straight up" to be along the positive *y* axis and "straight back" to be along the negative *x*-axis, I know one piece of the mass travels with $\vec{v}_1 = 3.00\text{ m/s}\,\hat{y}$ and the other travels with $\vec{v}_2 = -2.00\text{ m/s}\,\hat{x}$. Calculate the velocity of the third piece, $\vec{v}_3$.

Note that all three pieces have the same mass, $m = \frac{1}{3}M$.

**SKETCH:**




**RESEARCH:** Note that at the peak height, $v_y$ for the ball (before exploding) is zero. Then the initial momentum of the ball prior to exploding is $\vec{p} = M(v_0 \cos\theta)\,\hat{x}$. Find $\vec{v}_3$ by conservation of momentum. Specifically, $p_{ix} = p_{fx}$ and $p_{iy} = p_{fy}$.

**SIMPLIFY:** Along *x*, $p_{ix} = p_{fx}$ so:

$$Mv_0 \cos\theta_0 = mv_{1x} + mv_{2x} + mv_{3x} = m(-v_2 + v_{3x}) \Rightarrow v_{3x} = \frac{M}{m}v_0 \cos\theta_0 + v_2 = 3v_0 \cos\theta_0 + v_2$$

Along *y*, $p_{iy} = p_{fy} \Rightarrow 0 = mv_{1y} + mv_{2y} + mv_{3y} \Rightarrow v_{3y} = -v_{1y}$. Then $v_3 = \sqrt{v_{3x}^2 + v_{3y}^2}$ and $\theta = \tan^{-1}(v_{3y} / v_{3x})$ with respect to the horizontal.

**CALCULATE:** $v_{3x} = 3(10.0 \text{ m/s})\cos 40.0° + 2.00 \text{ m/s} = 24.98 \text{ m/s}$,

$v_{3y} = -v_{1y} = -3.00 \text{ m/s}$, $v_3 = \sqrt{(24.98 \text{ m/s})^2 + (3.00 \text{ m/s})^2} = 25.16 \text{ m/s}$, and

$\theta = \tan^{-1}\left(\dfrac{3.00 \text{ m/s}}{24.98 \text{ m/s}}\right) = 6.848°$.

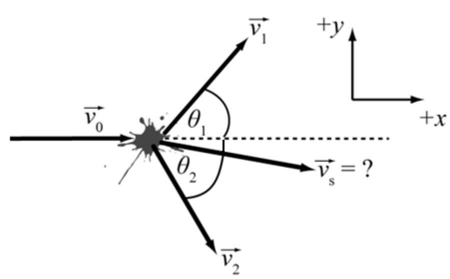


**ROUND:** $v_3 = 25.2 \text{ m/s}$ and $\theta = 6.85°$ below the horizontal.

**DOUBLE-CHECK:** The speeds of the first two fragments are small; it makes sense that the third fragment should have a larger speed to conserve the total momentum.

**7.101.** **THINK:** I know the skier's initial speed $v_0 = 22.0 \text{ m/s}$, the skier's mass $M = 61.0 \text{ kg}$, the mass of each ski $m = 1.50 \text{ kg}$ and the final velocity of each ski: $\vec{v}_1 = 25.0 \text{ m/s}$ at $\theta_1 = 12.0°$ to the left of the initial direction, and $\vec{v}_2 = 21.0 \text{ m/s}$ at $\theta_2 = 5.00°$ to the right of the initial direction. Calculate the magnitude and direction with respect to the initial direction of the skier's final velocity, $\vec{v}_s$.

**SKETCH:**




**RESEARCH:** The conservation of momentum requires $\sum_j (p_{fx})_j = \sum_j (p_{ix})_j$ and $\sum_j (p_{fy})_j = \sum_j (p_{iy})_j$. By conserving momentum in each direction, find $\vec{v}_s$. Take the initial direction to be along the *x*-axis.

**SIMPLIFY:** Then, $p_{ix} = m_{\text{total}} v_0 = (M + 2m)v_0$, and take $p_{ix} = p_{fx}$ in the equation $p_{fx} = Mv_{sx} + mv_{1x} + mv_{2x} = Mv_{sx} + mv_1 \cos\theta_1 + mv_2 \cos\theta_2$.

$$(M + 2m)v_0 = Mv_{sx} + m(v_1 \cos\theta_1 + v_2 \cos\theta_2) \Rightarrow v_{sx} = \frac{1}{M}\left((M + 2m)v_0 - m(v_1 \cos\theta_1 + v_2 \cos\theta_2)\right).$$

Similarly,

$$p_{iy} = 0 = p_{fy} = Mv_{sy} + mv_{1y} - mv_{2y} = Mv_{sy} + m(v_1 \sin\theta_1 - v_2 \sin\theta_2) \Rightarrow v_{sy} = \frac{m}{M}(v_2 \sin\theta_2 - v_1 \sin\theta_1)$$

With $v_{sx}$ and $v_{sy}$ known, get the direction with respect to the initial direction from $\theta_s = \tan^{-1}(v_{sy}/v_{sx})$. The magnitude of the velocity is $v_s = \sqrt{v_{sx}^2 + v_{sy}^2}$ .

**CALCULATE:**

$$v_{sx} = \frac{1}{61.0\text{ kg}}\Big(\big(61.0\text{ kg} + 2(1.50\text{ kg})\big)(22.0\text{ m/s}) - (1.50\text{ kg})\big((25.0\text{ m/s})\cos 12.0° + (21.0\text{ m/s})\cos 5.00°\big)\Big)$$
$$= 21.9662\text{ m/s}$$

$$v_{sy} = \frac{1.50\text{ kg}}{61.0\text{ kg}}\big((21.0\text{ m/s})\sin 5.00° - (25.0\text{ m/s})\sin 12.0°\big) = -0.08281\text{ m/s}$$

$$v_s = \sqrt{(21.9662\text{ m/s})^2 + (-0.08281\text{ m/s})^2} = 21.9664\text{ m/s}$$

$\theta_s = \tan^{-1}\left(\frac{-0.08281\text{ m/s}}{21.9662\text{ m/s}}\right) = -0.2160°$, where the negative indicates that $\theta_s$ lies below the $x$-axis, or to the right of the initial direction.

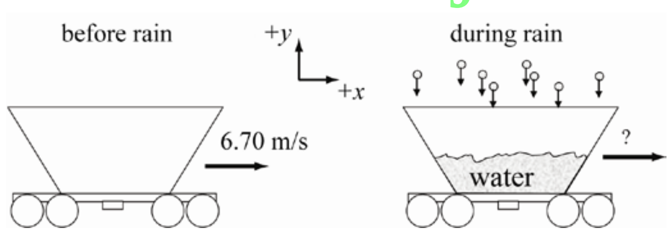
**ROUND:** $\vec{v}_s = 22.0\text{ m/s}$ at $0.216°$ to the right of the initial direction.

**DOUBLE-CHECK:** As the skier's mass is much greater than the mass of the two skis, it is reasonable that the skier carries the majority of the final momentum.

**7.102.** **THINK:** I know the car's initial speed $v_0 = 6.70\text{ m/s}$ and mass $m_c = 1.18 \cdot 10^5\text{ kg}$. There is no friction or air resistance. I want (a) the speed of the car $v_1$ after collecting $m_w = 1.62 \cdot 10^4\text{ kg}$ of water, and (b) the speed of the car $v_2$ after all the water has drained out, assuming an initial speed of $v_0 = 6.70\text{ m/s}$.



**SKETCH:**




**RESEARCH:** (a) Because the water enters the car completely in the vertical direction, it contributes mass but no horizontal momentum. Use conservation of momentum to determine the car's subsequent speed. $\Delta p = 0$. (b) The water drains out vertically in the moving frame of the car, which means that right after leaving the car the water has the same speed as the car. Therefore the speed of the car does not get changed at all by the draining water. No further calculation is necessary for part (b); the final speed of the car is the initial speed of 6.70 m/s.

**SIMPLIFY:**

(a) The initial mass is that of just the car. If I think of the water colliding perfectly inelastically with the car, the final mass is $m_c + m_w$.

$$p_f = p_i \Rightarrow (m_c + m_w)v_1 = m_c v_0 \Rightarrow v_1 = m_c v_0 / (m_c + m_w)$$

**CALCULATE:**

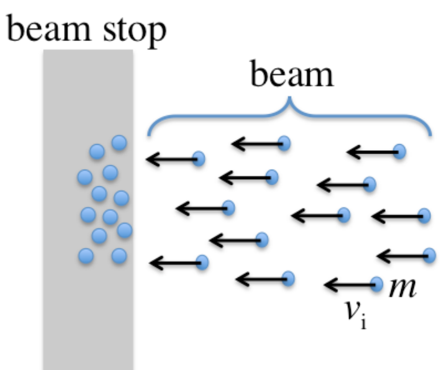
(a) $v_1 = \dfrac{(1.18 \cdot 10^5\text{ kg})(6.70\text{ m/s})}{1.18 \cdot 10^5\text{ kg} + 1.62 \cdot 10^4\text{ kg}} = 5.8912\text{ m/s}$

**ROUND:** With three significant figures in $v_0$, (a) $v_1 = 5.89\text{ m/s}$ and (b) $v_2 = 6.70\text{ m/s}$

**DOUBLE-CHECK:** The car slows down when the water is added. So why does the car not speed up when the water is drained? In the first case $v$ decreased when mass was added to the car, because the water had no initial horizontal velocity component. But when the water was drained from the car the water did have the same initial velocity component as the car, which is where the essential difference lies.

**7.103.** **THINK:** At first this looks like a complicated problem involving nuclear physics, because it describes beams, nuclei, rare isotopes, and beam stops. However, all that the beam stop does is to stop the nuclei hitting it, i.e. it sets the final speed to 0. Since we are given the mass and initial speed of each nucleus, we can find its initial momentum. Since the final momentum is zero, we therefore know the impulse that the beam stop receives from the collision with an individual nucleus.

**SKETCH:**




**RESEARCH:** The magnitude of the impulse a given nucleus receives from the beam stop is $J = |mv_f - mv_i|$, where the final speed is zero. From momentum conservation we know that the magnitude of the impulse that the beam stop receives form the nucleus is the same. The average force is defined as $F_{ave} = J_{total} / \Delta t$, where the total impulse is the combined impulse of all nuclei hitting the beam stop in a given time interval: $J_{total} = J\dfrac{dn}{dt}\Delta t$, and $\dfrac{dn}{dt}$ is the rate of nuclei per second given in the problem text.



**SIMPLIFY:** The average force is

$$F_{ave} = J_{total} / \Delta t = \left( J\frac{dn}{dt}\Delta t \right) / \Delta t = J\frac{dn}{dt} = |mv_f - mv_i|\frac{dn}{dt} = mv_i\frac{dn}{dt}$$

**CALCULATE:**

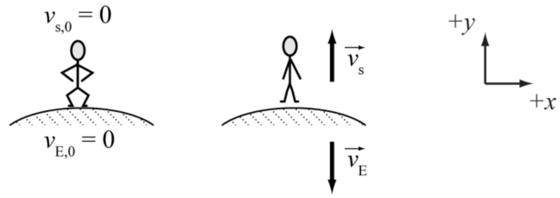
$$F_{ave} = (8.91 \cdot 10^{-26}\,\text{kg})(0.247 \cdot 2.998 \cdot 10^{8}\,\text{m/s})(7.25 \cdot 10^{5}\,/\,\text{s}) = 4.78348 \cdot 10^{-12}\,\text{kg m/s}^2$$

**ROUND:** We round to three significant figures and state $F_{ave} = 4.78$ pN as our final answer.

**DOUBLE-CHECK:** Our first check should almost always by to make sure that the units and order of magnitude of our answer work out. The units we found, kg m/s$^2$, are indeed the same as the force-unit, N. The magnitude, however, may at first seem surprising, because a pico-Newton is an incredibly small force. The reason why this force is so small is that the mass of the individual nuclei is so incredibly small.

**7.104.** **THINK:** I know the student's mass $m_s = 60.0$ kg, her average force $F_{av} = 770.$ N, the time $\Delta t = 0.250$ s, and the Earth's mass $m_E = 5.98 \cdot 10^{24}$ kg. I want to know the student's momentum after the impulse, $p_s$, the Earth's momentum after the impulse, $p_E$, the speed of the Earth after the impulse, $v_E$, the fraction of the total kinetic energy produced by the student's legs that goes to the Earth, $K_E/K_s$, and the maximum height of the student, $h$.

**SKETCH:** Consider the student-Earth system:




**RESEARCH:** In the student-Earth system, momentum is conserved; $\Delta p = 0$. Find the change in the student's momentum from $\Delta p = F_{av}\Delta t$ and then find the Earth's momentum and speed from momentum conservation. To find $K_E/K_s$ calculate $K_E$ and $K_s$ using $K = \frac{1}{2}mv^2$. Using energy conservation I can find $h$ for the student from $\Delta K_s + \Delta U = 0$.

**SIMPLIFY:** To find $p_s$: $\Delta p = F_{av}\Delta t$, with $p_{s,0} = 0$ ($v_i = 0$). $p_s = F_{av}\Delta t$. To find $p_E$: $\Delta p_{system} = 0 \;\Rightarrow\; p_s - p_{s,0} + p_E - p_{E,0} = 0$. Then $p_E = -p_s$. To find $v_E$: $p = mv \;\Rightarrow\; v_E = \frac{|p_E|}{m_E}$. (only want the speed, not the velocity) To find $K_E/K_s$: $K_E = \frac{1}{2}m_E v_E^2 = \frac{1}{2}\frac{p_E^2}{m_E} = \frac{1}{2}\frac{p_s^2}{m_E}$, and $K_s = \frac{1}{2}m_s v_s^2 = \frac{1}{2}\frac{p_s^2}{m_s}$. Then $\frac{K_E}{K_s} = \frac{(1/2)(p_s^2/m_E)}{(1/2)(p_s^2/m_s)} = \frac{m_s}{m_E}$. To find $h$: Note the kinetic energy of the Earth is negligible. Then $\Delta K + \Delta u = 0$ becomes $K_{s,f} - K_{s,i} + U_{s,f} - U_{s,i} = 0 \Rightarrow U_{s,f} = K_{s,i} \Rightarrow m_s gh = \frac{1}{2}\frac{p_s^2}{m_s} \Rightarrow h = \frac{p_s^2}{2gm_s^2}$.



**CALCULATE:** $p_s = F_{av}\Delta t = (770.\text{ N})(0.250\text{ s}) = 192.5\text{ kg m/s}$, $p_E = -p_S = -192.5\text{ kg m/s}$,

$v_E = \frac{|p_E|}{m_E} = \frac{192.5\text{ kg m/s}}{5.98\cdot 10^{24}\text{ kg}} = 3.2191\cdot 10^{-23}\text{ m/s}$, $\frac{K_E}{K_s} = \frac{m_s}{m_E} = \frac{60.0\text{ kg}}{5.98\cdot 10^{24}\text{ kg}} = 1.003\cdot 10^{-23}$, and

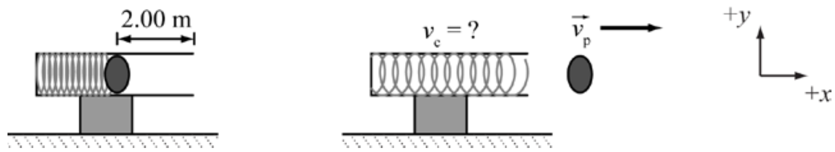
$$h = \frac{p_s^2}{2gm_s^2} = \frac{1}{2(9.81\text{ m/s}^2)}\left(\frac{192.5\text{ kg m/s}}{60.0\text{ kg}}\right) = 0.1635\text{ m}.$$

**ROUND:** To three significant figures, $p_s = 193\text{ kg m/s}$, $p_E = -193\text{ kg m/s}$, $v_E = 3.22\cdot 10^{-23}\text{ m/s}$, $K_E/K_s = 1.00\cdot 10^{-23}$ and $h = 0.164$ m.

**DOUBLE-CHECK:** Because the mass of the Earth is so large, its resulting velocity due to momentum conservation, and therefore its kinetic energy, should be negligible compared to the student's. The height $h$ is reasonable considering the time $\Delta t$ the student's $F_{av}$ acts over.

**7.105.** **THINK:** I want (a) the cannon's velocity $\vec{v}_c$ when the potato has been launched, and (b) the initial and final mechanical energy. There is no friction in the potato-cannon-ice system. Let the cannon's mass be $m_c = 10.0$ kg, the potato's mass be $m_p = 0.850$ kg, the cannon's spring constant be $k_c = 7.06\cdot 10^3$ N/m, the spring's compression be $\Delta x = 2.00$ m, the cannon and the potato's initial velocities be $\vec{v}_{c,0} = \vec{v}_{p,0} = 0$, and the potato's launch velocity be $\vec{v}_p = 175\text{ m/s}\,\hat{x}$. Take "horizontally to the right" to be the positive $\hat{x}$ direction.

**SKETCH:**




**RESEARCH:**

(a) Use the conservation of momentum $\Delta\vec{p} = 0$ to determine $\vec{v}_c$ when the potato, cannon and ice are considered as a system. Since the ice does not move, we can neglect the ice in the system and only consider the momenta of the potato and cannon.

(b) The total mechanical energy, $E_{mec}$, is conserved since the potato-cannon-ice system is isolated. That is, $E_{mec,f} = E_{mec,i}$. The value of $E_{mec,i}$ can be found by considering the spring potential energy of the cannon.

**SIMPLIFY:**

(a) $\Delta\vec{p} = 0 \Rightarrow \vec{p}_p - \vec{p}_{p,0} + \vec{p}_c - \vec{p}_{c,0} = 0 \Rightarrow \vec{p}_c = -\vec{p}_p \Rightarrow m_c\vec{v}_c = -m_p\vec{v}_p \Rightarrow \vec{v}_c = -\dfrac{m_p}{m_c}\vec{v}_p$

(b) $E_{mec,f} = E_{mec,i} = u_{s,i} = \dfrac{1}{2}k_c(\Delta x)^2$

**CALCULATE:**

(a) $\vec{v}_c = -\left(\dfrac{0.850 \text{ kg}}{10.0 \text{ kg}}\right)(175 \text{ m/s})\,\hat{x} = -14.875 \text{ m/s}\,\hat{x}$

(b) $E_{mec,f} = E_{mec,i} = \dfrac{1}{2}(7.06\cdot 10^3 \text{ N/m})(2.00 \text{ m})^2 = 14120 \text{ J}$

**ROUND:** With three significant figures for all given values,



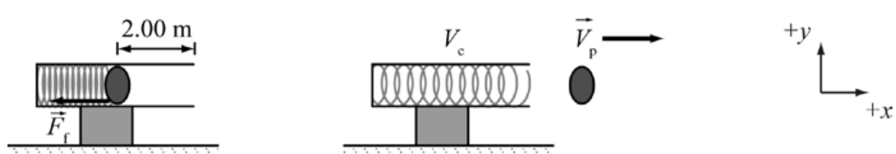
(a) $\vec{v}_c = -14.9 \text{ m/s}\,\hat{x}$, or $\vec{v}_c = 14.9 \text{ m/s}$ horizontally to the left.

(b) $E_{mec,f} = E_{mec,i} = 14.1 \text{ kJ}$.

**DOUBLE-CHECK:** Note $\vec{v}_c$ and $\vec{v}_p$ are directed opposite of each other, and $v_c < v_p$, as expected. Also, if $E_{mec,f}$ had been determined by considering the kinetic energies of the potato and the cannon, the same value would have been found.

**7.106.** **THINK:** The cannon has mass $m_c = 10.0$ kg and the potato has mass $m_p = 0.850$ kg. The cannon's spring constant $k_c = 7.06\cdot 10^3$ N/m and $\Delta x = 2.00$ m. The initial and final speeds of the potato are $v_i = 0$ and $v_f = 165$ m/s respectively. In this case there is friction between the potato and the cannon.

**SKETCH:**




**RESEARCH:**

(a) I will use the conservation of momentum: $p_i = p_f$.

(b) I will use $K = \dfrac{1}{2}mv^2$ and $U_s = \dfrac{1}{2}k(\Delta x)^2$.

(c) I will use $W = \Delta E$.

**SIMPLIFY:**

(a) $p_i = p_f = 0 \Rightarrow 0 = m_c v_c + m_p v_p \Rightarrow v_c = -\dfrac{m_p}{m_c}v_p$

(b) The total mechanical energy is $E_{\text{mec}} = \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta x)^2$. Before firing $\Delta x = 2.00$ m and $v = 0$ so $E_{\text{mec,i}} = \frac{1}{2}k(\Delta x)^2$. After firing $v_\text{c}$ and $v_\text{p}$ are non-zero and $\Delta x = 0$, so $E_{\text{mec,f}} = \frac{1}{2}m_\text{c}v_\text{c}^2 + \frac{1}{2}m_\text{p}v_\text{p}^2$.

(c) The work done by friction is $W_\text{f} = \Delta E = E_{\text{mec,f}} - E_{\text{mec,i}}$.

**CALCULATE:**

(a) $v_\text{c} = -\frac{0.850 \text{ kg}}{10.0 \text{ kg}}(165 \text{ m/s}) = -14.025 \text{ m/s}$

(b) $E_{\text{mec,i}} = \frac{1}{2}k_\text{c}(\Delta x)^2 = \frac{1}{2}(7.06 \cdot 10^3 \text{ N/m})(2.00 \text{ m})^2 = 14120 \text{ J}$,

$E_{\text{mec,f}} = \frac{1}{2}(10.0 \text{ kg})(-14.025 \text{ m/s})^2 + \frac{1}{2}(0.850 \text{ kg})(165 \text{ m/s})^2 = 12554.13 \text{ J}$

(c) $W_\text{f} = 12554.13 \text{ J} - 14120 \text{ J} = -1565.87 \text{ J}$

**ROUND:**

Round to three significant figures:

(a) $v_\text{c} = 14.0$ m/s , directed opposite to the direction of the potato.

(b) $E_{\text{mec,i}} = 14.1 \text{ kJ}$, $E_{\text{mec,f}} = 12.6 \text{ KJ}$

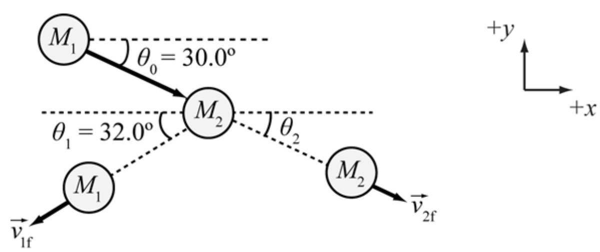
(c) $W_\text{f} = -1.56 \text{ KJ}$

**DOUBLE-CHECK:** The final mechanical energy is approximately 10% lower that the initial one. From the previous problem we know that the muzzle velocity in the absence of friction is 175 m/s. The muzzle velocity in this case is 165 m/s, about 5% lower. Since the kinetic energy is proportional to the square of the velocity, we should then expect a lowering of 10%. It also makes sense that $W_\text{f}$ is negative as it is due to a frictional force



**7.107.** **THINK:** There are two masses $M_1 = 1.00$ kg and $M_2 = 2.00$ kg and the initial and final speeds of $M_1$; $v_{1\text{i}} = 2.50$ m/s and $v_{1\text{f}} = 0.500$ m/s.

**SKETCH:**




**RESEARCH:** Use the conservation of momentum, $p_\text{i} = p_\text{f}$, in the *x*- and *y*-directions.

**SIMPLIFY:**

$p_{\text{ix}} = p_{\text{fx}} \Rightarrow M_1v_{1\text{i}}\cos\theta_0 = M_2v_{2\text{f}}\cos\theta_2 - M_1v_{1\text{f}}\cos\theta_1$

$p_{\text{iy}} = p_{\text{fy}} \Rightarrow -M_1v_{1\text{i}}\sin\theta_0 = -M_2v_{2\text{f}}\sin\theta_2 - M_1v_{1\text{f}}\sin\theta_1$

$$v_{2x} = v_{2\text{f}}\cos\theta_2 = \frac{M_1v_{1\text{i}}\cos\theta_0 + M_1v_{1\text{f}}\cos\theta_1}{M_2} = \frac{M_1}{M_2}(v_{1\text{i}}\cos\theta_0 + v_{1\text{f}}\cos\theta_1)$$

$$v_{2y} = v_{2\text{f}}\sin\theta_2 = \frac{M_1v_{1\text{i}}\sin\theta_0 - M_1v_{1\text{f}}\sin\theta_1}{M_2} = \frac{M_1}{M_2}(v_{1\text{i}}\sin\theta_0 - v_{1\text{f}}\sin\theta_1)$$

$$v_{2\text{f}} = \sqrt{v_{2x}^2 + v_{2y}^2} = \frac{M_1}{M_2}\sqrt{(v_{1\text{i}}\cos\theta_0 + v_{1\text{f}}\cos\theta_1)^2 + (v_{1\text{i}}\sin\theta_0 - v_{1\text{f}}\sin\theta_1)^2}$$

**CALCULATE:**

$$v_{2f} = \frac{1.00\text{ kg}}{2.00\text{ kg}}\sqrt{\left[(2.50\text{ m/s})\cos 30.0° + (0.50\text{ m/s})\cos 32.0°\right]^2 + \left[(2.50\text{ m/s})\sin 30.0° - (0.50\text{ m/s})\sin 32.0°\right]^2}$$
$$= 1.3851\text{ m/s}$$

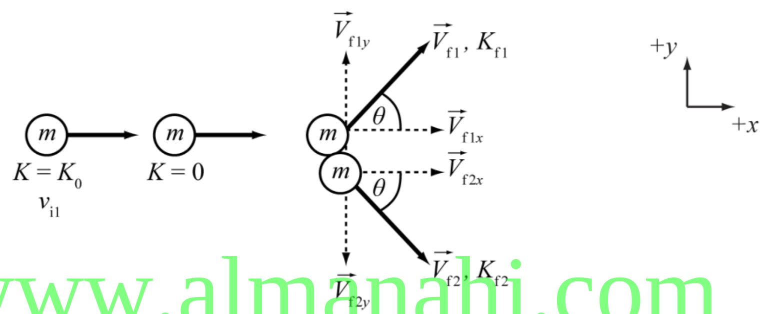
**ROUND:**

$v_{2f} = 1.39$ m/s

**DOUBLE-CHECK:** Initial kinetic energy is $\frac{1}{2}(1.00\text{ kg})(2.50\text{ m/s})^2 = 3.13\text{ J}$. Final kinetic energy is $\frac{1}{2}(1.00\text{ kg})(0.500\text{ m/s})^2 + \frac{1}{2}(2.00\text{ kg})(1.39\text{ m/s})^2 = 2.06\text{ J}$. Energy has been lost during the collision, as expected.

**7.108.** **THINK:** Since it is an elastic collision, kinetic energy and momentum are both conserved. Both particles are protons and therefore have equal masses. The first particle is deflected $\theta = 25^\circ$ from its path while the particle initially at rest is deflected by an angle $\phi$. The initial kinetic energy of the first particle is $K_0$.

**SKETCH:**






**RESEARCH:** $v_x = v\cos\theta$, $v_y = v\sin\theta$, $K_0 = K_{f1} + K_{f2}$, $K_i = \frac{1}{2}mv_i^2$, $\sum p_i = \sum p_f$, $\cos^2\theta + \sin^2\theta = 1$, $\cos(A+B) = \cos A\cos B - \sin A\sin B$

**SIMPLIFY:** $K_0 = K_{f1} + K_{f2}$, $K_{f1} = \frac{1}{2}mv_{f1}^2$, and $K_{f2} = \frac{1}{2}mv_{f2}^2$. So:

$$K_0 = \frac{1}{2}mv_{f1}^2 + \frac{1}{2}mv_{f2}^2 \Rightarrow v_{f1}^2 + v_{f2}^2 = \frac{2K_0}{m}. \tag{1}$$

$p_{xi} = p_{xf}$, $p_{yi} = p_{yf}$, and $K_0 = \frac{1}{2}mv_{i1}^2$. So

$$mv_{i1} = mv_{f1x} + mv_{f2x} \Rightarrow v_{i1} = v_{f1x} + v_{f2x} \Rightarrow \sqrt{\frac{2K_0}{m}} = v_{f1}\cos 25° + v_{f2}\cos\phi. \tag{2}$$

$$0 = mv_{f1y} - mv_{f2y} \Rightarrow v_{f1y} - v_{f2y} = 0 \Rightarrow v_{f1}\sin 25° - v_{f2}\sin\phi = 0 \tag{3}$$

Squaring equations (2) and (3) and taking the sum,

$$v_{f1}^2\left(\cos^2 25° + \sin^2 25°\right) + v_{f2}^2\left(\cos^2\phi + \sin^2\phi\right) + 2v_{1f}v_{2f}\left(\cos 25°\cos\phi - \sin 25°\sin\phi\right) = \frac{2K_0}{m}$$
$$v_{f1}^2 + v_{f2}^2 + 2v_{1f}v_{2f}\cos(25° + \phi) = \frac{2K_0}{m}$$

Subtracting equation (1),

$$2v_{1f}v_{2f}\cos(25° + \phi) = 0 \Rightarrow \cos(25° + \phi) = 0 \Rightarrow 25° + \phi = 90° \Rightarrow \phi = 65°$$

Therefore $v_{f1}\cos 25° + v_{f2}\cos 65° = \sqrt{2K_0/m}$ and $v_{f1}\sin 25° - v_{f2}\sin 65° = 0$. $v_{f1} = \dfrac{\sin 65°}{\sin 25°}v_{f2}$, and $v_{f2} = v_{f1}\dfrac{\sin 25°}{\sin 65°}$:

$$v_{f2}\left(\frac{\sin 65°\cos 25°}{\sin 25°}\right) + v_{f2}\cos 65° = v_{f2}\left(\frac{\sin 65°}{\tan 25°} + \cos 65°\right) = \sqrt{\frac{2K_0}{m}} \Rightarrow \frac{2K_0}{m\left(\dfrac{\sin 65°}{\tan 25°} + \cos 65°\right)^2} = v_{f2}^2$$

$$v_{f1}\cos 25° + v_{f1}\left(\frac{\sin 25°\cos 65°}{\sin 65°}\right) = v_{f1}\left(\frac{\sin 25°}{\tan 65°} + \cos 25°\right) = \sqrt{\frac{2K_0}{m}} \Rightarrow \frac{2K_0}{m\left(\dfrac{\sin 25°}{\tan 65°} + \cos 25°\right)^2} = v_{f1}^2$$

Therefore, $K_{f1} = \dfrac{1}{2}mv_{f1}^2 = \dfrac{K_0}{\left(\dfrac{\sin 25°}{\tan 65°} + \cos 25°\right)^2}$ and $K_{f2} = \dfrac{1}{2}mv_{f2}^2 = \dfrac{K_0}{\left(\dfrac{\sin 65°}{\tan 25°} + \cos 65°\right)^2}$.

**CALCULATE:** $K_{f1} = \dfrac{K_0}{1.2174}$, and $K_{f2} = \dfrac{K_0}{5.5989}$.

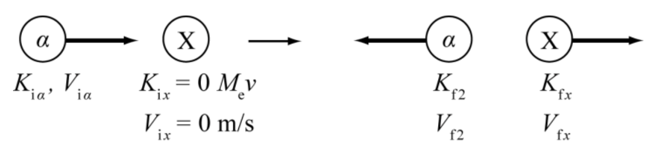
**ROUND:** $K_{f1} = 0.821K_0$ and $K_{f2} = 0.179K_0$.

**DOUBLE-CHECK:** The sum of the coefficients is 0.18+0.82 = 1, which means $K_0 = K_{f1} + K_{f2}$ which means that energy is conserved, so it makes sense.

**7.109.** **THINK:** Since the collision is elastic, momentum and kinetic energy are conserved. Also, since the alpha particle is backscattered, that means that is reflected $180^{\circ}$ back and therefore the collision can be treated as acting in one dimension. The initial and final energies of the alpha particle are given in units of MeV and not J, $K_{i\alpha} = 2.00$ MeV and $K_{f\alpha} = 1.59$ MeV. I can leave the energy in these units and not convert to Joules. $m_\alpha = 6.65 \cdot 10^{-27}$ kg.



**SKETCH:**




**RESEARCH:** $v_{f1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)v_{1i}$, $v_{f2} = \left(\dfrac{2m_1}{m_1 + m_2}\right)v_{1i}$, and $K_i = K_f$, $E = (1/2)mv^2$.

**SIMPLIFY:** $K_{i\alpha} + K_{iX} = K_{f\alpha} + K_{fx} \Rightarrow K_{fX} = K_{i\alpha} - K_{f\alpha}$, $E_{i\alpha} = \dfrac{1}{2}m_\alpha v_{i\alpha}^2 \Rightarrow v_{i\alpha} = \sqrt{\dfrac{2E_{i\alpha}}{m_\alpha}}$, and

$v_{fX\,\alpha} = \left(\dfrac{2m_\alpha}{m_\alpha + m_X}\right)v_i = \left(\dfrac{2m_\alpha}{m_\alpha + m_X}\right)\sqrt{\dfrac{2K_{i\alpha}}{m_\alpha}}$. Since $K_{fX} = \dfrac{1}{2}m_X v_{fX}^2$:

$$K_{i\alpha} - K_{f\alpha} = \frac{1}{2}m_X v_{fX}^2 = \frac{1}{2}m_X\left(\left(\frac{2m_\alpha}{m_\alpha + m_X}\right)\sqrt{\frac{2K_{i\alpha}}{m_\alpha}}\right)^2 = \frac{1}{2}m_X\left(\frac{4m_\alpha^2}{(m_\alpha + m_X)^2}\right)\left(\frac{2K_{i\alpha}}{m_\alpha}\right) = \frac{4m_X m_\alpha K_{i\alpha}}{m_\alpha^2 + 2m_\alpha m_X + m_X^2}.$$

This simplifies to $0 = \left(\dfrac{K_{i\alpha} - K_{f\alpha}}{K_{i\alpha}}\right)m_x^2 + \left(2\left(\dfrac{K_{i\alpha} - K_{f\alpha}}{K_{i\alpha}}\right)m_\alpha - 4m_\alpha\right)m_x + \left(\dfrac{K_{i\alpha} - K_{f\alpha}}{K_{i\alpha}}\right)m_\alpha^2$.

So, $m_X = \dfrac{\left(4 - 2\left(\dfrac{K_{i\alpha} - K_{f\alpha}}{K_{i\alpha}}\right)\right)m_\alpha \pm m_\alpha\sqrt{\left(2\left(\dfrac{K_{i\alpha} - K_{f\alpha}}{K_{i\alpha}}\right) - 4\right)^2 - 4\left(\dfrac{K_{i\alpha} - K_{f\alpha}}{K_{i\alpha}}\right)^2}}{2\left(\dfrac{K_{i\alpha} - K_{f\alpha}}{K_{i\alpha}}\right)}$ by the quadratic formula.

**CALCULATE:**

$$m_X = \frac{\left(4-2\left(\frac{2.00\text{ MeV}-1.59\text{ MeV}}{2.00\text{ MeV}}\right)\right)\left(6.65\cdot 10^{-27}\text{ kg}\right)}{2\left(\frac{2.00\text{ MeV}-1.59\text{ MeV}}{2.00\text{ MeV}}\right)} \pm \frac{\left(6.65\cdot 10^{-27}\text{ kg}\right)\sqrt{\left(2\left(\frac{2.00\text{ MeV}-1.59\text{ MeV}}{2.00\text{ MeV}}\right)-4\right)^2-4\left(\frac{2.00\text{ MeV}-1.59\text{ MeV}}{2.00\text{ MeV}}\right)^2}}{2\left(\frac{2.00\text{ MeV}-1.59\text{ MeV}}{2.00\text{ MeV}}\right)}$$

$$=1.1608\cdot 10^{-25}\text{ kg},\ 3.8098\cdot 10^{-28}\text{ kg}$$

**ROUND:** $m_X = 1.16\cdot 10^{-25}$ kg. Since the ratio of the masses of atom X and the alpha particle is $m_X / m_\alpha = (1.16\cdot 10^{-25}\text{ kg})/(6.65\cdot 10^{-27}\text{ kg}) = 17.5$, and since the alpha particle has 4 nucleons, expect atom X to have 70 nucleons. Consulting the table in Appendix B, germanium is a good guess. Another possibility would be zinc, but zinc-70 is a relatively rare isotope of zinc, whereas germanium-70 is a common isotope of germanium.

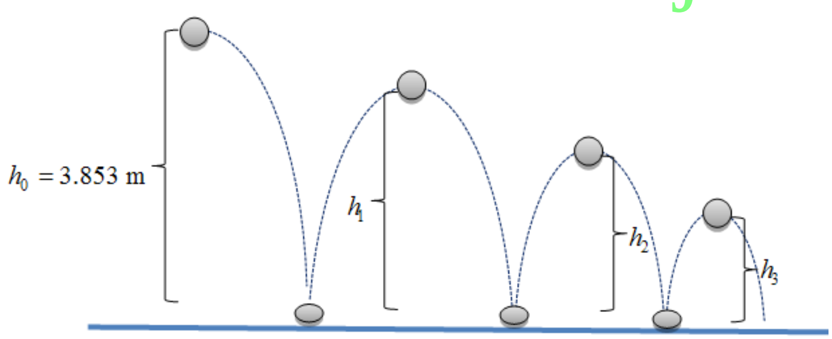
**DOUBLE-CHECK:** Since the alpha particle is reflected back, the mass of atom X must be greater than the mass of the alpha particle, so this value makes sense.

## Multi-Version Exercises

**7.110.** **THINK:** The coefficient of restitution is used to compute the height of the next bounce from the peak of the previous bounce. Since the ball was dropped (not thrown), assume that it started with no velocity, exactly as it would at the peak of a bounce.

**SKETCH:** The ball hits the floor three times:






**RESEARCH:** The coefficient of restitution is defined to be $\varepsilon = \sqrt{\frac{h_f}{h_i}}$. In this case, the ball bounces three times; it is necessary to find expressions relating $h_0$, $h_1$, $h_2$, and $h_3$.

**SIMPLIFY:** For the first bounce, $\varepsilon = \sqrt{\frac{h_1}{h_0}}$. For the second bounce, $\varepsilon = \sqrt{\frac{h_2}{h_1}}$, and for the third bounce, $\varepsilon = \sqrt{\frac{h_3}{h_2}}$. Squaring all three equations gives: $\varepsilon^2 = \frac{h_1}{h_0}$, $\varepsilon^2 = \frac{h_2}{h_1}$, and $\varepsilon^2 = \frac{h_3}{h_2}$. Now, solve for $h_3$ in terms of $h_2$ and ε: $h_3 = \varepsilon^2 h_2$. Similarly, solve for $h_2$ in terms of $h_1$ and ε, then for $h_1$ in terms of $h_0$ and ε, to get:

$$h_3 = \varepsilon^2 h_2$$
$$h_2 = \varepsilon^2 h_1$$
$$h_1 = \varepsilon^2 h_0$$

Finally, combine these three equations to get an expression for $h_3$ in terms of the values given in the problem, $h_1$ and $\varepsilon$.

$$\begin{aligned} h_3 &= \varepsilon^2 h_2 \\ &= \varepsilon^2\left(\varepsilon^2 h_1\right) \\ &= \varepsilon^2\left(\varepsilon^2\left(\varepsilon^2 h_0\right)\right) \\ &= \varepsilon^6 h_0. \end{aligned}$$

**CALCULATE:** The coefficient of restitution of the Super Ball is 0.8887 and the ball is dropped from a height of 3.853 m above the floor. So the height of the third bounce is:

$$\begin{aligned} h_3 &= \varepsilon^6 h_0 \\ &= (0.8887)^6\, 3.853 \text{ m} \\ &= 1.89814808 \text{ m} \end{aligned}$$

**ROUND:** The only numbers used here were the coefficient of restitution and the height. They were multiplied together and were given to four significant figures. Thus the answer should have four figures; the ball reached a maximum height of 1.898 m above the floor.

**DOUBLE-CHECK:** From experience with Super Balls, this seems reasonable. Double check by working backwards to find the maximum height of each bounce. If the ball bounced 1.898 m on the third bounce, then it reached a height of 1.898 m / $0.8887^2$ = 2.403177492 m on the second bounce and 2.403177492 m / $0.8887^2$ = 3.042814572 m on the first bounce. From there, the height at which the ball was dropped is computed to be 3.042814572 m / $0.8887^2$ = 3.852699416 m. When rounded to four decimal places, this gives an initial height of 3.853 m, which agrees with the number given in the problems and confirms that the calculations were correct.



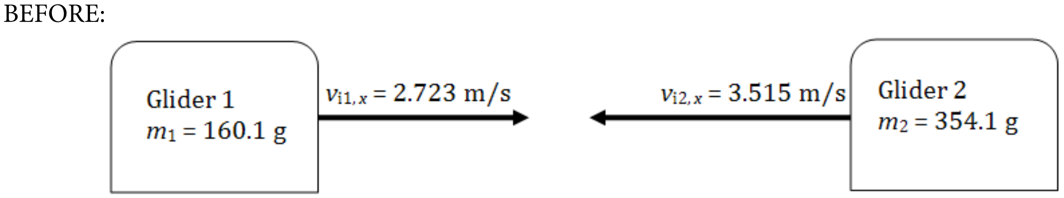
**7.111.** $h_0 = \dfrac{h_3}{\varepsilon^6} = \dfrac{2.234 \text{ m}}{(0.9115)^6} = 3.895 \text{ m}$

**7.112.** $\varepsilon = \sqrt[6]{\dfrac{h_3}{h_0}} = \sqrt[6]{\dfrac{2.621 \text{ m}}{3.935 \text{ m}}} = 0.9345$

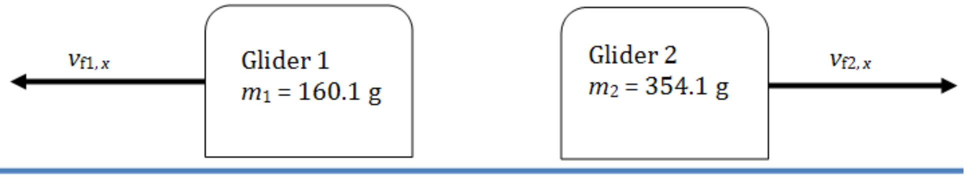
**7.113.** **THINK:** This problem uses the properties of conservation of energy. Since the masses and initial speeds of the gliders are given, it is possible to use the fact that the collision is totally elastic and the initial conditions to find the velocity of the glider after the collision.

**SKETCH:** The sketch shows the gliders before and after the collision. Note that the velocities are all in the $x$ – direction. Define the positive $x$ – direction to be to the right.

BEFORE:



AFTER:




**RESEARCH:** Since this is a one-dimensional, totally elastic collision, we know that the speed of the first glider after the collision is given by the equation:

$$v_{f1,x} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{i1\,x} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{i2\,x}$$

**SIMPLIFY:** Since the masses are given in grams and the velocities in meters per second, there is no need to convert any of the units in this problem. All of the values needed to compute the final velocity of Glider 1.

**CALCULATE:** The masses and velocities are given in the problem. Substitute them into the equation to get $v_{f1,x} = \left(\frac{160.1\text{ g} - 354.1\text{ g}}{160.1\text{ g} + 354.1\text{ g}}\right)(2.723\text{ m/s}) + \left(\frac{2 \cdot 354.1\text{ g}}{160.1\text{ g} + 354.1\text{ g}}\right) \cdot (-3.515\text{ m/s})$, so the velocity of Glider 1 after the collision is –5.868504473 m/s. The velocity is negative to indicate that the glider is moving to the left.

**ROUND:** The measured numbers in this problem all have four significant figures, so the final answer should also have four figures. This means that the final velocity of Glider 1 is 5.869 m/s to the left.

**DOUBLE-CHECK:** Though the speed of Glider 1 is greater after the collision than it was before the collision, which makes sense because Glider 2 was more massive and had a faster speed going into the collision. The problem can also be checked by calculating the speed of Glider 2 after the collision using the equation $v_{f2,x} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{i1\,x} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{i2\,x}$ and confirming that the energy before the collision is equal to the energy after the collision, $\frac{p_{f1}^2}{2m_1} + \frac{p_{f2}^2}{2m_2} = \frac{p_{i1}^2}{2m_1} + \frac{p_{i2}^2}{2m_2}$. Before the collision,



$$\begin{aligned}
\frac{p_{i1}^2}{2m_1} + \frac{p_{i2}^2}{2m_2} &= \frac{(m_1 v_{i1})^2}{2m_1} + \frac{(m_2 v_{i2})^2}{2m_2} \\
&= \frac{(160.1\text{ g} \cdot 2.723\text{ m/s})^2}{2 \cdot 160.1\text{ g}} + \frac{(354.1\text{ g} \cdot 3.515\text{ m/s})^2}{2 \cdot 354.1\text{ g}} \\
&= 2781.041643\text{ g} \cdot \text{m}^2/\text{s}^2
\end{aligned}$$

After the collision,

$$\frac{p_{f1}^2}{2m_1}+\frac{p_{f2}^2}{2m_2}$$

$$=\frac{(m_1v_{f1})^2}{2m_1}+\frac{(m_2v_{f2})^2}{2m_2}$$

$$=\frac{(m_1v_{f1})^2}{2m_1}+\frac{\left(m_2\left[\left(\frac{2m_1}{m_1+m_2}\right)v_{i1x}+\left(\frac{m_2-m_1}{m_1+m_2}\right)v_{i2x}\right]\right)^2}{2m_2}$$

$$=\frac{(160.1\text{ g}\cdot 5.869\text{ m/s})^2}{2\cdot 160.1\text{ g}}$$

$$+\frac{\left(354.1\text{ g}\cdot\left[\left(\frac{2\cdot 160.1\text{ g}}{160.1\text{ g}+354.1\text{ g}}\right)\cdot 2.723\text{ m/s}+\left(\frac{354.1\text{ g}-160.1\text{ g}}{160.1\text{ g}+354.1\text{ g}}\right)\cdot(-3.515\text{ m/s})\right]\right)^2}{2\cdot 354.1\text{ g}}$$

$$=2781.507234\text{ g}\cdot\text{m}^2/\text{s}^2$$

The energies before and after the collision are both close to 2781 g·m$^2$ / s$^2$, confirming that the values calculated for the speeds of the gliders were correct.

**7.114.** $$v_{f1}=\left(\frac{m_1-m_2}{m_1+m_2}\right)v_{i1}+\left(\frac{2m_2}{m_1+m_2}\right)v_{i2}$$

$$(m_1+m_2)v_{f1}=(m_1-m_2)v_{i1}+2m_2v_{i2}$$

$$m_1v_{f1}+m_2v_{f1}=m_1v_{i1}-m_2v_{i1}+2m_2v_{i2}$$

$$m_2v_{f1}+m_2v_{i1}-2m_2v_{i2}=m_1v_{i1}-m_1v_{f1}$$

$$m_2=\frac{m_1v_{i1}-m_1v_{f1}}{v_{f1}+v_{i1}-2v_{i2}}=\frac{m_1(v_{i1}-v_{f1})}{v_{i1}+v_{f1}-2v_{i2}}$$

$$m_2=\frac{(176.3\text{ g})(2.199\text{ m/s}-(-4.511\text{ m/s}))}{2.199\text{ m/s}-4.511\text{ m/s}-2(-3.301\text{ m/s})}=275.8\text{ g}$$

**7.115.** $$v_{f1}=\left(\frac{m_1-m_2}{m_1+m_2}\right)v_{i1}+\left(\frac{2m_2}{m_1+m_2}\right)v_{i2}$$

$$(m_1+m_2)v_{f1}=(m_1-m_2)v_{i1}+2m_2v_{i2}$$

$$m_1v_{f1}+m_2v_{f1}=m_1v_{i1}-m_2v_{i1}+2m_2v_{i2}$$

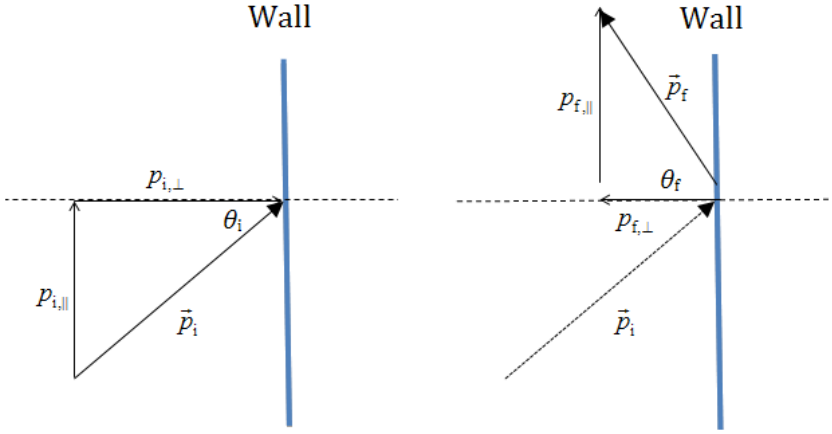
$$m_2v_{f1}+m_2v_{i1}-2m_2v_{i2}=m_1v_{i1}-m_1v_{f1}$$

$$m_1=\frac{m_2v_{f1}+m_2v_{i1}-2m_2v_{i2}}{v_{i1}-v_{f1}}=\frac{m_2(v_{f1}+v_{i1}-2v_{i2})}{v_{i1}-v_{f1}}$$

$$m_1=\frac{(277.3\text{ g})(-4.887\text{ m/s}+2.277\text{ m/s}-2(-3.789\text{ m/s}))}{2.277\text{ m/s}-(-4.887\text{ m/s})}=192.3\text{ g}$$

**7.116.** **THINK:** For this problem, it will help to think about the components of the momentum that are perpendicular to and parallel to the wall. After the collision, the momentum parallel to the wall is unchanged The perpendicular component is in the opposite direction and is multiplied by the coefficient of restitution after the collision with the wall.

**SKETCH:** Show the path of the racquetball before and after it hits the wall.




**RESEARCH:** The mass and initial speed can be used to calculate the initial momentum $\vec{p}_i = m\vec{v}$. The angle at which the racquetball hits the wall is used to calculate the parallel and perpendicular components from the initial momentum: $p_{i,\perp} = p_i \cos\theta_i$ and $p_{i,\parallel} = p_i \sin\theta_i$. The component of the momentum parallel to the wall is unchanged in the collision, so $p_{f,\parallel} = p_{i,\parallel}$. The component of the final momentum perpendicular to the wall has a magnitude equal to the coefficient of restitution times the component of the initial momentum parallel to the wall: $p_{f,\perp} = \varepsilon p_{i,\perp}$ in the opposite direction from $p_{i,\perp}$. With a little trigonometry, the final angle can be calculated from the perpendicular and parallel components of the final momentum: $\tan\theta_f = \dfrac{p_{f,\parallel}}{p_{f,\perp}}$.



**SIMPLIFY:** Since $\tan\theta_f = \dfrac{p_{f,\parallel}}{p_{f,\perp}}$, take the inverse tangent to find $\theta_f = \tan^{-1}\left(\dfrac{p_{f,\parallel}}{p_{f,\perp}}\right)$. Substitute $p_{f,\parallel} = p_{i,\parallel} = p_i \sin\theta_i$ and $p_{f,\perp} = \varepsilon p_i \cos\theta_i$ into the equation to get:

$$\theta_f = \tan^{-1}\left(\frac{p_i \sin\theta_i}{\varepsilon p_i \cos\theta_i}\right) = \tan^{-1}\left(\frac{1}{\varepsilon}\cdot\frac{\sin\theta_i}{\cos\theta_i}\right) = \tan^{-1}\left(\frac{1}{\varepsilon}\cdot\tan\theta_i\right)$$

**CALCULATE:** The exercise states that the initial angle is 43.53° and the coefficient of restitution is 0.8199. Using those values, the final angle is:

$$\theta_f = \tan^{-1}\left(\frac{\tan\theta_i}{\varepsilon}\right) = \tan^{-1}\left(\frac{\tan 43.53^\circ}{0.8199}\right) = 49.20289058^\circ$$

**ROUND:** The angle and coefficient of restitution are the only measured values used in these calculations, and both are given to four significant figures, so the final answer should also have four significant figures. The racquetball rebounds at an angle of 49.20° from the normal.

**DOUBLE-CHECK:** This answer is physically realistic. The component of the momentum does not change, but the perpendicular component is reduced by about one fifth, so the angle should increase. To double check the calculations, use the speed and mass of the racquetball to find the initial and final momentum: $p_{i,\parallel} = mv_{i,\parallel} = m\vec{v}_i \sin\theta_i$ and $p_{i,\perp} = mv_{i,\perp} = m\vec{v}_i \cos\theta_i$. Thus $p_{i,\parallel} = 437.9416827 \text{ g}\cdot\text{m/s}$ and

$p_{\text{i},\perp} = 461.0105692 \text{ g}\cdot\text{m/s}$. The parallel portion of the momentum is unchanged, and the perpendicular portion is the coefficient of restitution times the initial perpendicular momentum, giving a final parallel component of $p_{\text{f},\parallel} = 437.9416827 \text{ g}\cdot\text{m/s}$ and $p_{\text{f},\perp} = 377.9825657 \text{ g}\cdot\text{m/s}$. The final angle can be computed as $\theta_{\text{f}} = \tan^{-1}\left(\frac{p_{\text{f},\parallel}}{p_{\text{f},\perp}}\right)$ or 49.20°, which confirms the calculations.

**7.117.** $\tan\theta_{\text{f}} = \frac{1}{\varepsilon}\tan\theta_{\text{i}}$

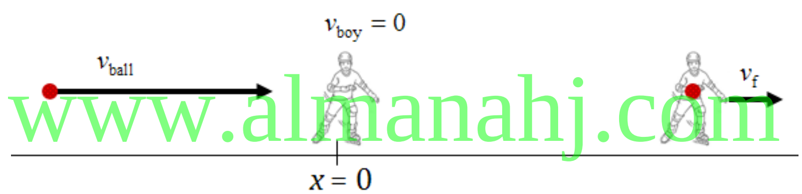
$$\varepsilon = \frac{\tan\theta_{\text{i}}}{\tan\theta_{\text{f}}} = \frac{\tan 48.67°}{\tan 55.75°} = 0.7742$$

**7.118.** $\tan\theta_{\text{f}} = \frac{1}{\varepsilon}\tan\theta_{\text{i}}$

$$\theta_{\text{i}} = \tan^{-1}\left(\varepsilon\tan\theta_{\text{f}}\right) = \tan^{-1}\left(0.8787\tan 57.24°\right) = 53.78°$$

**7.119.** **THINK:** When the boy catches the dodgeball, he holds on to it and does not let go. The boy and the ball stick together and have the same velocity after the collision, so this is a totally inelastic collision. This means that the final velocity of the boy and ball can be calculated from the initial velocities and masses of the boy and dodgeball.

**SKETCH:** Choose the $x$ – axis to run in the same direction as the dodgeball, with the origin at the boy's initial location.





**RESEARCH:** In a totally inelastic collision, the final velocity of both objects is given by $v_{\text{f}} = \frac{m_{\text{ball}}v_{\text{ball}} + m_{\text{boy}}v_{\text{boy}}}{m_{\text{ball}} + m_{\text{boy}}}$.

**SIMPLIFY:** Because the initial velocity of the boy $v_{\text{boy}} = 0$, the equation can be simplified to $v_{\text{f}} = \frac{m_{\text{ball}}v_{\text{ball}} + m_{\text{boy}}\cdot 0}{m_{\text{ball}} + m_{\text{boy}}} = \frac{m_{\text{ball}}v_{\text{ball}}}{m_{\text{ball}} + m_{\text{boy}}}$. Since the mass of the ball is given in grams and the mass of the boy is given in kilograms, it is necessary to multiply the mass of the ball by a conversion factor of $\frac{1 \text{ kg}}{1000 \text{ g}}$.

**CALCULATE:** The mass of the ball is 511.1 g, or $511.1 \text{ g}\cdot\frac{1 \text{ kg}}{1000 \text{ g}} = 0.5111 \text{ kg}$. The mass of the boy is 48.95 kg and the initial velocity of the dodgeball is 23.63 m/s. The final velocity is

$$\begin{aligned} v_{\text{f}} &= \frac{m_{\text{ball}}v_{\text{ball}}}{m_{\text{ball}} + m_{\text{boy}}} \\ &= \frac{0.5111 \text{ kg}\cdot 23.63 \text{ m/s}}{48.95 \text{ kg} + 0.5111 \text{ kg}} \\ &= 0.2441776062 \text{ m/s}. \end{aligned}$$

**ROUND:** The measured values in this problem are given to four significant figures, and the sum of the masses also has four significant figures, so the final answer should also have four significant figures. The final velocity of the boy and dodg ball is 0.2442 m/s in the same direction that the dodgeball was traveling initially.

**DOUBLE-CHECK:** This answer makes sense. The mass of the boy is much greater than the mass of the dodgeball, so a smaller speed of this massive system (boy plus dodgeball) will have the same momentum as the ball traveling much faster. To confirm that the answer is correct, check that the momentum after the collision is equal to the momentum before the collision. Before the collision, the boy is not moving so he has no momentum, and the dodgeball has a momentum of $p_x = mv_x = 0.511\text{ kg}\cdot 23.63\text{ m/s}$ or 12.075 kg · m/s. After the collision, the total momentum is $mv_x = (0.511\text{ kg} + 48.95\text{ kg})\cdot 0.2442\text{ m/s}$ or 12.078 kg · m/s. These agree within rounding error, so this confirms that the original calculation was correct.

**7.120.**

$$v_\text{f} = \frac{m_\text{ball} v_\text{ball}}{m_\text{ball} + m_\text{boy}}$$

$$v_\text{f}\left(m_\text{ball} + m_\text{boy}\right) = m_\text{ball} v_\text{ball}$$

$$v_\text{ball} = \frac{v_\text{f}\left(m_\text{ball} + m_\text{boy}\right)}{m_\text{ball}} = \frac{(0.2304\text{ m/s})(0.5131\text{ kg} + 53.53\text{ kg})}{0.5131\text{ kg}} = 24.27\text{ m/s}$$

**7.121.**

$$v_\text{f} = \frac{m_\text{ball} v_\text{ball}}{m_\text{ball} + m_\text{boy}}$$

$$v_\text{f}\left(m_\text{ball} + m_\text{boy}\right) = m_\text{ball} v_\text{ball}$$

$$v_\text{f} m_\text{ball} + v_\text{f} m_\text{boy} = m_\text{ball} v_\text{ball}$$

$$m_\text{boy} = \frac{m_\text{ball} v_\text{ball} - v_\text{f} m_\text{ball}}{v_\text{f}} = m_\text{ball}\frac{v_\text{ball} - v_\text{f}}{v_\text{f}} = (0.5151\text{ kg})\frac{24.91\text{ m/s} - 0.2188\text{ m/s}}{0.2188\text{ m/s}} = 58.13\text{ kg}$$

# Chapter 8: Systems of Particles and Extended Objects

## Concept Checks

**8.1.** b **8.2.** a **8.3.** d **8.4.** b **8.5.** a

## Multiple-Choice Questions

**8.1.** d **8.2.** b **8.3.** d **8.4.** b and d **8.5.** e **8.6.** a **8.7.** b **8.8.** d **8.9.** b **8.10.** e **8.11.** a **8.12.** c **8.13.** a **8.14.** b **8.15.** b **8.16.** b

## Conceptual Questions

**8.17.** It is reasonable to assume the explosion is entirely an internal force. This means the momentum, and hence the velocity of the center of mass remains unchanged. Therefore, the motion of the center of mass remains the same.

**8.18.** The length of the side of the cube is given as $d$. If the cubes have a uniform mass distribution, then the center of mass of each cube is at its geometric center. Let $m$ be the mass of a cube. The coordinates of the center of mass of the structure are given by:

$$X_{\text{cm}} = \frac{m\left(\frac{d}{2}+\frac{d}{2}+\frac{d}{2}+\frac{3d}{2}\right)}{4m} = \frac{3d}{4}, \quad Y_{\text{cm}} = \frac{m\left(\frac{d}{2}+\frac{d}{2}+\frac{d}{2}+\frac{3d}{2}\right)}{4m} = \frac{3d}{4} \quad \text{and} \quad Z_{\text{cm}} = \frac{m\left(\frac{d}{2}+\frac{d}{2}+\frac{d}{2}+\frac{3d}{2}\right)}{4m} = \frac{3d}{4}.$$

Therefore, the center of mass of the structure is located at $\vec{R} = \left(X_{\text{cm}},\, Y_{\text{cm}},\, Z_{\text{cm}}\right) = \left(\frac{3d}{4}, \frac{3d}{4}, \frac{3d}{4}\right)$.

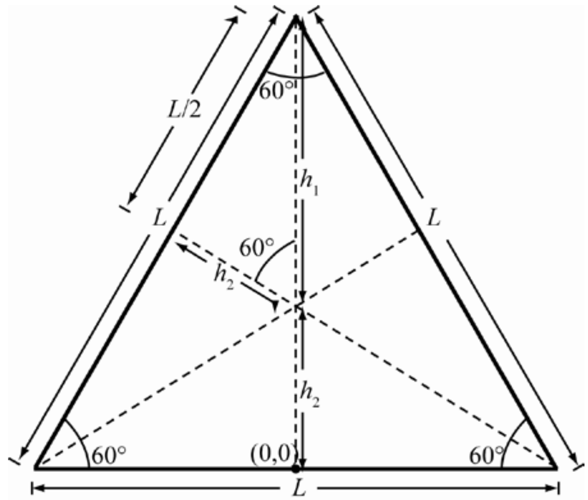


**8.19.** After the explosion, the motion of the center of mass should remain unchanged. Since both masses are equal, they must be equidistant from the center of mass. If the first piece has $x$-coordinate $x_1$ and the second piece has $x$-coordinate $x_2$, then $\left|X_{\text{cm}} - x_1\right| = \left|X_{\text{cm}} - x_2\right|$. For example, since the position of the center of mass is still 100 m, one piece could be at 90 m and the other at 110 m: $\left|100-90\right| = \left|100-110\right|$.

**8.20.** Yes, the center of mass can be located outside the object. Take a donut for example. If the donut has a uniform mass density, then the center of mass is located at its geometric center, which would be the center of a circle. However, at the donut's center, there is no mass, there is a hole. This means the center of mass can lie outside the object.

**8.21.** It is possible if, for example, there are outside forces involved. The kinetic energy of an object is proportional to the momentum squared $\left(K \propto p^2\right)$. So if $p$ increases, $K$ increases.

**8.22.** The intersection of the triangle's altitudes implies the triangle has a uniform mass density, meaning the center of mass is at the geometric center. To show this point by physical reasoning means using geometry to show where it is.

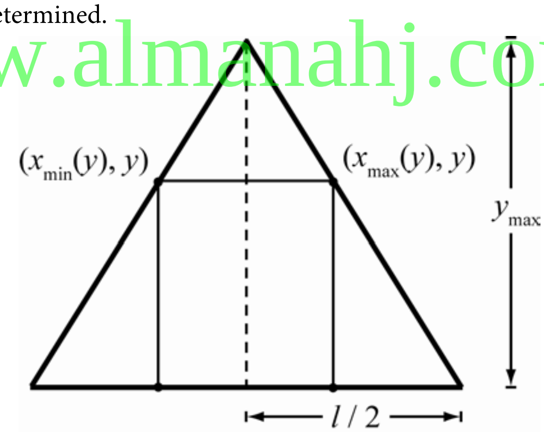



It can be seen that $h_1 \sin 60° = L/2$ and $h_1 \cos 60° = h_2$. Therefore,

$$h_1 = \frac{L}{2\sin 60°} = \frac{L}{2\left(\sqrt{3}/2\right)} = \frac{L}{\sqrt{3}} = \frac{L\sqrt{3}}{3} \Rightarrow h_2 = \frac{L\sqrt{3}}{3}\cos 60° = \frac{L\sqrt{3}}{3}\left(\frac{1}{2}\right) = \frac{L\sqrt{3}}{6}.$$

If the center of the bottom side of the triangle is (0, 0), then the center of mass is located at $\left(0, h_2\right) = \left(0, L\sqrt{3}/6\right)$. To calculate by direct measurement, note that due to symmetry by the choice of origin, the $x$ coordinate of the center of mass is in the middle of the $x$ axis. Therefore, $X_{\text{cm}} = 0$, which means only $Y_{\text{cm}}$ must be determined.






Clearly, the $x$ value of a point along the side of the triangle is dependent on the value of $y$ for that point, meaning $x$ is a function of $y$. When $y$ is zero, $x$ is $L/2$ and when $x$ is zero, $y$ is $y_{\max} = h_1 + h_2 = L\sqrt{3}/2$. The change in $x$ should be linear with change in $y$, so $x = my + b$, where $m = \frac{\Delta x}{\Delta y} = \frac{\left(L/2\right) - 0}{0 - \left(L\sqrt{3}/2\right)} = -\frac{1}{\sqrt{3}}$.

Therefore, $\frac{L}{2} = -\frac{0}{\sqrt{3}} + b = 0 + b \Rightarrow b = \frac{L}{2}$ and $0 = -\frac{L\sqrt{3}}{2\sqrt{3}} + b = -\frac{L}{2} + b \Rightarrow b = \frac{L}{2}$. The equation for $x$ is then given by $x\left(y\right) = -\frac{y}{\sqrt{3}} + \frac{L}{2}$. Since the mass density is uniform, the geometry of the triangle can be considered. $Y_{\text{cm}} = \frac{1}{A}\iint y\,dA$, where $A = \frac{L^2\sqrt{3}}{4}$ and $dA = dxdy$.

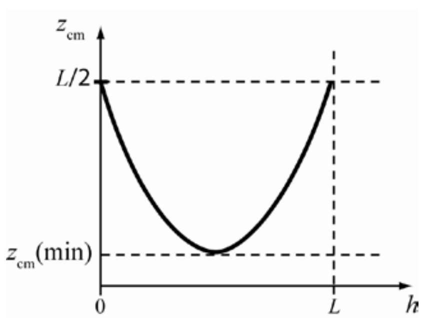
The integral then becomes:

$Y_{\text{cm}} = \frac{4}{L^2\sqrt{3}} \int_{y_{\min}}^{y_{\max}} y dy \int_{x_{\min}(y)}^{x_{\max}(y)} dx = \frac{4}{L^2\sqrt{3}} \int_0^{\frac{L\sqrt{3}}{2}} y\left(x_{\max}(y) - x_{\min}(y)\right)dy.$ Due to symmetry, $x_{\max}(y) = -x_{\min}(y)$ and $x_{\max}(y) = x(y).$ Therefore,

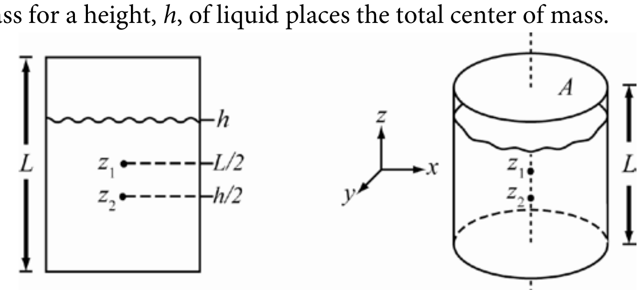
$$\begin{aligned} Y_{\text{cm}} &= \frac{8}{L^2\sqrt{3}} \int_0^{\frac{L\sqrt{3}}{2}} yx(y)dy = \frac{8}{L^2\sqrt{3}} \int_0^{\frac{L\sqrt{3}}{2}} \left(\frac{-y^2}{\sqrt{3}} + \frac{yL}{2}\right)dy \\ &= \frac{8}{L^2\sqrt{3}}\left[\frac{y^2L}{4} - \frac{y^3}{3\sqrt{3}}\right]_0^{\frac{L\sqrt{3}}{2}} = \frac{8}{L^2\sqrt{3}}\left[\frac{3L^3}{16} - \frac{L^3}{8}\right] = \frac{8}{L^2\sqrt{3}}\left[\frac{L^3}{16}\right] \\ &= \frac{L}{2\sqrt{3}} = \frac{L\sqrt{3}}{6}. \end{aligned}$$

The center of mass is located at $R = (X_{\text{cm}}, Y_{\text{cm}}) = \left(0, \frac{L\sqrt{3}}{6}\right).$ This is consistent with reasoning by geometry.

**8.23.** (a) The empty can and the liquid should each have their centers of mass at their geometric centers, so initially the center of mass of both is at the center of the can (assuming the can is filled completely with soda). Assuming the liquid drains out uniformly, only the height changes and the cross sectional area remains constant, so the center of mass is initially at *L*/2 and changes only in height. As liquid drains, its mass *M* will drop by $\Delta M$ but the mass of the can, *m*, remains the same. As liquid drains, its center of mass will also fall such that if the liquid is at a height *h*, $0 < h < L$, its center of mass is at *h*/2. As long as $M - \Delta M > m$, the center of mass of both will also fall to some height $h'$, $h/2 < h' < L$. Once $M - \Delta M < m$, the center of mass of both will begin to increase again until $M - \Delta M = 0$ and the center of mass is that of just the can at *L*/2. A sketch of the height of the center of mass of both as a function of liquid height is shown below.




(b) In order to determine the minimum value of the center of mass in terms of *L*, *M* and *m*, first consider where the center of mass for a height, *h*, of liquid places the total center of mass.




$Z_1$ is the center of mass of the can. $Z_2$ is the center of mass of the liquid. Notice the center of mass moves along the *z* axis only. *A* is the cross sectional area of the can in the *xy* plane. $\rho_M$ is the density of the liquid. *h* is the height of the liquid.

The coordinate of the center of mass is given by

$$Z_{\text{cm}}=\frac{\frac{mL}{2}+\frac{Mh}{2}}{m+M}.$$

When $h = L$, $Z_{\text{cm}}=L/2$. When $h<L$, $h=\alpha L$, where $0\le\alpha<1$. In other words, the height of the liquid is a fraction, $\alpha$, of the initial height, $L$. Initially the mass of the liquid is $M=\rho V=\rho AL$. When $h(\alpha)=\alpha L$, the mass of the liquid is $M(\alpha)=\rho Ah(\alpha)=\alpha\rho AL=\alpha M$. This means the center of mass for some value of $\alpha$ is

$$Z_{\text{cm}}(\alpha)=\frac{\frac{mL}{2}+\frac{M(\alpha)h(\alpha)}{2}}{m+M(\alpha)}=\frac{\frac{mL}{2}+\frac{\alpha^2ML}{2}}{m+\alpha M}=\frac{L}{2}\left(\frac{1+b\alpha^2}{1+b\alpha}\right).$$

where $b = M/m$ and $M$ is the initial mass of the liquid. In order to determine the minimum value of $Z_{\text{cm}}$, $Z_{\text{cm}}(\alpha)$ must be minimized in terms of $\alpha$ to determine where $\alpha_{\text{min}}$ occurs and then determine $Z_{\text{cm}}(\alpha_{\text{min}})$.

$$\frac{dZ_{\text{cm}}(\alpha)}{d\alpha}=a\frac{d}{d\alpha}\left(\frac{1+b\alpha^2}{1+b\alpha}\right)=a\left[\frac{b^2\alpha^2+2b\alpha-b}{(1+b\alpha)^2}\right]\text{, where } a = L/2.$$

When $dZ_{\text{cm}}(\alpha)/d\alpha=0\Rightarrow b^2\alpha^2+2b\alpha-b=0$. Using the quadratic equation, $\alpha=\frac{-1\pm\sqrt{1+b}}{b}$. Since $b>0$ and $\alpha>0$, $\alpha_{\text{min}}=\frac{-1+\sqrt{1+b}}{b}$. Therefore, $Z_{\text{cm}}(\alpha_{\text{min}})=a\left(\frac{1+b\alpha_{\text{min}}{}^2}{1+b\alpha_{\text{min}}}\right)=2a\left(\frac{1+b-\sqrt{1+b}}{b\sqrt{1+b}}\right)$.



$$Z_{\text{cm}}(\alpha_{\text{min}})=\frac{L\left(M+m-m\sqrt{1+\frac{M}{m}}\right)}{M\sqrt{1+\frac{M}{m}}}$$

If it is assumed that soda has a similar density to water and the can is made of aluminum, then the ratio of $M/m\approx 30$, giving a minimum $Z_{\text{cm}}$ of about $L/6$.

**8.24.** (a) If the astronaut throws both at the same time, he gains their momentum of them moving at a velocity, $v$. If he throws one first at a velocity, $v$, he will recoil back at a velocity, $v'$. So when he throws the second item, he will gain its momentum at a velocity of $v-v'$, which is less than $v$. So he gains less momentum from throwing the second item after the first than if he throws both items at the same time. Therefore, he obtains maximum speed when he throws both at the same time.

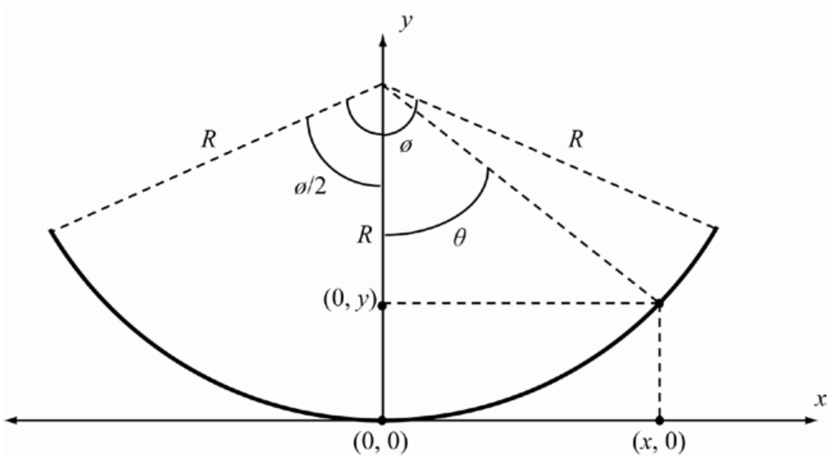
(b) If the astronaut throws the heavier object (tool box) first, it will give the astronaut a large velocity, $v'$, so when he throws the lighter object (hammer), it will have a small velocity of $v-v'$. So its momentum contribution will be very small. However, if he throws the lighter item first, $v'$ will be smaller in this scenario, so the momentum of the box will be dependent on $v-v'$, which is greater and contributes a large amount of momentum to the astronaut, giving him a larger velocity. Therefore, throwing the lighter object first will maximize his velocity.

(c) The absolute maximum velocity is when both items are thrown at the same time. Initially the momentum is zero and after the toss, the astronaut travels with velocity, $v'$ and the box and hammer travel with velocity, $v$ in the opposite direction.

$$\vec{p}_{\text{i}}=\vec{p}_{\text{f}}\Rightarrow 0=Mv'-\left(\frac{M}{2}+\frac{M}{4}\right)v\Rightarrow v'=\frac{3}{4}v$$

Therefore, the maximum velocity is ¾ of the velocity at which he throws the two items.

**8.25.** Let the angle $\theta$ sweep through from $-\phi/2$ to $\phi/2$. Keeping $R$ constant as $\theta$ increases, the length of the rod, $l = R\theta$, increases and in turn the mass, $m = \lambda l$, increases. Since the mass is uniformly distributed, the center of mass should be in the same location. So rather than bending a rod of constant length where $\theta$ and $R$ change, keep $R$ constant and change $\theta$ and $l$. Use Cartesian coordinates to determine the center of mass. Since the center of mass is a function of $\theta$, it must be determined how the coordinates change with the angle $\theta$.




$$y = R - R\cos\theta,\quad x = R\sin\theta,\quad m = \lambda R\phi,\quad dm = \lambda R\, d\theta$$

$$X_{\text{cm}} = \frac{1}{m}\int x\,dm = \frac{1}{\lambda R\phi}\int_{-\phi/2}^{\phi/2} R\sin\theta\,\lambda R\,d\theta = \frac{R}{\phi}\int_{-\phi/2}^{\phi/2}\sin\theta\,d\theta = \left[-\frac{R}{\phi}\cos\theta\right]_{-\frac{\phi}{2}}^{\frac{\phi}{2}} = -\frac{R}{\phi}\left(\cos\frac{\phi}{2} - \cos\left(-\frac{\phi}{2}\right)\right) = 0$$



$$Y_{\text{cm}} = \frac{1}{m}\int y\,dm = \frac{1}{\lambda R\phi}\int_{-\phi/2}^{\phi/2}(R - R\cos\theta)\lambda R\,d\theta = \frac{R}{\phi}\int_{-\phi/2}^{\phi/2}(1-\cos\theta)\,d\theta = \left[\frac{R}{\phi}(\theta - \sin\theta)\right]_{-\frac{\phi}{2}}^{\frac{\phi}{2}}$$

$$= \frac{R}{\phi}\left(\frac{\phi}{2} - \left(-\frac{\phi}{2}\right)\right) - \frac{R}{\phi}\left(\sin\left(\frac{\phi}{2}\right) - \sin\left(-\frac{\phi}{2}\right)\right) = R - \frac{2R\sin\left(\frac{\phi}{2}\right)}{\phi}$$

$$\vec{R}_{\text{cm}} = (X_{\text{cm}}, Y_{\text{cm}}) = \left(0, R - \frac{2R\sin(\phi/2)}{\phi}\right)$$

**8.26.** As eggs $A$, $B$ and/or $C$ are removed, the center of mass will shift down and to the left. To determine the overall center of mass, use the center of the eggs as their center position, such that eggs $A$, $B$ and $C$ are located respectively at

$$\left(\frac{d}{2},\frac{d}{2}\right),\quad \left(\frac{3d}{2},\frac{d}{2}\right),\quad \left(\frac{5d}{2},\frac{d}{2}\right).$$

Since all of the eggs are of the same mass, $m$, and proportional to $d$, $m$ and $d$ can be factored out of the equations for $X_{\text{cm}}$ and $Y_{\text{cm}}$.

(a) $$X_{\text{cm}} = \frac{md}{11m}\left(2\left(-\frac{5}{2}\right) + 2\left(-\frac{3}{2}\right) + 2\left(-\frac{1}{2}\right) + \frac{1}{2} + 2\left(\frac{3}{2}\right) + 2\left(\frac{5}{2}\right)\right) = -\frac{d}{22},\quad Y_{\text{cm}} = \frac{md}{11m}\left(6\left(-\frac{1}{2}\right) + 5\left(\frac{1}{2}\right)\right) = -\frac{d}{22}$$

$$\vec{R}_{\text{cm}} = \left(-\frac{d}{22}, -\frac{d}{22}\right)$$

(b) $X_{\text{cm}} = \dfrac{md}{11m}\left(2\left(-\dfrac{5}{2}\right)+2\left(-\dfrac{3}{2}\right)+2\left(-\dfrac{1}{2}\right)+2\left(\dfrac{1}{2}\right)+\dfrac{3}{2}+2\left(\dfrac{5}{2}\right)\right)=-\dfrac{3d}{22},\ Y_{\text{cm}}=\dfrac{md}{11m}\left(6\left(-\dfrac{1}{2}\right)+5\left(\dfrac{1}{2}\right)\right)=-\dfrac{d}{22}$

$\vec{R}_{\text{cm}}=\left(-\dfrac{3d}{22},-\dfrac{d}{22}\right)$

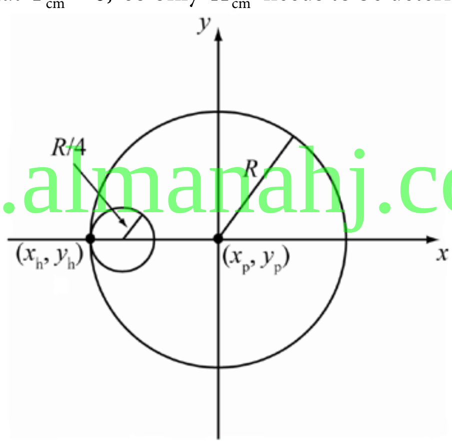
(c) $X_{\text{cm}} = \dfrac{md}{11m}\left(2\left(-\dfrac{5}{2}\right)+2\left(-\dfrac{3}{2}\right)+2\left(-\dfrac{1}{2}\right)+2\left(\dfrac{1}{2}\right)+2\left(\dfrac{3}{2}\right)+\dfrac{5}{2}\right)=-\dfrac{5d}{22},\ Y_{\text{cm}}=\dfrac{md}{11m}\left(6\left(-\dfrac{1}{2}\right)+5\left(\dfrac{1}{2}\right)\right)=-\dfrac{d}{22}$

$\vec{R}_{\text{cm}}=\left(-\dfrac{5d}{22},-\dfrac{d}{22}\right)$

(d) $X_{\text{cm}} = \dfrac{md}{9m}\left(2\left(-\dfrac{5}{2}\right)+2\left(-\dfrac{3}{2}\right)+2\left(-\dfrac{1}{2}\right)+\dfrac{1}{2}+\dfrac{3}{2}+\dfrac{5}{2}\right)=-\dfrac{d}{2},\qquad Y_{\text{cm}}=\dfrac{md}{9m}\left(6\left(-\dfrac{1}{2}\right)+3\left(\dfrac{1}{2}\right)\right)=-\dfrac{d}{6}$

$\vec{R}_{\text{cm}}=\left(-\dfrac{d}{2},-\dfrac{d}{6}\right)$

**8.27.** The center of the pizza is at $(0,0)$ and the center of the piece cut out is at $(-3R/4,0)$. Assume the pizza and the hole have a uniform mass density (though the hole is considered to have a negative mass). Then the center of mass can be determined from geometry. Also, because of symmetry of the two circles and their $y$ position, it can be said that $Y_{\text{cm}}=0$, so only $X_{\text{cm}}$ needs to be determined.






$A_{\text{p}}=\pi R^2,\ A_{\text{h}}=\pi\left(\dfrac{R}{4}\right)^2=\pi\dfrac{R^2}{16},\quad (x_{\text{p}},y_{\text{p}})=(0,0),\quad (x_{\text{h}},y_{\text{h}})=\left(-\dfrac{3}{4}R,0\right)$

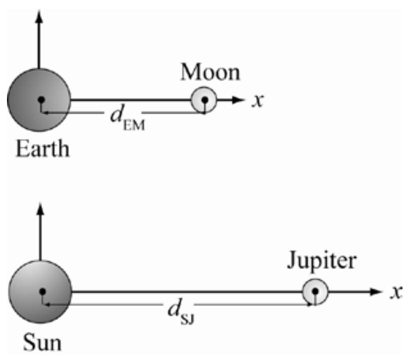
$$X_{\text{cm}}=\frac{x_{\text{p}}A_{\text{p}}-x_{\text{h}}A_{\text{h}}}{A_{\text{p}}-A_{\text{h}}}=\frac{0-\left(-\frac{3}{4}R\right)\left(\frac{\pi R^2}{16}\right)}{\pi R^2-\frac{\pi R^2}{16}}=\frac{R}{20},\quad \vec{R}_{\text{cm}}=\left(\frac{R}{20},0\right)$$

**8.28.** Since the overall mass of the hourglass does not change and the center of mass must move from the top half to the bottom half, then the center of mass velocity, $v_{\text{cm}}$, must be non-zero and pointing down. As the sand flows from the top part of the hourglass to the lower part, $v_{\text{cm}}$ changes with time. The magnitude of $v_{\text{cm}}$ is larger when the sand has just started to flow than just before all the sand has flowed through. Thus $dv_{\text{cm}}/dt=a_{\text{cm}}$ must be in the opposite direction from $v_{\text{cm}}$, which is the upward direction. The scale must supply the force required to produce this upward acceleration, so the hourglass weighs more when the sand is flowing than when the sand is stationary. You can find a published solution to a similar version of this problem at the following reference: K.Y. Shen and Bruce L. Scott, American Journal of Physics, **53**, 787 (1985).

## Exercises

**8.29.** **THINK:** Determine (a) the distance, $d_1$, from the center of mass of the Earth-Moon system to the geometric center of the Earth and (b) the distance, $d_2$, from the center of mass of the Sun-Jupiter system to the geometric center of the Sun. The mass of the Earth is approximately $m_{\text{E}} = 5.9742 \cdot 10^{24}$ kg and the mass of the Moon is approximately $m_{\text{M}} = 7.3477 \cdot 10^{22}$ kg. The distance between the center of the Earth to the center of the Moon is $d_{\text{EM}} = 384,400$ km. Also, the mass of the Sun is approximately $m_{\text{S}} = 1.98892 \cdot 10^{30}$ kg and the mass of Jupiter is approximately $m_{\text{J}} = 1.8986 \cdot 10^{27}$ kg. The distance between the center of the Sun and the center of Jupiter is $d_{\text{SJ}} = 778,300,000$ km.

**SKETCH:**




**RESEARCH:** Determine the center of mass of the two object system from $\vec{R} = \dfrac{\vec{r}_1 m_1 + \vec{r}_2 m_2}{m_1 + m_2}$. By considering the masses on the $x$-axis (as sketched), the one dimensional equation can be used for $x$. Assuming a uniform, spherically symmetric distribution of each planet's mass, they can be modeled as point particles. Finally, by placing the Earth (Sun) at the origin of the coordinate system, the center of mass will be determined with respect to the center of the Earth (Sun), i.e. $d_1 (d_2) = x$.



**SIMPLIFY:**

(a) $d_1 = x = \dfrac{x_1 m_{\text{E}} + x_2 m_{\text{M}}}{m_{\text{E}} + m_{\text{M}}} = \dfrac{d_{\text{EM}} m_{\text{M}}}{m_{\text{E}} + m_{\text{M}}}$

(b) $d_2 = x = \dfrac{x_1 m_{\text{S}} + x_2 m_{\text{J}}}{m_{\text{S}} + m_{\text{J}}} = \dfrac{d_{\text{SJ}} m_{\text{J}}}{m_{\text{S}} + m_{\text{J}}}$

**CALCULATE:**

(a) $d_1 = \dfrac{(384,400 \text{ km})(7.3477 \cdot 10^{22} \text{ kg})}{(5.9742 \cdot 10^{24} \text{ kg}) + (7.3477 \cdot 10^{22} \text{ kg})} = \dfrac{2.8244559 \cdot 10^{28} \text{ km} \cdot \text{kg}}{6.047677 \cdot 10^{24} \text{ kg}} = 4670.3 \text{ km}$

(b) $d_2 = \dfrac{(7.783 \cdot 10^{8} \text{ km})(1.8986 \cdot 10^{27} \text{ kg})}{(1.98892 \cdot 10^{30} \text{ kg}) + (1.8986 \cdot 10^{27} \text{ kg})} = 742247.6 \text{ km}$

**ROUND:**

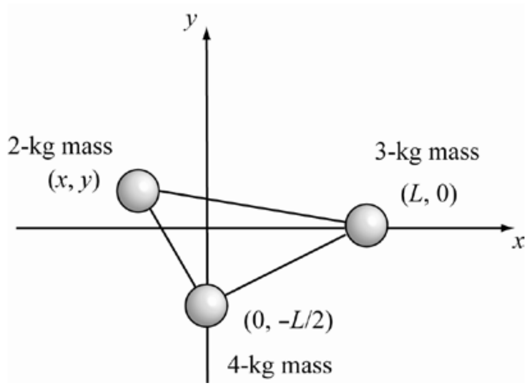
(a) $d_{\text{EM}}$ has four significant figures, so $d_1 = 4670.$ km.

(b) $d_{\text{SJ}}$ has four significant figures, so $d_2 = 742,200$ km.

**DOUBLE-CHECK:** In each part, the distance $d_1 / d_2$ is much less than half the separation distance $d_{\text{EM}} / d_{\text{SJ}}$. This makes sense as the center of mass should be closer to the more massive object in the two body system.

**8.30. THINK:** The center of mass coordinates for the system are $(L/4, -L/5)$. The masses are $m_1 = 2$ kg, $m_2 = 3$ kg and $m_3 = 4$ kg. The coordinates for $m_2$ are $(L, 0)$ and the coordinates for $m_3$ are $(0, -L/2)$. Determine the coordinates for $m_1$.

**SKETCH:**




**RESEARCH:** The $x$ and $y$ coordinates for $m_1$ can be determined from the equations for the center of mass in each dimension:

$$X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i \text{ and } Y = \frac{1}{M}\sum_{i=1}^{n} y_i m_i.$$

**SIMPLIFY:** $X = \dfrac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} \Rightarrow x_1 = \dfrac{1}{m_1}\left(X\left(m_1 + m_2 + m_3\right) - x_2 m_2 - x_3 m_3\right)$

Similarly, $y_1 = \dfrac{1}{m_1}\left(Y\left(m_1 + m_2 + m_3\right) - y_2 m_2 - y_3 m_3\right)$.



**CALCULATE:** $x_1 = \left(\dfrac{1}{2\text{ kg}}\right)\left(\dfrac{L}{4}\left(2\text{ kg} + 3\text{ kg} + 4\text{ kg}\right) - L\left(3\text{ kg}\right) - 0\left(4\text{ kg}\right)\right) = -\dfrac{3}{8}L$

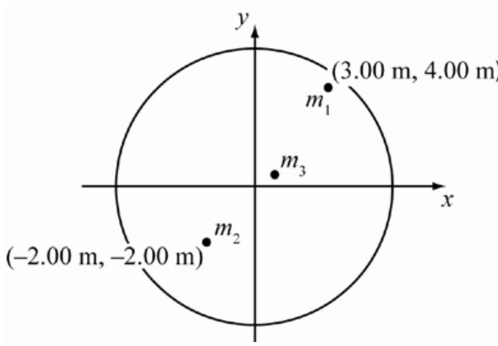
$y_1 = \left(\dfrac{1}{2\text{ kg}}\right)\left(-\dfrac{L}{5}\left(2\text{ kg} + 3\text{ kg} + 4\text{ kg}\right) - 0\left(3\text{ kg}\right) - \left(-\dfrac{L}{2}\right)\left(4\text{ kg}\right)\right) = \dfrac{1}{10}L$

**ROUND:** Rounding is not necessary since the initial values and the results are fractions, so $m_1$ is located at $(-3L/8, L/10)$.

**DOUBLE-CHECK:** The coordinates for $m_1$ are reasonable: since $X_{cm}$ is positive and $Y_{cm}$ is negative and both coordinates have comparatively small values (and thus the center of mass is close to the origin), it makes sense that $x$ will be negative to balance the 3-kg mass and $y$ will be positive to balance the 4-kg mass.

**8.31. THINK:** The mass and location of the first acrobat are known to be $m_1 = 30.0$ kg and $\vec{r}_1 = (3.00\text{ m}, 4.00\text{ m})$. The mass and location of the second acrobat are $m_2 = 40.0$ kg and $\vec{r}_2 = (-2.00\text{ m}, -2.00\text{ m})$. The mass of the third acrobat is $m_3 = 20.0$ kg. Determine the position of the third acrobat, $\vec{r}_3$, when the center of mass (com) is at the origin.

**SKETCH:**




**RESEARCH:** Let $M$ be the sum of the three masses. The coordinates of $m_3$ can be determined from the center of mass equations for each dimension,

$$X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i \text{ and } Y = \frac{1}{M}\sum_{i=1}^{n} y_i m_i.$$

**SIMPLIFY:** Since $X = 0$, $X = \frac{1}{M}\left(x_1m_1 + x_2m_2 + x_3m_3\right) = 0 \Rightarrow x_3 = \frac{\left(-x_1m_1 - x_2m_2\right)}{m_3}$. Similarly, with $Y = 0$, $y_3 = \frac{\left(-y_1m_1 - y_2m_2\right)}{m_3}$.

**CALCULATE:** $x_3 = \frac{\left(-(3.00 \text{ m})(30.0 \text{ kg}) - (-2.00 \text{ m})(40.0 \text{ kg})\right)}{20.0 \text{ kg}} = -0.500 \text{ m}$,

$y_3 = \frac{\left(-(4.00 \text{ m})(30.0 \text{ kg}) - (-2.00 \text{ m})(40.0 \text{ kg})\right)}{20.0 \text{ kg}} = -2.00 \text{ m}$

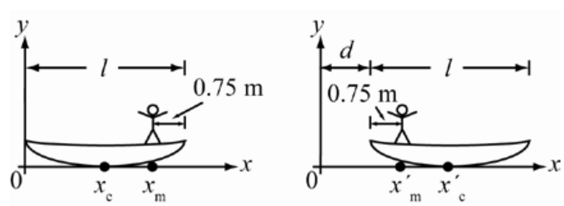


**ROUND:** $\vec{r}_3 = \left(-0.500 \text{ m}, -2.00 \text{ m}\right)$

**DOUBLE-CHECK:** The resulting location is similar to the locations of the other acrobats.

**8.32.** **THINK:** The man's mass is $m_\text{m} = 55$ kg and the canoe's mass is $m_\text{c} = 65$ kg. The canoe's length is $l = 4.0$ m. The man moves from 0.75 m from the back of the canoe to 0.75 m from the front of the canoe. Determine how far the canoe moves, $d$.

**SKETCH:**




**RESEARCH:** The center of mass position for the man and canoe system does not change in our external reference frame. To determine $d$, the center of mass location must be determined before the canoe moves. Then the new location for the canoe after the man moves can be determined given the man's new position and the center of mass position. Assume the canoe has a uniform density such that its center of mass location is at the center of the canoe, $x_\text{c} = 2.0$ m. The man's initial position is $x_\text{m} = l - 0.75 \text{ m} = 3.25$ m. After moving, the canoe is located at $x'_\text{c}$ and the man is located at $x'_\text{m} = x'_\text{c} + a$. $a$ is the relative position of the man with respect to the canoe's center of mass and $a = -l/2 + 0.75 \text{ m} = -1.25$ m. Then the distance the canoe moves is $d = x'_\text{c} - x_\text{c}$.

**SIMPLIFY:**

$$X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i.$$

The center of mass is $X = \frac{1}{M}\left(x_{\text{m}} m_{\text{m}} + x_{\text{c}} m_{\text{c}}\right)$. After moving,

$X = \frac{1}{M}\left(x'_{\text{m}} m_{\text{m}} + x'_{\text{c}} m_{\text{c}}\right) = \frac{1}{M}\left(\left(x'_{\text{c}} + a\right) m_{\text{m}} + x'_{\text{c}} m_{\text{c}}\right)$. Since $X$ does not change, the equations can be equated:

$$\frac{1}{M}\left(\left(x'_{\text{c}} + a\right) m_{\text{m}} + x'_{\text{c}} m_{\text{c}}\right) = \frac{1}{M}\left(x_{\text{m}} m_{\text{m}} + x_{\text{c}} m_{\text{c}}\right)$$

This implies $x_{\text{m}} m_{\text{m}} + x_{\text{c}} m_{\text{c}} = x'_{\text{c}} m_{\text{m}} + x'_{\text{c}} m_{\text{c}} + a m_{\text{m}} \;\Rightarrow\; x'_{\text{c}} = \frac{x_{\text{m}} m_{\text{m}} + x_{\text{c}} m_{\text{c}} - a m_{\text{m}}}{m_{\text{m}} + m_{\text{c}}}$.

**CALCULATE:** $x'_c = \frac{(3.25 \text{ m})(55.0 \text{ kg}) + (2.00 \text{ m})(65.0 \text{ kg}) - (-1.25 \text{ m})(55.0 \text{ kg})}{55.0 \text{ kg} + 65.0 \text{ kg}} = 3.1458 \text{ m}$

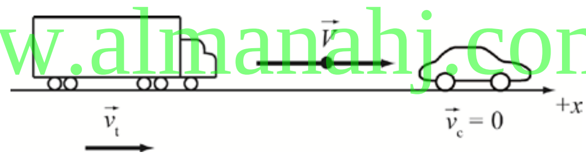
Then $d = 3.1458 \text{ m} - 2.00 \text{ m} = 1.1458 \text{ m}$.

**ROUND:** As each given value has three significant figures, $d$ = 1.15 m.

**DOUBLE-CHECK:** This distance is less than the distance traveled by the man (2.5 m), as it should be to preserve the center of mass location.

**8.33.** **THINK:** The mass of the car is $m_{\text{c}} = 2.00 \text{ kg}$ and its initial speed is $v_{\text{c}} = 0$. The mass of the truck is $m_{\text{t}} = 3.50 \text{ kg}$ and its initial speed is $v_{\text{t}} = 4.00 \text{ m/s}$ toward the car. Determine (a) the velocity of the center of mass, $\vec{V}$, and (b) the velocities of the truck, $\vec{v}'_{\text{t}}$ and the car, $\vec{v}'_{\text{c}}$ with respect to the center of mass.

**SKETCH:**






**RESEARCH:**

(a) The velocity of the center of mass can be determined from $\vec{V} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{v}_i$.

Take $\vec{v}_{\text{t}}$ to be in the positive $x$-direction.

(b) Generally, the relative velocity, $\vec{v}\,'$, of an object with velocity, $\vec{v}$, in the lab frame is given by $\vec{v}\,' = \vec{v} - \vec{V}$, where $\vec{V}$ is the velocity of the relative reference frame. Note the speeds of the car and the truck relative to the center of mass do not change after their collision, but the relative velocities change direction; that is, $\vec{v}'_{\text{t}}(\text{before collision}) = -\vec{v}'_{\text{t}}(\text{after collision})$ and similarly for the car's relative velocity.

**SIMPLIFY:**

(a) Substituting $\vec{v}_{\text{c}} = 0$ and $M = m_{\text{c}} + m_{\text{t}}$, $\vec{V} = \frac{1}{M}\left(m_{\text{c}}\vec{v}_{\text{c}} + m_{\text{t}}\vec{v}_{\text{t}}\right)$ becomes $\vec{V} = \frac{\left(m_{\text{t}}\vec{v}_{\text{t}}\right)}{\left(m_{\text{c}} + m_{\text{t}}\right)}$.

(b) $\vec{v}'_{\text{t}}$ and $\vec{v}'_{\text{c}}$ before the collision are $\vec{v}'_{\text{t}} = \vec{v}_{\text{t}} - \vec{V}$ and $\vec{v}'_{\text{c}} = \vec{v}_{\text{c}} - \vec{V} = -\vec{V}$.

**CALCULATE:**

(a) $\vec{V} = \frac{(3.50 \text{ kg})(4.00\hat{x} \text{ m/s})}{(3.50 \text{ kg} + 2.00 \text{ kg})} = 2.545\hat{x} \text{ m/s}$

(b) $\vec{v}'_{\text{t}} = (4.00\hat{x} \text{ m/s}) - (2.545\hat{x} \text{ m/s}) = 1.4545\hat{x} \text{ m/s}$, $\vec{v}'_{\text{c}} = -2.545\hat{x} \text{ m/s}$

**ROUND:** There are three significant figures for each given value, so the results should be rounded to the same number of significant figures.

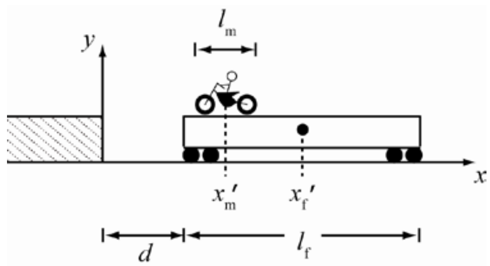
(a) $\vec{V} = 2.55\hat{x} \text{ m/s}$

(b) Before the collision, $\vec{v}_t' = 1.45\hat{x}$ m/s and $\vec{v}_c' = -2.55\hat{x}$ m/s. This means that after the collision, the velocities with respect to the center of mass become $\vec{v}_t' = -1.45\hat{x}$ m/s and $\vec{v}_c' = 2.55\hat{x}$ m/s.

**DOUBLE-CHECK:** $\vec{V}$ is between the initial velocity of the truck and the initial velocity of the car, as it should be.

**8.34.** **THINK:** The motorcycle with rider has a mass of $m_{\text{m}} = 350.$ kg. The flatcar's mass is $m_{\text{f}} = 1500.$ kg. The length of the motorcycle is $l_{\text{m}} = 2.00$ m and the length of the flatcar is $l_{\text{f}} = 20.0$ m. The motorcycle starts at one of end of the flatcar. Determine the distance, *d*, that the flatcar will be from the platform when the motorcycle reaches the end of the flatcar.

**SKETCH:** After the motorcycle and rider drive down the platform:




**RESEARCH:** The flatcar-motorcycle center of mass stays in the same position while the motorcycle moves. First, the center of mass must be determined before the motorcycle moves. Then the new location of the flatcar's center of mass can be determined given the center of mass for the system and the motorcycle's final position. Then the distance, *d*, can be determined. Assume that the motorcycle and rider's center of mass and the flatcar's center of mass are located at their geometric centers. Take the initial center of mass position for the motorcycle to be $x_{\text{m}} = l_{\text{f}} - l_{\text{m}}/2$, and the initial center of mass for the flatcar to be $x_{\text{f}} = l_{\text{f}}/2$. The final position of the center of mass for the motorcycle will be $x_{\text{m}}' = d + l_{\text{m}}/2$, and the final position for the flatcar will be $x_{\text{f}}' = d + l_{\text{f}}/2$. Then *d* can be determined from



$$X = \frac{1}{M}\sum_{i=1}^{n} x_i m_i.$$

**SIMPLIFY:** Originally, $X = \frac{1}{M}(x_{\text{m}}m_{\text{m}} + x_{\text{f}}m_{\text{f}})$. After the motorcycle moves, $X = \frac{1}{M}(x_{\text{m}}'m_{\text{m}} + x_{\text{f}}'m_{\text{f}})$.

As the center of mass remains constant, the two expressions can be equated:

$$\frac{1}{M}(x_{\text{m}}m_{\text{m}} + x_{\text{f}}m_{\text{f}}) = \frac{1}{M}(x_{\text{m}}'m_{\text{m}} + x_{\text{f}}'m_{\text{f}})$$

$$x_{\text{m}}m_{\text{m}} + x_{\text{f}}m_{\text{f}} = \left(d + \frac{1}{2}l_{\text{m}}\right)m_{\text{m}} + \left(d + \frac{1}{2}l_{\text{f}}\right)m_{\text{f}}$$

$$x_{\text{m}}m_{\text{m}} + x_{\text{f}}m_{\text{f}} = d(m_{\text{m}} + m_{\text{f}}) + \frac{1}{2}l_{\text{m}}m_{\text{m}} + \frac{1}{2}l_{\text{f}}m_{\text{f}}$$

$$d = \frac{\left(x_{\text{m}} - \frac{1}{2}l_{\text{m}}\right)m_{\text{m}} + \left(x_{\text{f}} - \frac{1}{2}l_{\text{f}}\right)m_{\text{f}}}{m_{\text{m}} + m_{\text{f}}}$$

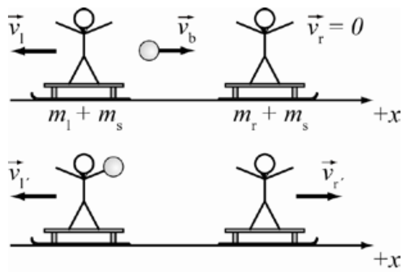
$x_{\text{m}} = l_{\text{f}} - \frac{l_{\text{m}}}{2}$ and $x_{\text{f}} = \frac{l_{\text{f}}}{2}$, therefore $d = \frac{(l_{\text{f}} - l_{\text{m}})m_{\text{m}}}{m_{\text{m}} + m_{\text{f}}}$.

**CALCULATE:** $d = \frac{(20.0\text{ m} - 2.00\text{ m})(350.\text{ kg})}{350.\text{ kg} + 1500.\text{ kg}} = 3.4054$ m

**ROUND:** $m_{\text{m}}$ has three significant figures, so the result should be rounded to $d$ = 3.41 m.
**DOUBLE-CHECK:** It is reasonable that the distance moved is less than length of the flatcar.

**8.35.** **THINK:** The mass of the sled is $m_s = 10.0$ kg, the mass of the ball is $m_b = 5.00$ kg, and the mass of the student on the left is $m_l = 50.0$ kg. His relative ball-throwing speed is $v_{bl} = 10.0$ m/s. The mass of the student on the right is $m_r = 45.0$ kg and his relative ball-throwing speed is $v_{br} = 12.0$ m/s. Determine (a) the speed of the student on the left, $v_l$, after first throwing the ball, (b) the speed of the student on the right, $v_r$, after catching the ball, (c) the speed of the student on the left after catching the pass, $v_l'$, and (d) the speed of the student on the right after throwing the pass, $v_r'$.

**SKETCH:**




**RESEARCH:** Momentum is conserved between each student and ball system. For each step, use $\vec{P}_i = \vec{P}_f$. In addition, the relative velocity of the ball is the difference between its velocity in the lab frame and the velocity of the student in the lab frame who has thrown it. That is, $\vec{v}_{bl} = \vec{v}_b - \vec{v}_l$ and $\vec{v}_{br} = \vec{v}_b - \vec{v}_r$. Recall each student begins at rest.

**SIMPLIFY:**

(a) Determine $v_l$ after the ball is first thrown:

$$\vec{P}_i = \vec{P}_f \Rightarrow 0 = (m_s + m_l)\vec{v}_l + m_b\vec{v}_b \Rightarrow 0 = (m_s + m_l)\vec{v}_l + m_b(\vec{v}_{bl} + \vec{v}_l) \Rightarrow \vec{v}_l = -\frac{m_b\vec{v}_{bl}}{m_s + m_l + m_b}.$$



(b) Determine $\vec{v}_r$ after the student catches the ball. The velocity of the ball, $\vec{v}_b$, in the lab frame is needed. From part (a), $\vec{v}_l$ is known. Then $\vec{v}_b = \vec{v}_{bl} + \vec{v}_l$. So, $\vec{v}_b$ is known before it is caught. Now, for the student on the right catching the ball,

$$\vec{P}_i = \vec{P}_f \Rightarrow m_b\vec{v}_b = (m_b + m_r + m_s)\vec{v}_r \Rightarrow \vec{v}_r = \frac{m_b\vec{v}_b}{m_b + m_r + m_s}.$$

(c) Now the student on the right throws the ball and the student on the left catches it. To determine $\vec{v}_l'$, the velocity of the ball after it is thrown, $\vec{v}_b'$, is needed. It is known that $\vec{v}_{br} = \vec{v}_b - \vec{v}_r$. Then to determine $\vec{v}_b'$, consider the situation when the student on the right throws the ball. For the student on the right: $P_i = P_f \Rightarrow (m_s + m_r + m_b)\vec{v}_r = m_b\vec{v}_b' + (m_r + m_s)\vec{v}_r'$, where $\vec{v}_r$ is known from part (b) and $\vec{v}_{br} = \vec{v}_b' - \vec{v}_r' \Rightarrow \vec{v}_r' = \vec{v}_b' - \vec{v}_{br}$. Then, the fact that $(m_s + m_r + m_b)\vec{v}_r = m_b\vec{v}_b' + (m_r + m_s)(\vec{v}_b' - \vec{v}_{br})$ implies $\vec{v}_b' = \dfrac{(m_s + m_r + m_b)\vec{v}_r + (m_r + m_s)\vec{v}_{br}}{m_b + m_r + m_s}$. With $\vec{v}_b'$ known, consider the student on the left catching this ball: $P_i = P_f \Rightarrow m_b\vec{v}_b' + (m_l + m_s)\vec{v}_l = (m_b + m_l + m_s)\vec{v}_l'$. $\vec{v}_l$ is known from part (a) and $\vec{v}_b'$ has just been determined, so $\vec{v}_l' = \dfrac{m_b\vec{v}_b' + (m_l + m_s)\vec{v}_l}{m_b + m_l + m_s}$.

(d) $\vec{v}_{br} = \vec{v}_b' - \vec{v}_r' \Rightarrow \vec{v}_r' = \vec{v}_b' - \vec{v}_{br}$ and $\vec{v}_b'$ has been determined in part (c).

**CALCULATE:**

(a) $\vec{v}_l = -\dfrac{(5.00 \text{ kg})(10.0 \text{ m/s})}{10.0 \text{ kg} + 50.0 \text{ kg} + 5.00 \text{ kg}} = -0.76923 \text{ m/s}$

(b) $\vec{v}_b = 10.0 \text{ m/s} - 0.769 \text{ m/s} = 9.231 \text{ m/s}$, $\vec{v}_r = \dfrac{(5.00 \text{ kg})(9.23077 \text{ m/s})}{5.00 \text{ kg} + 45.0 \text{ kg} + 10.0 \text{ kg}} = 0.76923 \text{ m/s}$

(c) The ball is thrown to the left, or along the $-\hat{x}$ axis by the student on the right. That is, $\vec{v}_{\text{br}} = -12.0$ m/s.

$$\vec{v}_{\text{b}}' = \frac{(10.0 \text{ kg} + 45.0 \text{ kg} + 5.00 \text{ kg})(0.769 \text{ m/s}) + (45.0 \text{ kg} + 10.0 \text{ kg})(-12.0 \text{ m/s})}{5.00 \text{ kg} + 45.0 \text{ kg} + 10.0 \text{ kg}} = -10.23100 \text{ m/s}$$

$$\vec{v}_{\text{l}}' = \frac{(5.00 \text{ kg})(-10.2310 \text{ m/s}) + (50.0 \text{ kg} + 10.0 \text{ kg})(-0.769 \text{ m/s})}{5.00 \text{ kg} + 50.0 \text{ kg} + 10.0 \text{ kg}} = -1.49685 \text{ m/s}$$

(d) $\vec{v}_{\text{r}}' = (-10.231 \text{ m/s}) - (-12.0 \text{ m/s}) = 1.769 \text{ m/s}$

**ROUND:**

(a) $\vec{v}_{\text{l}} = -0.769$ m/s (to the left)

(b) $\vec{v}_{\text{r}} = 0.769$ m/s (to the right)

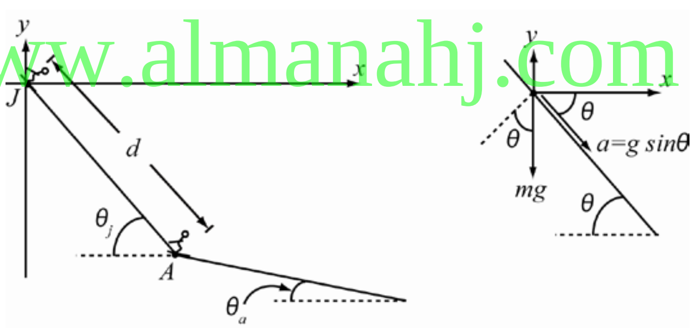
(c) $\vec{v}_{\text{l}}' = -1.50$ m/s (to the left)

(d) $\vec{v}_{\text{r}}' = 1.77$ m/s (to the right)

**DOUBLE-CHECK:** Before rounding, $|\vec{v}_{\text{l}}'| > |\vec{v}_{\text{l}}| > 0$ (where the initial speed was zero) and $|\vec{v}_{\text{r}}'| > |\vec{v}_{\text{r}}| > 0$, as expected.

**8.36.** **THINK:** Jack's mass is $m_{\text{J}} = 88.0$ kg. Jack's initial position is taken as $(0,0)$ and the angle of his slope is $\theta_{\text{J}} = 35.0°$. The distance of his slope is $d$ = 100. m. Annie's mass is $m_{\text{A}} = 64.0$ kg. Her slope angle is $\theta_{\text{A}} = 20.0°$. Take her initial position to be $(d\cos\theta_{\text{J}}, -d\sin\theta_{\text{J}})$. Determine the acceleration, velocity and position vectors of their center of mass as functions of time, before Jack reaches the less steep section.

**SKETCH:**






**RESEARCH:** To determine the acceleration, velocity and position vectors for the center of mass, the vectors must be determined in each direction. Assuming a constant acceleration, the familiar constant acceleration equations can be used. In addition,

$$\vec{R} = \frac{1}{M}\sum_{i=1}^{n}\vec{r}_i m_i, \quad \vec{V} = \frac{d\vec{R}}{dt} = \frac{1}{M}\sum_{i=1}^{n}\vec{v}_i m_i, \quad \vec{A} = \frac{d\vec{V}}{dt} = \frac{1}{M}\sum_{i=1}^{n}\vec{a}_i m_i,$$

where each equation can be broken into its vector components.

**SIMPLIFY:** The magnitude of the net acceleration of each skier is $a = g\sin\theta$ down the incline of angle, $\theta$. In the $x$-direction, $a_{\text{J}x} = (g\sin\theta_{\text{J}})\cos\theta_{\text{J}}$ and $a_{\text{A}x} = (g\sin\theta_{\text{A}})\cos\theta_{\text{A}}$. In the $y$-direction, $a_{\text{J}y} = -(g\sin\theta_{\text{J}})\sin\theta_{\text{J}} = -g\sin^2\theta_{\text{J}}$ and $a_{\text{A}y} = -(a\sin\theta_{\text{A}})\sin\theta_{\text{A}} = -g\sin^2\theta_{\text{A}}$. Then,

$$A_x = \frac{1}{M}(m_{\text{J}}a_{\text{J}x} + m_{\text{A}}a_{\text{A}x}) = \frac{g}{M}(m_{\text{J}}\sin\theta_{\text{J}}\cos\theta_{\text{J}} + m_{\text{A}}\sin\theta_{\text{A}}\cos\theta_{\text{A}}), \text{ where } M = m_{\text{J}} + m_{\text{A}} \text{ and}$$

$$A_y = \frac{1}{M}(m_{\text{J}}a_{\text{J}y} + m_{\text{A}}a_{\text{A}y}) = -\frac{g}{M}(m_{\text{J}}\sin^2\theta_{\text{J}} + m_{\text{A}}\sin^2\theta_{\text{A}}).$$

Each skier starts from rest. In the $x$-direction, $v_{\text{J}x} = a_{\text{J}x}t = g\sin\theta_{\text{J}}\cos\theta_{\text{J}}t$ and $v_{\text{A}x} = a_{\text{A}x}t = g\sin\theta_{\text{A}}\cos\theta_{\text{A}}t$. In the $y$-direction, $v_{\text{J}y} = a_{\text{J}y}t = -g\sin^2\theta_{\text{J}}t$ and $v_{\text{A}y} = a_{\text{A}y}t = -g\sin^2\theta_{\text{A}}t$.

Then,

$$V_x = \frac{1}{M}\left(m_{\rm J}v_{{\rm J}x} + m_{\rm A}v_{{\rm A}x}\right) = \frac{g}{M}\left(m_{\rm J}\sin\theta_{\rm J}\cos\theta_{\rm J} + m_{\rm A}\sin\theta_{\rm A}\cos\theta_{\rm A}\right)t = A_x t \text{ and}$$

$$V_y = \frac{1}{M}\left(m_{\rm J}v_{{\rm J}y} + m_{\rm A}v_{{\rm A}y}\right) = -\frac{g}{M}\left(m_{\rm J}\sin^2\theta_{\rm J} + m_{\rm A}\sin^2\theta_{\rm A}\right)t = A_y t.$$

The position in the $x$-direction is given by:

$$x_{\rm J} = \frac{1}{2}a_{{\rm J}x}t^2 + x_{\rm J0} = \frac{1}{2}g\sin\theta_{\rm J}\cos\theta_{\rm J}t^2 \text{ and } x_{\rm A} = \frac{1}{2}a_{{\rm A}x}t^2 + x_{\rm A0} = \frac{1}{2}g\sin\theta_{\rm A}\cos\theta_{\rm A}t^2 + d\cos\theta_{\rm J}.$$

In the $y$-direction,

$$y_{\rm J} = \frac{1}{2}a_{{\rm J}y}t^2 + y_{\rm J0} = -\frac{1}{2}g\sin^2\theta_{\rm J}t^2 \text{ and } y_{\rm A} = \frac{1}{2}a_{{\rm A}y}t^2 + y_{\rm A0} = -\frac{1}{2}g\sin^2\theta_{\rm A}t^2 - d\sin\theta_{\rm J}.$$

Then,

$$X = \frac{1}{M}\left(m_{\rm J}x_{\rm J} + m_{\rm A}x_{\rm A}\right) = \frac{1}{M}\left(\frac{1}{2}m_{\rm J}g\sin\theta_{\rm J}\cos\theta_{\rm J}t^2 + \frac{1}{2}m_{\rm A}g\sin\theta_{\rm A}\cos\theta_{\rm A}t^2 + m_{\rm A}d\cos\theta_{\rm J}\right) = \frac{1}{2}A_x t^2 + \frac{m_{\rm A}}{M}d\cos\theta_{\rm J}$$

$$Y = \frac{1}{M}\left(m_{\rm J}y_{\rm J} + m_{\rm A}y_{\rm A}\right) = -\frac{1}{M}\left(\frac{1}{2}m_{\rm J}g\sin^2\theta_{\rm J}t^2 + \frac{1}{2}m_{\rm A}g\sin^2\theta_{\rm A}t^2 + m_{\rm A}d\sin\theta_{\rm J}\right) = \frac{1}{2}A_y t^2 - \frac{m_{\rm A}}{M}d\sin\theta_{\rm J}.$$

**CALCULATE:**

$$A_x = \frac{\left(9.81 \text{ m/s}^2\right)}{88.0 \text{ kg} + 64.0 \text{ kg}}\left(\left(88.0 \text{ kg}\right)\sin 35.0°\cos 35.0° + \left(64.0 \text{ kg}\right)\sin 20.0°\cos 20.0°\right) = 3.996 \text{ m/s}^2$$

$$A_y = -\frac{\left(9.81 \text{ m/s}^2\right)}{88.0 \text{ kg} + 64.0 \text{ kg}}\left(\left(88.0 \text{ kg}\right)\sin^2\left(35.0°\right) + \left(64.0 \text{ kg}\right)\sin^2\left(20.0°\right)\right) = -2.352 \text{ m/s}^2$$

$$V_x = \left(3.996 \text{ m/s}^2\right)t,\; V_y = \left(-2.352 \text{ m/s}^2\right)t$$



$$X = \frac{1}{2}\left(3.996 \text{ m/s}^2\right)t^2 + \frac{64.0 \text{ kg}}{\left(88.0 \text{ kg} + 64.0 \text{ kg}\right)}\left(100. \text{ m}\right)\cos\left(35.0°\right) = \left(1.998 \text{ m/s}^2\right)t^2 + 34.49 \text{ m}$$

$$Y = \frac{1}{2}\left(-2.352 \text{ m/s}^2\right)t^2 - \frac{64.0 \text{ kg}}{\left(88.0 \text{ kg} + 64.0 \text{ kg}\right)}\left(100. \text{ m}\right)\sin\left(35.0°\right) = \left(-1.176 \text{ m/s}^2\right)t^2 - 24.1506 \text{ m}$$

**ROUND:** Rounding to three significant figures, $A_x = 4.00 \text{ m/s}^2$, $A_y = -2.35 \text{ m/s}^2$, $V_x = \left(4.00 \text{ m/s}^2\right)t$ and $V_y = \left(-2.35 \text{ m/s}^2\right)t$, $X = \left(2.00 \text{ m/s}^2\right)t^2 + 34.5 \text{ m}$ and $Y = \left(-1.18 \text{ m/s}^2\right)t^2 - 24.2 \text{ m}$.

**DOUBLE-CHECK:** The acceleration of the center of mass is not time dependent.

**8.37.** **THINK:** The proton's mass is $m_{\rm p} = 1.6726 \cdot 10^{-27}$ kg and its initial speed is $v_{\rm p} = 0.700c$ (assumed to be in the lab frame). The mass of the tin nucleus is $m_{\rm sn} = 1.9240 \cdot 10^{-25}$ kg (assumed to be at rest). Determine the speed of the center of mass, $v$, with respect to the lab frame.

**SKETCH:** A sketch is not necessary.

**RESEARCH:** The given speeds are in the lab frame. To determine the speed of the center of mass use $V = \frac{1}{M}\sum_{i=1}^{n} m_i v_i$.

**SIMPLIFY:** $V = \frac{1}{m_{\rm p} + m_{\rm sn}}\left(m_{\rm p}v_{\rm p} + m_{\rm sn}v_{\rm sn}\right) = \frac{m_{\rm p}v_{\rm p}}{m_{\rm p} + m_{\rm sn}}$

**CALCULATE:** $V = \frac{\left(1.6726 \cdot 10^{-27} \text{ kg}\right)\left(0.700c\right)}{\left(1.6726 \cdot 10^{-27} \text{ kg}\right) + \left(1.9240 \cdot 10^{-25} \text{ kg}\right)} = 0.0060329c$

**ROUND:** Since $v_{\rm p}$ has three significant figures, the result should be rounded to $V = 0.00603c$.

**DOUBLE-CHECK:** Since $m_{\rm sn}$ is at rest and $m_{\rm sn} >> m_{\rm p}$, it is expected that $V << v_{\rm p}$.

**8.38.** **THINK:** Particle 1 has a mass of $m_1 = 2.0 \text{ kg}$, a position of $\vec{r}_1 = (2.0 \text{ m}, 6.0 \text{ m})$ and a velocity of $\vec{v}_1 = (4.0 \text{ m/s}, 2.0 \text{ m/s})$. Particle 2 has a mass of $m_2 = 3.0 \text{ kg}$, a position of $\vec{r}_2 = (4.0 \text{ m}, 1.0 \text{ m})$ and a velocity of $\vec{v}_2 = (0, 4.0 \text{ m/s})$. Determine (a) the position $\vec{R}$ and the velocity $\vec{V}$ for the system's center of mass and (b) a sketch of the position and velocity vectors for each particle and for the center of mass.
**SKETCH:** To be provided in the calculate step, part (b).

**RESEARCH:** To determine $\vec{R}$, use $X = \frac{1}{M}(x_1 m_1 + x_2 m_2)$ and $Y = \frac{1}{M}(y_1 m_1 + y_2 m_2)$. To determine $\vec{V}$, use $V_x = \frac{1}{M}(v_{1x} m_1 + v_{2x} m_2)$ and $V_y = \frac{1}{M}(v_{1y} m_1 + v_{2y} m_2)$.
**SIMPLIFY:** It is not necessary to simplify.
**CALCULATE:**

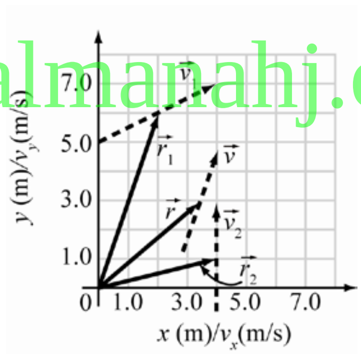
(a) $X = \frac{1}{2.00 \text{ kg} + 3.00 \text{ kg}}\left((2.00 \text{ m})(2.00 \text{ kg}) + (4.00 \text{ m})(3.00 \text{ kg})\right) = 3.20 \text{ m}$

$$Y = \frac{1}{2.00 \text{ kg} + 3.00 \text{ kg}}\left((6.00 \text{ m})(2.00 \text{ kg}) + (1.00 \text{ m})(3.00 \text{ kg})\right) = 3.00 \text{ m}$$

$$V_x = \frac{1}{2.00 \text{ kg} + 3.00 \text{ kg}}\left((4.00 \text{ m/s})(2.00 \text{ kg}) + 0(3.00 \text{ kg})\right) = 1.60 \text{ m/s}$$

$$V_y = \frac{1}{2.00 \text{ kg} + 3.00 \text{ kg}}\left((2.00 \text{ m/s})(2.00 \text{ kg}) + (4.00 \text{ m/s})(3.00 \text{ kg})\right) = 3.20 \text{ m/s}$$

(b)

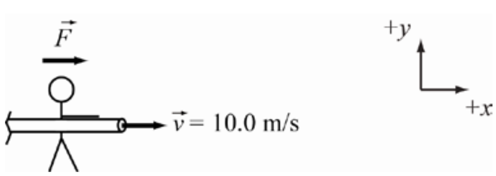





**ROUND:** Each given value has three significant figures, so the results should be rounded to *X* = 3.20 m, *Y* = 3.00 m, $V_x = 1.60$ m/s and $V_y = 3.20$ m/s.

**DOUBLE-CHECK:** $\vec{R}$ should point between $\vec{r}_1$ and $\vec{r}_2$, and $\vec{V}$ should point between $\vec{v}_1$ and $\vec{v}_2$.

**8.39.** **THINK:** The radius of the hose is *r* = 0.0200 m and the velocity of the spray is *v* = 10.0 m/s. Determine the horizontal force, $\vec{F}_f$, required of the fireman to hold the hose stationary.
**SKETCH:**




**RESEARCH:** By Newton's third law, the force exerted by the fireman is equal in magnitude to the force exerted by the hose. The thrust force of the hose can be determined from $\vec{F}_{\text{thrust}} = -\vec{v}_c dm/dt$. To determine $dm/dt$, consider the mass of water exiting the hose per unit time.
**SIMPLIFY:** The volume of water leaving the hose is this velocity times the area of the hose's end. That is,

$$\frac{dV_{\mathrm{w}}}{dt} = Av = \pi r^2 v.$$

With $\rho_{\mathrm{w}} = m / V_{\mathrm{w}}$, $\frac{dm}{dt} = \rho_{\mathrm{w}} \frac{dV_{\mathrm{w}}}{dt} = \rho_{\mathrm{w}} \pi r^2 v$. Now, by Newton's third law, $\vec{F}_{\mathrm{f}} = -\vec{F}_{\mathrm{thrust}}$, so

$\vec{F}_{\mathrm{f}} = \vec{v}_{\mathrm{c}} \frac{dm}{dt} = \vec{v}_{\mathrm{c}} \rho_{\mathrm{w}} \pi r^2 v$. Since $v_{\mathrm{c}}$ is in fact $v$, $F_{\mathrm{f}} = \rho_{\mathrm{w}} \pi r^2 v^2$.

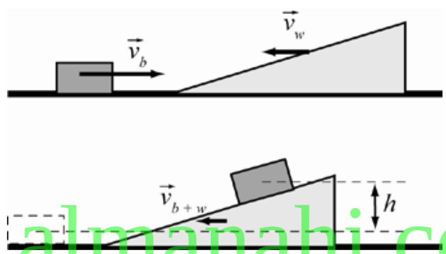
**CALCULATE:** $F_{\mathrm{f}} = \pi \left(1000 \text{ kg/m}^3\right)\left(0.0200 \text{ m}\right)^2 \left(10.0 \text{ m/s}\right)^2 = 125.7 \text{ N}$

**ROUND:** Since $v$ has three significant figures, $\vec{F}_{\mathrm{f}} = 126 \text{ N}$ in the direction of the water's velocity.

**DOUBLE-CHECK:** The result has units of force. Also, this is a reasonable force with which to hold a fire hose.

**8.40.** **THINK:** The block's mass is $m_{\mathrm{b}} = 1.2$ kg. It has an initial velocity is $\vec{v}_{\mathrm{b}} = 2.5$ m/s (with the positive *x* axis being the right direction). The wedge's mass is $m_{\mathrm{w}}$ and its initial velocity is $\vec{v}_{\mathrm{w}} = -1.1$ m/s. Their final velocity when the wedge stops moving is $\vec{v}_{\mathrm{b+w}}$. Determine (a) $m_{\mathrm{w}}$, if the block's center of mass rises by $h = 0.37$ m and (b) $\vec{v}_{\mathrm{b+w}}$.

**SKETCH:**






**RESEARCH:** Momentum is conserved. As this is an elastic collision, and there are only conservative forces, mechanical energy is also conserved. Use $P_{\mathrm{i}} = P_{\mathrm{f}}$, $\Delta K + \Delta U = 0$, $K = mv^2/2$ and $U = mgh$ to determine $m_{\mathrm{w}}$ and ultimately $\vec{v}_{\mathrm{b+w}}$.

**SIMPLIFY:** It will be useful to determine an expression for $\vec{v}_{\mathrm{b+w}}$ first:

$$\vec{P}_{\mathrm{i}} = \vec{P}_{\mathrm{f}} \Rightarrow m_{\mathrm{b}}\vec{v}_{\mathrm{b}} + m_{\mathrm{w}}\vec{v}_{\mathrm{w}} = \left(m_{\mathrm{b}} + m_{\mathrm{w}}\right)\vec{v}_{\mathrm{b+w}} \Rightarrow \vec{v}_{\mathrm{b+w}} = \frac{m_{\mathrm{b}}\vec{v}_{\mathrm{b}} + m_{\mathrm{w}}\vec{v}_{\mathrm{w}}}{m_{\mathrm{b}} + m_{\mathrm{w}}}.$$

(a) From the conservation of mechanical energy:

$$\Delta K + \Delta U = K_{\mathrm{f}} - K_{\mathrm{i}} + U_{\mathrm{f}} - U_{\mathrm{i}} = 0 \Rightarrow \frac{1}{2}\left(m_{\mathrm{b}} + m_{\mathrm{w}}\right)\vec{v}_{\mathrm{b+w}}^{\,2} - \frac{1}{2}m_{\mathrm{b}}\vec{v}_{\mathrm{b}}^{\,2} - \frac{1}{2}m_{\mathrm{w}}\vec{v}_{\mathrm{w}}^{\,2} + m_{\mathrm{b}}gh = 0$$

$$\Rightarrow \frac{1}{2}\left(m_{\mathrm{b}} + m_{\mathrm{w}}\right)\frac{\left(m_{\mathrm{b}}\vec{v}_{\mathrm{b}} + m_{\mathrm{w}}\vec{v}_{\mathrm{w}}\right)^2}{\left(m_{\mathrm{b}} + m_{\mathrm{w}}\right)^2} - \frac{1}{2}m_{\mathrm{b}}\vec{v}_{\mathrm{b}}^{\,2} - \frac{1}{2}m_{\mathrm{w}}\vec{v}_{\mathrm{w}}^{\,2} + m_{\mathrm{b}}gh = 0$$

$$\Rightarrow \frac{\left(m_{\mathrm{b}}^2\vec{v}_{\mathrm{b}}^{\,2} + 2m_{\mathrm{b}}m_{\mathrm{w}}\vec{v}_{\mathrm{b}}\vec{v}_{\mathrm{w}} + m_{\mathrm{w}}^2\vec{v}_{\mathrm{w}}^{\,2}\right)}{2\left(m_{\mathrm{b}} + m_{\mathrm{w}}\right)} - \frac{1}{2}m_{\mathrm{b}}\vec{v}_{\mathrm{b}}^{\,2} - \frac{1}{2}m_{\mathrm{w}}\vec{v}_{\mathrm{w}}^{\,2} + m_{\mathrm{b}}gh = 0$$

Multiply the expression by $2\left(m_{\mathrm{b}} + m_{\mathrm{w}}\right)$:

$$m_{\mathrm{b}}^2\vec{v}_{\mathrm{b}}^{\,2} + 2m_{\mathrm{b}}m_{\mathrm{w}}\vec{v}_{\mathrm{b}}\vec{v}_{\mathrm{w}} + m_{\mathrm{w}}^2\vec{v}_{\mathrm{w}}^{\,2} - m_{\mathrm{b}}\vec{v}_{\mathrm{b}}^{\,2}\left(m_{\mathrm{b}} + m_{\mathrm{w}}\right) - m_{\mathrm{w}}\vec{v}_{\mathrm{w}}^{\,2}\left(m_{\mathrm{b}} + m_{\mathrm{w}}\right) + 2m_{\mathrm{b}}gh\left(m_{\mathrm{b}} + m_{\mathrm{w}}\right) = 0$$

$$\Rightarrow m_{\mathrm{b}}^2\vec{v}_{\mathrm{b}}^{\,2} + 2m_{\mathrm{b}}m_{\mathrm{w}}\vec{v}_{\mathrm{b}}\vec{v}_{\mathrm{w}} + m_{\mathrm{w}}^2\vec{v}_{\mathrm{w}}^{\,2} - m_{\mathrm{b}}^2\vec{v}_{\mathrm{b}}^{\,2} - m_{\mathrm{b}}m_{\mathrm{w}}\vec{v}_{\mathrm{b}}^{\,2} - m_{\mathrm{w}}m_{\mathrm{b}}\vec{v}_{\mathrm{w}}^{\,2} + m_{\mathrm{w}}^2\vec{v}_{\mathrm{w}}^{\,2} + 2m_{\mathrm{b}}^2gh + 2m_{\mathrm{b}}m_{\mathrm{w}}gh = 0$$

$$\Rightarrow 2m_{\mathrm{b}}m_{\mathrm{w}}\vec{v}_{\mathrm{b}}\vec{v}_{\mathrm{w}} - m_{\mathrm{b}}m_{\mathrm{w}}\vec{v}_{\mathrm{b}}^{\,2} - m_{\mathrm{b}}m_{\mathrm{w}}\vec{v}_{\mathrm{w}}^{\,2} + 2m_{\mathrm{b}}^2gh + 2m_{\mathrm{b}}m_{\mathrm{w}}gh = 0$$

$$\Rightarrow m_{\mathrm{w}} = -\frac{2m_{\mathrm{b}}^2gh}{2m_{\mathrm{b}}\vec{v}_{\mathrm{b}}\vec{v}_{\mathrm{w}} - m_{\mathrm{b}}\vec{v}_{\mathrm{b}}^{\,2} - m_{\mathrm{b}}\vec{v}_{\mathrm{w}}^{\,2} + 2m_{\mathrm{b}}gh} = \frac{2m_{\mathrm{b}}gh}{\vec{v}_{\mathrm{b}}^{\,2} + \vec{v}_{\mathrm{w}}^{\,2} - 2\vec{v}_{\mathrm{b}}\vec{v}_{\mathrm{w}} - 2gh}.$$

(b) With $m_w$ known, $\vec{v}_{b+w} = \dfrac{m_b\vec{v}_b + m_w\vec{v}_w}{m_b + m_w}$.

**CALCULATE:**

(a) $m_w = \dfrac{2(1.20 \text{ kg})(9.81 \text{ m/s}^2)0.370 \text{ m}}{(2.5 \text{ m/s})^2 + (-1.10 \text{ m/s})^2 - 2(2.50 \text{ m/s})(-1.10 \text{ m/s}) - 2(9.81 \text{ m/s}^2)(0.370 \text{ m})}$

$= \dfrac{8.712 \text{ kg}\cdot\text{m}^2/\text{s}^2}{6.25 \text{ m}^2/\text{s}^2 + 1.21 \text{ m}^2/\text{s}^2 + 5.5 \text{ m}^2/\text{s}^2 - 7.2594 \text{ m}^2/\text{s}^2} = 1.528 \text{ kg}$

(b) $\vec{v}_{b+w} = \dfrac{(1.20 \text{ kg})(2.50 \text{ m/s}) + (1.528 \text{ kg})(-1.10 \text{ m/s})}{1.20 \text{ kg} + 1.528 \text{ kg}} = 0.4835 \text{ m/s}$

**ROUND:** Each given value has three significant figures, so the results should be rounded to: $m_w = 1.53 \text{ kg}$ and $\vec{v}_{b+w} = 0.484 \text{ m/s}$ to the right.

**DOUBLE-CHECK:** These results are reasonable given the initial values.

**8.41.** **THINK:** For rocket engines, the specific impulse is $J_{\text{spec}} = \dfrac{J_{\text{tot}}}{W_{\text{expended fuel}}} = \dfrac{1}{W_{\text{expended fuel}}}\int_{t_0}^{t} F_{\text{thrust}}(t')dt'$.

(a) Determine $J_{\text{spec}}$ with an exhaust nozzle speed of $v$.

(b) Evaluate and compare $J_{\text{spec}}$ for a toy rocket with $v_{\text{toy}} = 800. \text{ m/s}$ and a chemical rocket with $v_{\text{chem}} = 4.00 \text{ km/s}$.

**SKETCH:** Not applicable.

**RESEARCH:** It is known that $\vec{F}_{\text{thrust}} = -v_c dm/dt$. Rewrite $W_{\text{expended fuel}}$ as $m_{\text{expended}}g$. With the given definition, $J_{\text{spec}}$ can be determined for a general $v$, and for $v_{\text{toy}}$ and $v_{\text{chem}}$.



**SIMPLIFY:** $J_{\text{spec}} = \dfrac{1}{m_{\text{expended}}g}\int_{m_0}^{m} -v\,dm = -\dfrac{v}{m_{\text{expended}}g}(m - m_0)$. Now, $m - m_0 = -m_{\text{expended}}$, so $J_{\text{spec}} = \dfrac{v}{g}$.

**CALCULATE:** $J_{\text{spec, toy}} = \dfrac{v_{\text{toy}}}{g} = \dfrac{800. \text{ m/s}}{(9.81 \text{ m/s}^2)} = 81.55 \text{ s}$, $J_{\text{spec, chem}} = \dfrac{v_{\text{chem}}}{g} = \dfrac{4.00\cdot 10^3 \text{ m/s}}{(9.81 \text{ m/s}^2)} = 407.75 \text{ s}$

$\dfrac{J_{\text{spec, toy}}}{J_{\text{spec, chem}}} = \dfrac{v_{\text{toy}}}{v_{\text{chem}}} = \dfrac{800. \text{ m/s}}{4.00\cdot 10^3 \text{ m/s}} = 0.200$

**ROUND:**

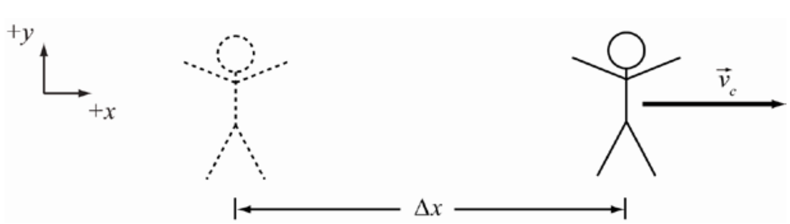
(a) $J_{\text{spec, toy}} = 81.6 \text{ s}$

(b) $J_{\text{spec, chem}} = 408 \text{ s}$ and $J_{\text{spec, toy}} = 0.200 J_{\text{spec, chem}}$.

**DOUBLE-CHECK:** The units of the results are units of specific impulse. Also, as expected $J_{\text{spec, toy}} < J_{\text{spec, chem}}$.

**8.42.** **THINK:** The astronaut's total mass is $m$ = 115 kg. The rate of gas ejection is $dm/dt = 7.00 \text{ g/s} = 0.00700 \text{ kg/s}$ and the leak speed is $v_c = 800. \text{ m/s}$. After $\Delta t = 6.00 \text{ s}$, how far has the astronaut moved from her original position, $\Delta x$?

**SKETCH:**



**RESEARCH:** Assume that the astronaut starts from rest and the acceleration is constant. $\Delta x$ can be determined from $\Delta x = (v_i + v_f)\Delta t / 2$. To determine $v_f$, use the rocket-velocity equation $v_f - v_i = v_c \ln(m_i / m_f)$. The loss of mass can be determined from $\Delta m = \frac{dm}{dt}\Delta t$.

**SIMPLIFY:** Since $v_i = 0$, $v_f = v_c \ln(m_i / m_f)$, where $m_i = m$ and $m_f = m - \Delta m = m - \frac{dm}{dt}\Delta t$. Then,

$$v_f = v_c \ln\left(\frac{m}{m - \frac{dm}{dt}\Delta t}\right) \text{ and } \Delta x = \frac{1}{2} v_f \Delta t.$$

**CALCULATE:** $v_f = (800.\text{ m/s})\ln\left(\frac{115\text{ kg}}{115\text{ kg} - (0.00700\text{ kg/s})(6.00\text{ s})}\right) = 0.29223\text{ m/s}$

$$\Delta x = \frac{1}{2}(0.29223\text{ m/s})(6.00\text{ s}) = 0.87669\text{ m}$$

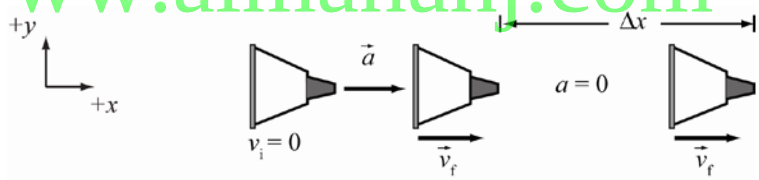
**ROUND:** The problem values have three significant figures, so the results should be rounded to $v_f = 0.292\text{ m/s}$ $\Delta x = 0.877\text{ m}$.

**DOUBLE-CHECK:** Considering how such a small amount of the total mass has escaped, this is a reasonable distance to have moved.

**8.43.** **THINK:** The mass of the payload is $m_p = 5190.0\text{ kg}$, and the fuel mass is $m_f = 1.551 \cdot 10^5\text{ kg}$. The fuel exhaust speed is $v_c = 5.600 \cdot 10^3\text{ m/s}$. How long will it take the rocket to travel a distance $\Delta x = 3.82 \cdot 10^8\text{ m}$ after achieving its final velocity, $v_f$? The rocket starts accelerating from rest.



**SKETCH:**




**RESEARCH:** The rocket's travel speed, $v_f$, can be determined from $v_f - v_i = v_c \ln(m_i / m_f)$. Then $\Delta t$ can be determined from $\Delta x = v\Delta t$.

**SIMPLIFY:** $v_f = v_c \ln\left(\frac{m_p + m_f}{m_p}\right)$, and $\Delta t = \Delta x / v_f$.

**CALCULATE:** $v_f = (5.600 \cdot 10^3\text{ m/s})\ln\left(\frac{5190.0\text{ kg} + 1.551 \cdot 10^5\text{ kg}}{5190.0\text{ kg}}\right) = 19209\text{ m/s}$,

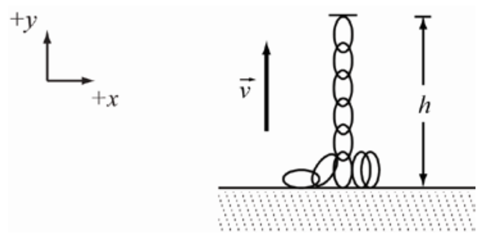
$$\Delta t = \frac{3.82 \cdot 10^8\text{ m}}{19209\text{ m/s}} = 19886\text{ s}$$

**ROUND:** $\Delta x$ has three significant figures, so the result should be rounded to $\Delta t = 19{,}886\text{ s} = 5.52\text{ h}$.

**DOUBLE-CHECK:** This is a reasonable time for a rocket with such a large initial velocity to reach the Moon from the Earth.

**8.44.** **THINK:** The linear density of the chain is $\lambda = 1.32\text{ kg/m}$, and the speed at which one end of the chain is lifted is $v = 0.47\text{ m/s}$. Determine (a) the net force acting on the chain, $F_{net}$ and (b) the force, $F$, applied to the end of the chain when $h = 0.15\text{ m}$ has been lifted off the table.

**SKETCH:**




**RESEARCH:**

(a) Since the chain is raised at a constant rate, $v$, the net force is the thrust force, $F_{\text{thrust}} = v_{\text{c}}dm/dt$. Since the chain's mass in the air is increasing, $F_{net} = v\,dm/dt$.

(b) The applied force can be determined by considering the forces acting on the chain and the net force determined in part (a): $F_{\text{net}} = \sum F_i$.

**SIMPLIFY:**

(a) $F_{\text{net}} = v\dfrac{dm}{dt} = v\lambda\dfrac{dh}{dt} = v\lambda v = v^2\lambda$

(b) $F_{\text{net}} = F_{\text{applied}} - mg \Rightarrow F_{\text{applied}} = F_{\text{net}} + mg = v^2\lambda + mg = v^2\lambda + \lambda hg$

**CALCULATE:**

(a) $F_{\text{net}} = (0.470 \text{ m/s})^2(1.32 \text{ kg/m}) = 0.2916 \text{ N}$

(b) $F_{\text{applied}} = 0.2916 \text{ N} + (1.32 \text{ kg/m})(0.150 \text{ m})(9.81 \text{ m/s}^2) = 0.2916 \text{ N} + 1.942 \text{ N} = 2.234 \text{ N}$

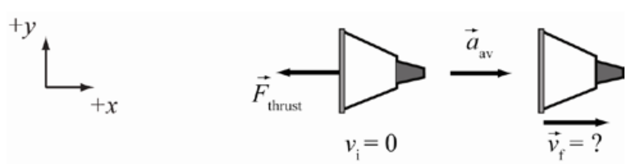
**ROUND:** $v$ and $h$ each have three significant figures, so the results should be rounded to $F_{\text{net}} = 0.292 \text{ N}$ and $F_{\text{applied}} = 2.23 \text{ N}$.



**DOUBLE-CHECK:** These forces are reasonable to determine for this system. Also, $F_{\text{net}} < F_{\text{applied}}$.

**8.45.** **THINK:** The thrust force is $\vec{F}_{\text{thrust}} = 53.2 \cdot 10^6 \text{ N}$ and the propellant velocity is $v = 4.78 \cdot 10^3 \text{ m/s}$. Determine (a) $dm/dt$, (b) the final speed of the spacecraft, $v_{\text{s}}$, given $v_{\text{i}} = 0$, $m_{\text{i}} = 2.12 \cdot 10^6 \text{ kg}$ and $m_{\text{f}} = 7.04 \cdot 10^4 \text{ kg}$ and (c) the average acceleration, $a_{\text{av}}$ until burnout.

**SKETCH:**




**RESEARCH:**

(a) To determine $dm/dt$, use $\vec{F}_{\text{thrust}} = -v_{\text{c}}dm/dt$.

(b) To determine $v_{\text{f}}$, use $v_{\text{f}} - v_{\text{i}} = v_{\text{c}}\ln(m_{\text{i}}/m_{\text{f}})$.

(c) $\Delta v$ is known from part (b). $\Delta t$ can be determined from the equivalent ratios,

$\dfrac{dm}{dt} = \dfrac{\Delta m}{\Delta t}$, where $\Delta m = m_{\text{i}} - m_{\text{f}}$.

**SIMPLIFY:**

(a) Since $\vec{F}_{\text{thrust}}$ and $\vec{v}_{\text{c}}$ are in the same direction, the equation can be rewritten as:

$$F_{\text{thrust}} = v_{\text{c}}\frac{dm}{dt} \Rightarrow \frac{dm}{dt} = \frac{F_{\text{thrust}}}{v_{\text{c}}}.$$

(b) $v_i = 0 \Rightarrow v_f = v_c \ln\left(\dfrac{m_i}{m_f}\right)$

(c) $\dfrac{dm}{dt} = \dfrac{\Delta m}{\Delta t} \Rightarrow \Delta t = \dfrac{\Delta m}{dm/dt}$, $a_{av} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_f}{\Delta m}\left(\dfrac{dm}{dt}\right)$ $\left(v_i = 0\right)$

**CALCULATE:**

(a) $\dfrac{dm}{dt} = \dfrac{\left(53.2 \cdot 10^6 \text{ N}\right)}{\left(4.78 \cdot 10^3 \text{ m/s}\right)} = 11129.7 \text{ kg/s}$

(b) $v_f = \left(4.78 \cdot 10^3 \text{ m/s}\right)\ln\left(\dfrac{2.12 \cdot 10^6 \text{ kg}}{7.04 \cdot 10^4 \text{ kg}}\right) = 1.6276 \cdot 10^4 \text{ m/s}$

(c) $a_{av} = \dfrac{\left(1.6276 \cdot 10^4 \text{ m/s}\right)}{\left(2.12 \cdot 10^6 \text{ kg} - 7.04 \cdot 10^4 \text{ kg}\right)}\left(11129.7 \text{ kg/s}\right) = 88.38 \text{ m/s}^2$

**ROUND:** Each given value has three significant figures, so the results should be rounded to $dm/dt = 11100 \text{ kg/s}$, $v_f = 1.63 \cdot 10^4 \text{ m/s}$ and $a_{av} = 88.4 \text{ m/s}^2$.

**DOUBLE-CHECK:** The results all have the correct units. Also, the results are reasonable for a spaceship with such a large thrust force.

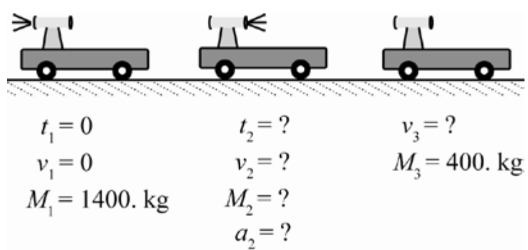
**8.46.** **THINK:** The mass of the cart with an empty water tank is $m_c = 400. \text{ kg}$. The volume of the water tank is $V = 1.00 \text{ m}^3$. The rate at which water is ejected in SI units is

$$dV/dt = \left(200. \frac{\text{L}}{\text{min}}\right)\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 0.003333 \text{ m}^3/\text{s}.$$



The muzzle velocity is $v_c = 25.0$ m/s. Determine (a) the time, $t_2$, to switch from backward to forward so the cart ends up at rest (it starts from rest), (b) the mass of the cart, $M_2$, and the velocity, $v_2$, at the time, $t_2$, (c) the thrust, $F_{\text{thrust}}$, of the rocket and (d) the acceleration, $a_2$, of the cart just before the valve is switched. Note the mass of the cart increases by 1000. kg when the water tank is full, as $m_w = \rho V = \left(1000. \text{ kg/m}^3\right)\left(1.00 \text{ m}^3\right)$. That is, the initial mass is $M_1 = 1400. \text{ kg}$.

**SKETCH:**




**RESEARCH:**

(a) $t_2$ can be determined from the ratio, $\dfrac{M_1 - M_2}{t_2 - t_1} = \dfrac{dm}{dt}$, with $t_1 = 0$. Note that, $dm/dt = \rho dV/dt$. $M_2$ can be determined from $v_f - v_i = v_c \ln\left(m_i/m_f\right)$. When the cart stops moving, the water tank is empty and the total mass is $M_3 = 400$ kg.

(b) Using the mass determined in part (a), $v_2$ can be determined from $v_f - v_i = v_c \ln\left(m_i/m_f\right)$.

(c) Use $\vec{F}_{\text{thrust}} = -\vec{v}_c dm/dt$.

(d) Since $\vec{F}_{\text{thrust}} = M\vec{a}_{\text{net}}$, $a_2$ can be determined from this equation.

**SIMPLIFY:**

(a) Consider the first leg of the trip before the valve is switched:

$$v_2 - v_1 = v_c \ln(M_1 / M_2) \Rightarrow v_2 = v_c \ln(M_1 / M_2).$$

In the second leg, $v_c$ changes direction, and the similar equation is

$$v_3 - v_2 = -v_c \ln(M_2 / M_3) \Rightarrow v_2 = v_c \ln(M_2 / M_3).$$

Then it must be that $\ln(M_2 / M_3) = \ln(M_2 / M_3)$, or $M_1 / M_2 = M_2 / M_3$. Then $M_2 = \sqrt{M_3 M_1}$. Now,

$$\frac{M_1 - M_2}{t_2} = \frac{dm}{dt} = \rho \frac{dV}{dt} \Rightarrow t_2 = \frac{M_1 - M_2}{\rho \frac{dV}{dt}} = \frac{M_1 - \sqrt{M_3 M_1}}{\rho \frac{dV}{dt}}.$$

(b) From above, $M_2 = \sqrt{M_3 M_1}$, $v_2 = v_c \ln\left(\frac{M_1}{M_2}\right)$.

(c) $\vec{F}_{\text{thrust}} = -\vec{v}_c \frac{dm}{dt} = -\vec{v}_c \rho \frac{dV}{dt}$

(d) $\vec{a}_2 = \frac{\vec{F}_{\text{thrust}}}{M_2}$

**CALCULATE:**

(a) $t_2 = \frac{1400.\text{ kg} - \sqrt{(400.\text{ kg})(1400.\text{ kg})}}{(1000.\text{ kg/m}^3)(0.003333\text{ m}^3\text{/s})} = 195.5\text{ s}$

(b) $M_2 = \sqrt{(400.\text{ kg})(1400.\text{ kg})} = 748.33\text{ kg}$, $v_2 = (25.0\text{ m/s})\ln\left(\frac{(1400.\text{ kg})}{(748.33\text{ kg})}\right) = 15.66\text{ m/s}$



(c) Before the valve is switched, $v_c$ is directed backward, i.e. $\vec{v}_c = -25.0\text{ m/s}$. Then $\vec{F}_{\text{thrust}} = -(-25.0\text{ m/s})(1000.\text{ kg/m}^3)(0.003333\text{ m}^3\text{/s}) = 83.33\text{ N}$ forward. After the valve is switched, $\vec{F}_{\text{thrust}}$ is directed backward, i.e. $\vec{F}_{\text{thrust}} = -83.33\text{ N}$.

(d) Before the valve is switched, $\vec{a}_2 = \frac{83.33\text{ N}}{748.33\text{ kg}} = 0.111355\text{ m/s}^2$.

**ROUND:**

Rounding to three significant figures:

(a) $t_2 = 196\text{ s}$

(b) $M_2 = 748\text{ kg}$ and $v_2 = 15.7\text{ m/s}$.

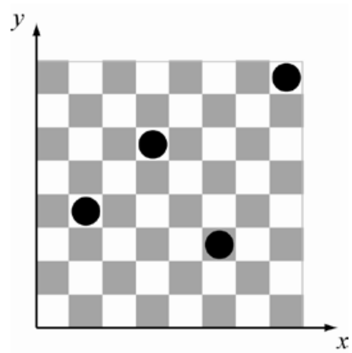
(c) $\vec{F}_{\text{thrust}} = -83.3\text{ N}$

(d) $\vec{a}_2 = 0.111\text{ m/s}^2$

**DOUBLE-CHECK:** All the units for the results are appropriate. Also, the results are of reasonable orders of magnitude.

**8.47.** **THINK:** The checkerboard has dimensions 32.0 cm by 32.0 cm. Its mass is $m_b = 100.\text{ g}$ and the mass of each of the four checkers is $m_c = 20.0\text{ g}$. Determine the center of mass of the system. Note the checkerboard is 8 by 8 squares, thus the length of the side of each square is 32.0 cm/8 = 4.00 cm. From the figure provided, the following *x-y* coordinates can be associated with each checker's center of mass: $m_1 : (22.0\text{ cm}, 10.0\text{ cm})$, $m_2 : (6.00\text{ cm}, 14.0\text{ cm})$, $m_3 : (14.0\text{ cm}, 22.0\text{ cm})$, $m_4 : (30.0\text{ cm}, 30.0\text{ cm})$. Assuming a uniform density distribution, the checkerboard's center of mass is located at $(x_b, y_b) = (16.0\text{ cm}, 16.0\text{ cm})$.

**SKETCH:**




**RESEARCH:** To determine the system's center of mass, use the following equations: $X=\frac{1}{M}\sum_{i=1}^{n}x_i m_i$ and

$Y=\frac{1}{M}\sum_{i=1}^{n}y_i m_i$.

**SIMPLIFY:** $M=m_\text{b}+4m_\text{c}$

$X=\frac{1}{M}\left(x_\text{b}m_\text{b}+m_\text{c}\left(x_1+x_2+x_3+x_4\right)\right)$, $Y=\frac{1}{M}\left(y_\text{b}m_\text{b}+m_\text{c}\left(y_1+y_2+y_3+y_4\right)\right)$

**CALCULATE:** $M=100.\text{ g}+4(20.0\text{ g})=180.\text{ g}$

$$X=\frac{1}{180.\text{ g}}\left(16.0\text{ cm}(100.0\text{ g})+20.0\text{ g}\left(22.0\text{ cm}+6.00\text{ cm}+14.0\text{ cm}+30.0\text{ cm}\right)\right)=16.889\text{ cm}$$

$$Y=\frac{1}{180.\text{ g}}\left(16.0\text{ cm}(100.\text{ g})+20.0\text{ g}\left(10.0\text{ cm}+14.0\text{ cm}+22.0\text{ cm}+30.0\text{ cm}\right)\right)=17.33\text{ cm}$$



**ROUND:** $X=16.9$ cm and $Y=17.3$ cm. The answer is (16.9 cm,17.3 cm).

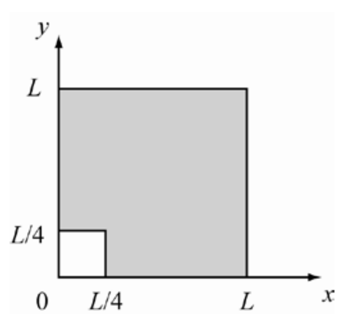
**DOUBLE-CHECK:** $m_\text{b}>m_\text{c}$, so it is reasonable to expect the system's center of mass to be near the board's center of mass.

**8.48.** **THINK:** The total mass of the plate is $M_\text{tot}=0.205$ kg. The dimensions of the plate are $L$ by $L$, $L=5.70$ cm. The dimensions of the smaller removed plate are $L/4$ by $L/4$. The mass of the smaller removed plate is

$$\frac{M_\text{tot}}{A_\text{tot}}=\frac{m_\text{s}}{A_\text{s}}\Rightarrow m_\text{s}=A_\text{s}\frac{M_\text{tot}}{A_\text{tot}}=\left(\frac{L}{4}\right)^2\frac{M_\text{tot}}{L^2}=\frac{1}{16}M_\text{tot}.$$

Determine the distance from the bottom left corner of the plate to the center of mass after the smaller plate is removed. Note the mass of the plate with the void is $m_\text{p}=M_\text{tot}-m_\text{s}=15M_\text{tot}/16$.

**SKETCH:**



**RESEARCH:** The center of mass in each dimension is $X=\frac{1}{M}\sum_{i=1}^{n}x_i m_i$ and $y=\frac{1}{M}\sum_{i=1}^{n}y_i m_i$. The center of mass of the plate with the void, $(X_\text{p},Y_\text{p})$, can be determined by considering the center of mass of the total system as composed of the smaller plate of mass $m_\text{s}$ and the plate with the void of mass $m_\text{p}$. Note the center of mass of the total system is at the total plate's geometric center, $(X,Y)=(L/2,L/2)$, assuming uniform density. Similarly, the center of mass of the smaller plate is at its center $(X_\text{s},Y_\text{s})=(L/8,L/8)$. The distance of the center of mass of the plate from the origin is then $d=\sqrt{X_\text{p}^2+Y_\text{p}^2}$.

**SIMPLIFY:** $X=\frac{1}{M_\text{tot}}\left(X_\text{p}m_\text{p}+X_\text{s}m_\text{s}\right)$, and $X_\text{p}=\frac{\left(XM_\text{tot}-X_\text{s}m_\text{s}\right)}{M_\text{tot}-\frac{1}{16}M_\text{tot}}=\frac{L\left(\frac{1}{2}M_\text{tot}-\frac{1}{8}\left(\frac{1}{16}M_\text{tot}\right)\right)}{\frac{15}{16}M_\text{tot}}=\frac{21}{40}L.$

Similarly, $Y_\text{p}=\frac{\left(YM_\text{tot}-Y_\text{s}m_\text{s}\right)}{m_\text{p}}=\frac{L\left(\frac{1}{2}M_\text{tot}-\frac{1}{8}\left(\frac{1}{16}M_\text{tot}\right)\right)}{\frac{15}{16}M_\text{tot}}=\frac{21}{40}L.$

**CALCULATE:** $X_\text{p}=Y_\text{p}=\frac{21}{40}(5.70\text{ cm})=2.9925\text{ cm}$

$d=\sqrt{(2.9925\text{ cm})^2+(2.9925\text{ cm})^2}=4.232\text{ cm}$

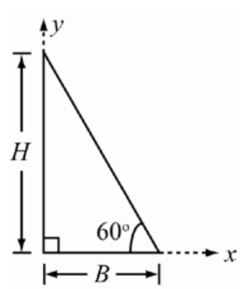
**ROUND:** Since $L$ has three significant figures, the result should be rounded to $d=4.23$ cm.

**DOUBLE-CHECK:** It is expected that the center of mass for the plate with the void would be further from the origin than the center of mass for the total plate.



**8.49.** **THINK:** The height is $H$ = 17.3 cm and the base is $B$ = 10.0 cm for a flat triangular plate. Determine the $x$ and $y$-coordinates of its center of mass. Since it is not stated otherwise, we assume that the mass density of this plate is constant.

**SKETCH:**




**RESEARCH:** Assuming the mass density is constant throughout the object, the center of mass is given by $\vec{R}=\frac{1}{A}\int_A \vec{r}\,dA$, where $A$ is the area of the object. The center of mass can be determined in each dimension.

The $x$ coordinate and the $y$ coordinate of the center of mass are given by $X=\frac{1}{A}\int_A x\,dA$ and $Y=\frac{1}{A}\int_A y\,dA$, respectively. The area of the triangle is $A=HB/2$.

**SIMPLIFY:** The expression for the area of the triangle can be substituted into the formulae for the center of mass to get

$$X=\frac{2}{HB}\int_A x\,dA \text{ and } Y=\frac{2}{HB}\int_A y\,dA.$$

In the $x$-direction we have to solve the integral:

$$\int_A x dA = \int_0^B \int_0^{y_m(x)} x dy dx = \int_0^B x dx \int_0^{y_m(x)} dy = \int_0^B x y_m(x) dx = \int_0^B xH(1-x/B)dx = H\int_0^B x-\left(x^2/B\right)dx$$
$$= H\left(\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3/B\right)\Big|_0^B = \tfrac{1}{2}HB^2 - \tfrac{1}{3}HB^2 = \tfrac{1}{6}HB^2$$

Note that in this integration procedure the maximum for the $y$–integration depends on the value of $x$: $y_m(x) = H(1-x/B)$. Therefore we arrive at

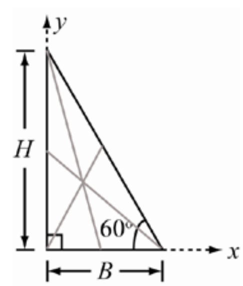
$$X = \frac{2}{HB}\int_A x\,dA = \frac{2}{HB}\cdot\frac{HB^2}{6} = \tfrac{1}{3}B$$

In the same way we can find that $Y = \tfrac{1}{3}H$.

**CALCULATE:** $X_{\text{com}} = \tfrac{1}{3}(10.0\text{ cm}) = 3.33333\text{ cm}$, $Y_{\text{com}} = \tfrac{1}{3}(17.3\text{ cm}) = 5.76667\text{ cm}$

**ROUND:** Three significant figures were provided in the question, so the results should be written $X = 3.33\text{ cm}$ and $Y = 5.77\text{ cm}$.

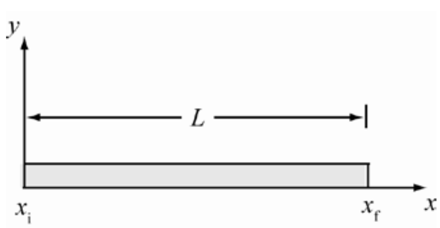
**DOUBLE-CHECK:** Units of length were calculated for both $X$ and $Y$, which is dimensionally correct. We also find that the center of mass coordinates are inside the triangle, which always has to be true for simple geometrical shape without holes in it. Finally, we can determine the location of the center of mass for a triangle geometrically by connecting the center of each side to the opposite corner with a straight line (see drawing). The point at which these three lines intersect is the location of the center of mass. You can see from the graph that this point has to be very close to our calculated result of $(\tfrac{1}{3}B, \tfrac{1}{3}H)$.




**8.50.** **THINK:** The linear density function for a 1.00 m long rod is $\lambda(x) = 100.\text{ g/m} + 10.0x\text{ g/m}^2$. One end of the rod is at $x_{\text{i}} = 0\text{ m}$ and the other end is situated at $x_{\text{f}} = 1.00\text{ m}$. The total mass, $M$ of the rod and the center of mass coordinate are to be determined.



**SKETCH:**




**RESEARCH:**

(a) The linear density of the rod is given by $\lambda(x) = dm/dx$. This expression can be rearranged to get $\lambda(x)dx = dm$. An expression for $\lambda(x)$ was given so both sides can be integrated to solve for $M$.

(b) The center of mass coordinate is given by $X_{\text{com}} = \dfrac{1}{M}\int x dm$.

**SIMPLIFY:**

(a) Integrate both of sides of the linear density function to get:

$$\int_{x_{\text{i}}}^{x_{\text{f}}}\left(100.\text{ g/m} + 10.0x\text{ g/m}^2\right)dx = \int_0^M dm \Rightarrow \left[100.x\text{ g/m} + 5.0x^2\text{ g/m}^2\right]_{x_{\text{i}}}^{x_{\text{f}}} = M.$$

(b) Substitute $dm = \lambda(x)dx$ into the expression for $X_{\text{com}}$ to get

$$X_{\text{com}} = \frac{1}{M}\int_{x_{\text{i}}}^{x_{\text{f}}} x\lambda(x)dx.$$

The value calculated in part (a) for $M$ can later be substituted. Substitute $\lambda(x)=100\text{ g/m}+10.0x\text{ g/m}^2$ into the expression for $X_{\text{com}}$ to get

$$X_{\text{com}}=\frac{1}{M}\int_{x_i}^{x_f}\left(100.x\text{ g/m}+10.0x^2\text{ g/m}^2\right)dx\Rightarrow\left[\frac{1}{M}\left(50.0x^2\text{ g/m}+\frac{10.0}{3}x^3\text{ g/m}^2\right)\right]_{x_i}^{x_f}.$$

**CALCULATE:**

(a) $M=100.\text{ g/m}(1\text{ m})+5.0\text{ g}\dfrac{(1\text{ m})^2}{\text{m}^2}=105\text{ g}$

(b) $X_{\text{com}}=\dfrac{1}{105\text{ g}}\left(50.0(1\text{ m})^2\text{ g/m}+\dfrac{10.0}{3}(1\text{ m})^3\text{ g/m}^2\right)=0.50793651\text{ m}$

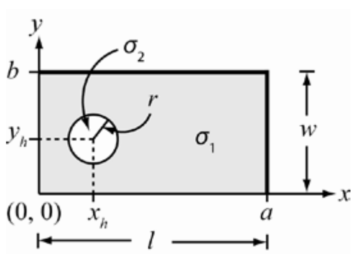
**ROUND:**
Rounding to three significant figures
(a) $M=105\text{ g}$
(b) $X_{\text{com}}=0.508\text{ m}$

**DOUBLE-CHECK:** The correct units were calculated for the mass and the center of mass so the results are dimensionally correct. Our result for the location of the center of mass of the rod, 50.8 cm, is just larger than the geometric center of the rod, 50.0 cm. This makes sense because the density of the rod increases slightly with increasing distance.

**8.51.** **THINK:** The area density for a thin, rectangular plate is given as $\sigma_1=1.05\text{ kg/m}^2$. Its length is $a$ = 0.600 m and its width is $b$ = 0.250 m. The lower left corner of the plate is at the origin. A circular hole of radius, $r$ = 0.0480 m is cut out of the plate. The hole is centered at the coordinates $x_\text{h}=0.068\text{ m}$ and $y_\text{h}=0.068\text{ m}$. A round disk of radius, $r$ is used to plug the hole. The disk, $D$, has a uniform area density of $\sigma_2=5.32\text{ kg/m}^2$. The distance from the origin to the modified plate's center of mass, $R$, is to be determined.



**SKETCH:**




**RESEARCH:** The center of mass, $R$, of an object can be defined mathematically as $R=\dfrac{1}{M}\sum_{i=1}^{n}\vec{r}_i m_i$ (1). In this equation, $M$ is the total mass of the system. The vector $\vec{r}_i$ denotes the position of the $i^{\text{th}}$ object's center of mass and $m_\text{i}$ is the mass of that object. To solve this problem, the center of mass of the plate, $R_\text{p}$, and the center of mass of the disk, $R_{\text{D,}}$ must be determined. Then equation (1) can be used to determine the distance from the origin to the modified center of mass, $R$. First, consider the rectangular plate, $P$, which has the hole cut in it. The position of the center of mass, $R_\text{p}$, is not known. The mass of P can be denoted $m_\text{p}$. Consider the disk of material, $d$, that was removed (which has a uniform area density of $\sigma_1$), and denote its center of mass as $R_\text{d}$ and its mass as $m_\text{d}$. Next, define $S$ as the system of the rectangular plate, $P$, and the disc of removed material, $d$. The mass of $S$ can be denoted $m_\text{s}=m_\text{p}+m_\text{d}$. The center of mass of $S$ is $R_\text{s}=(a/2)\hat{x}+(b/2)\hat{y}$. $m_\text{p}$ and $m_\text{d}$ are not known but it is known that they have uniform area density of $\sigma_1$. The uniform area density is given by $\sigma=m/A$. Therefore, $m_\text{p}=\sigma_1 A_\text{p}$ and

$m_d = \sigma_1 A_d$, where $A_p$ is the area of the plate minus the area of the hole and $A_d$ is the area of the disk, *d*. The expressions for these areas are $A_p = ab - \pi r^2$ and $A_d = \pi r^2$. Substituting these area expressions into the expressions for $m_p$ and $m_d$ gives $m_p = \sigma_1\left(ab - \pi r^2\right)$ and $m_d = \sigma_1 \pi r^2$. So the center of mass of the system is given by:

$$\vec{R}_s = \frac{\left(x_h \hat{x} + y_h \hat{y}\right) m_d + \vec{R}_p m_p}{\sigma_1\left(ab - \pi r^2\right) + \sigma_1 \pi r^2} \quad (2).$$

Now, consider the disk, *D*, that is made of the material of uniform area density, $\sigma_2$. Define its center of mass as $\vec{R}_D = x_h \hat{x} + y_h \hat{y}$. Also, define its mass as $m_D = \sigma_2 \pi r^2$.

**SIMPLIFY:** Rearrange equation (2) to solve for $\vec{R}_p$:

$$\vec{R}_p m_p = \vec{R}_s \sigma_1 ab - \left(x_h \hat{x} + y_h \hat{y}\right) m_d \Rightarrow \vec{R}_p = \frac{\vec{R}_s \sigma_1 ab - \left(x_h \hat{x} + y_h \hat{y}\right) m_d}{m_p}.$$

Now, substitute the values for $\vec{R}_s$, $m_d$ and $m_p$ into the above equation to get:

$$\vec{R}_p = \frac{\left(\frac{a}{2}\hat{x} + \frac{b}{2}\hat{y}\right)\sigma_1 ab - \left(x_h \hat{x} + y_h \hat{y}\right)\sigma_1 \pi r^2}{\sigma_1\left(ab - \pi r^2\right)}.$$

Once $\vec{R}_p$ is solved, it can be substituted into the expression for $\vec{R}$ to get $\vec{R} = \dfrac{\vec{R}_p m_p + \vec{R}_D m_D}{m_p + m_D}$. Use the distance formula $R = \sqrt{R_x^{\,2} + R_y^{\,2}}$.

**CALCULATE:**

$$\vec{R}_p = \frac{\left(\frac{0.600}{2}\hat{x} + \frac{0.250}{2}\hat{y}\right)\left(\left(1.05 \text{ kg/m}^3\right)\left(0.600 \text{ m}\right)\left(0.250 \text{ m}\right)\right) - \left(0.068\hat{x} + 0.068\hat{y}\right)\left(1.05 \text{ kg/m}^3\right)\pi\left(0.0480 \text{ m}\right)^2}{\left(1.05 \text{ kg/m}^3\right)\left(\left(0.600 \text{ m}\right)\left(0.250 \text{ m}\right) - \pi\left(0.0480 \text{ m}\right)^2\right)}$$

$$= \left(0.31176\hat{x} + 0.12789\hat{y}\right) \text{ m}$$

$$\vec{R} = \frac{\left(0.31176\hat{x} + 0.12789\hat{y}\right) \text{ m}\left(0.1499 \text{ kg}\right) + \left(0.068\hat{x} + 0.068\hat{y}\right) \text{ m}\left(0.038507 \text{ kg}\right)}{0.1499 \text{ kg} + 0.038507 \text{ kg}}$$

$$= \left(0.26194\hat{x} + 0.11565\hat{y}\right) \text{ m}$$

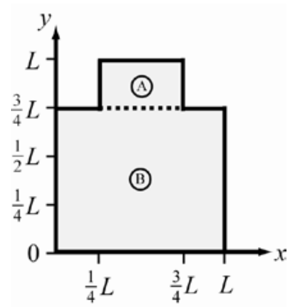
Then, the distance to the origin is given by $R = \sqrt{\left(0.26194 \text{ m}\right)^2 + \left(0.11565 \text{ m}\right)^2} = 0.28633 \text{ m}$.

**ROUND:** Densities are given to three significant figures. For dimensions the subtraction rule applies, where all dimensions are known to three decimal places. The result should be rounded to $R = 0.286$ m.

**DOUBLE-CHECK:** The position of the center of mass for the modified system is shifted closer to the position of the disk, *D*, which has an area density of $5.32 \text{ kg/m}^2$. This is reasonable because the disk has a much higher area density than the rest of the plate. Also, the results are reasonable considering the given values.

**8.52.** **THINK:** The object of interest is a uniform square metal plate with sides of length, $L$ = 5.70 cm and mass, $m$ = 0.205 kg. The lower left corner of the plate sits at the origin. Two squares with side length, $L/4$ are removed from each side at the top of the square. Determine the *x*-coordinate and the *y*-coordinate of the center of mass, denoted $X_{com}$ and $Y_{com}$, respectively.

**SKETCH:**




**RESEARCH:** Because the square is uniform, the equations for $X_{\text{com}}$ and $Y_{\text{com}}$ can be expressed by

$$X_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \text{ and } Y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i.$$

$M$ is the total mass of the system. In this problem it will be useful to treat the system as if it were made up of two uniform metal rectangles, $R_A$ and $R_B$.

(a) The center of mass $x$-coordinate for rectangle $A$ is $x_{\text{A}} = (L/2)\hat{x}$. The center of mass $x$-coordinate for rectangle $B$ is $x_{\text{B}} = (L/2)\hat{x}$.

(b) The center of mass $y$-coordinate for rectangle $A$ is $y_{\text{A}} = (7L/8)\hat{y}$. The center of mass $y$-coordinate for rectangle $B$ is $y_{\text{B}} = (3L/8)\hat{y}$. Both rectangles have the same uniform area density, $\sigma$. The uniform area density is given by $\sigma = m_{\text{A}}/A_{\text{A}} = m_{\text{B}}/A_{\text{B}}$. Therefore, $m_{\text{A}} = m_{\text{B}}A_{\text{A}}/A_{\text{B}}$. The areas are given by the following expressions:

$$A_{\text{A}} = \left(\frac{L}{4}\right)\left(\frac{L}{2}\right) = \frac{L^2}{8} \text{ and } A_{\text{B}} = \left(\frac{3L}{4}\right)L = \frac{3L^2}{4}.$$



**SIMPLIFY:**

(a) $X_{\text{com}} = \dfrac{x_{\text{A}}m_{\text{A}} + x_{\text{B}}m_{\text{B}}}{m_{\text{A}} + m_{\text{B}}}$

Substitute the expression for $m_{\text{A}}$ into the above equation to get:

$$X_{\text{com}} = \frac{x_{\text{A}}m_{\text{B}}\dfrac{A_{\text{A}}}{A_{\text{B}}} + x_{\text{B}}m_{\text{B}}}{m_{\text{B}}\dfrac{A_{\text{A}}}{A_{\text{B}}} + m_{\text{B}}} = \frac{x_{\text{A}}\left(\dfrac{A_{\text{A}}}{A_{\text{B}}}\right) + x_{\text{B}}}{\dfrac{A_{\text{A}}}{A_{\text{B}}} + 1}.$$

Then substitute the expressions for $x_{\text{A}}$, $x_{\text{B}}$, $A_{\text{A}}$ and $A_{\text{B}}$ to get:

$$X_{\text{com}} = \frac{\dfrac{L}{2}\left(\dfrac{L^2/8}{3L^2/4}\right) + \dfrac{L}{2}}{\dfrac{L^2/8}{3L^2/4} + 1} = \frac{\dfrac{L}{12} + \dfrac{L}{2}}{\dfrac{1}{6} + 1} = \frac{\dfrac{7L}{12}}{\dfrac{7}{6}} = \frac{1}{2}L.$$

(b) The same procedure can be used to solve for the $y$-coordinate of the center of mass:

$$Y_{\text{com}} = \frac{y_{\text{A}}\left(\dfrac{A_{\text{A}}}{A_{\text{B}}}\right) + y_{\text{B}}}{\dfrac{A_{\text{A}}}{A_{\text{B}}} + 1} = \frac{\dfrac{7L}{8}\left(\dfrac{1}{6}\right) + \dfrac{3L}{8}}{\dfrac{7}{6}} = \frac{\dfrac{7L}{48} + \dfrac{18L}{48}}{\dfrac{7}{6}} = \frac{25L}{48}\left(\frac{6}{7}\right) = \frac{25L}{56}.$$

**CALCULATE:**

(a) $X_{\text{com}} = \dfrac{1}{2}(5.70 \text{ cm}) = 2.85 \text{ cm}$

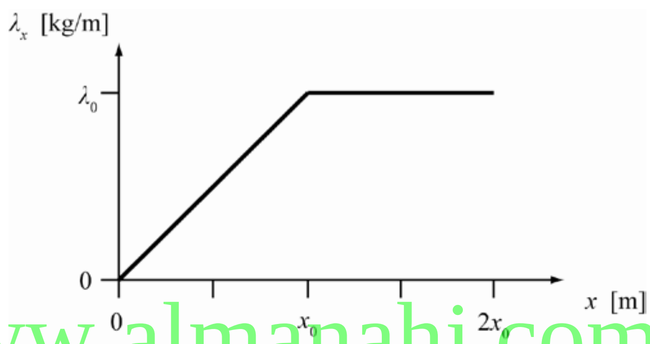
(b) $Y_{\text{com}} = \frac{25}{56}(5.70 \text{ cm}) = 2.5446 \text{ cm}$

**ROUND:** Three significant figures were provided in the question, so the results should be rounded to $X_{\text{com}} = 2.85 \text{ cm}$ and $Y_{\text{com}} = 2.54 \text{ cm}$.

**DOUBLE-CHECK:** Units of distance were calculated, which is expected when calculating the center of mass coordinates. The squares were removed uniformly at the top of the large square, so it makes sense that the *x*-coordinate of the center of mass stays at *L*/2 by symmetry and the *y*-coordinate of the center of mass is shifted slightly lower.

**8.53.** **THINK:** The linear mass density, $\lambda(x)$, is provided in the graph. Determine the location for the center of mass, $X_{\text{com}}$, of the object. From the graph, it can be seen that $\lambda(x) = \begin{Bmatrix} \frac{\lambda_0}{x_0}x, \ 0 \le x < x_0 \\ \lambda_0, \ x_0 \le x \le 2x_0 \end{Bmatrix}$.

**SKETCH:**






**RESEARCH:** The linear mass density, $\lambda(x)$, depends on *x*. To determine the center of mass, use the equation $X_{\text{com}} = \frac{1}{M}\int_L x\lambda(x)dx$. The mass of the system, M, can be determined using the equation $M = \int_L \lambda(x)dx$. In order to evaluate the center of mass of the system, two separate regions must be considered; the region from $x = 0$ to $x = x_0$ and the region from $x = x_0$ to $x = 2x_0$. The equation for $X_{\text{com}}$ can be expanded to $X_{\text{com}} = \frac{1}{M}\int_0^{x_0} x\frac{\lambda_0}{x_0}xdx + \frac{1}{M}\int_{x_0}^{2x_0} \lambda_0 xdx$. The equation for $M$ is $M = \int_0^{x_0} \frac{\lambda_0}{x_0}xdx + \int_{x_0}^{2x_0} \lambda_0 dx$.

**SIMPLIFY:** Simplify the expression for *M* first and then substitute it into the expression for $X_{\text{com}}$.

$$M = \int_0^{x_0} \frac{\lambda_0}{x_0}xdx + \int_{x_0}^{2x_0} \lambda_0 dx = \left[\frac{1}{2}\frac{\lambda_0}{x_0}x^2\right]_0^{x_0} + \left[x\lambda_0\right]_{x_0}^{2x_0} = \frac{1}{2}\lambda_0 x_0 + 2x_0\lambda_0 - x_0\lambda_0 = \frac{3}{2}x_0\lambda_0.$$

Substitute the above expression into the equation for $X_{\text{com}}$ to get:

$$X_{\text{com}} = \frac{2}{3x_0\lambda_0}\left[\int_0^{x_0} x^2\frac{\lambda_0}{x_0}dx + \int_{x_0}^{2x_0} \lambda_0 xdx\right] = \frac{2}{3x_0\lambda_0}\left[\frac{1}{3}\lambda_0 x_0^2 + 2\lambda_0 x_0^2 - \frac{1}{2}\lambda_0 x_0^2\right] = \frac{2}{3x_0\lambda_0}\left[\lambda_0 x_0^2\left(\frac{2}{6} + \frac{12}{6} - \frac{3}{6}\right)\right]$$

$$= \frac{2}{3x_0\lambda_0}\left(\frac{11}{6}\lambda_0 x_0^2\right) = \frac{11x_0}{9}.$$

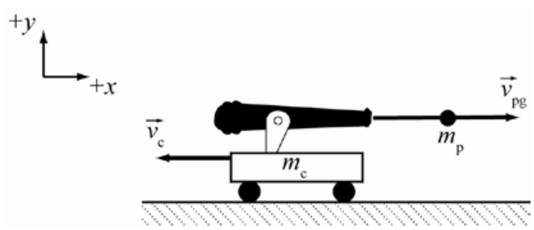
**CALCULATE:** This step does not apply.

**ROUND:** This step does not apply.

**DOUBLE-CHECK:** The units for the result are units of length, so the answer is dimensionally correct. It is reasonable that the calculated value is closer to the denser end of the object.

**8.54.** **THINK:** The mass of the cannon is $m_c = 750$ kg and the mass of the projectile is $m_p = 15$ kg. The total mass of the cannon and projectile system is $M = m_c + m_p$. The speed of the projectile is $v_p = 250$ m/s with respect to the muzzle just after the cannon has fired. The cannon is on wheels and can recoil with negligible friction. Determine the speed of the projectile with respect to the ground, $v_{pg}$.

**SKETCH:**




**RESEARCH:** The problem can be solved by considering the conservation of linear momentum. The initial momentum is $\vec{P}_i = 0$ because the cannon and projectile are both initially at rest. The final momentum is $\vec{P}_f = m_c\vec{v}_c + m_p\vec{v}_{pg}$. The velocity of the recoiling cannon is $v_c$. The equation for the conservation of momentum is $\vec{P}_i = \vec{P}_f$. The velocity of the projectile with respect to the cannon's muzzle can be represented as $\vec{v}_p = \vec{v}_{pg} - \vec{v}_c$. Take $\vec{v}_{pg}$ to be in the positive $x$-direction.

**SIMPLIFY:** Rearrange the above equation so that it becomes $\vec{v}_c = \vec{v}_{pg} - \vec{v}_p$. Then substitute this expression into the conservation of momentum equation:

$$P_i = P_f \Rightarrow 0 = m_c v_c + m_p v_{pg} \Rightarrow 0 = m_c\left(v_{pg} - v_p\right) + m_p v_{pg} \Rightarrow v_{pg}\left(m_c + m_p\right) = m_c v_p \Rightarrow v_{pg} = \frac{m_c v_p}{\left(m_c + m_p\right)}.$$



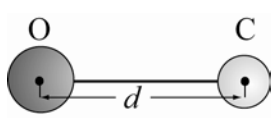
**CALCULATE:** $v_{pg} = \dfrac{(750 \text{ kg})(250 \text{ m/s})}{(750 \text{ kg} + 15 \text{ kg})} = 245.098$ m/s

**ROUND:** The least number of significant figures provided in the question is three, so the result should be rounded to $v_{pg} = 245$ m/s.

**DOUBLE-CHECK:** The units of speed are correct for the result. The velocity calculated for the projectile with respect to the ground is slower than its velocity with respect to the cannon's muzzle, which is what is expected.

**8.55.** **THINK:** The mass of a carbon atom is $m_C = 12.0$ u and the mass of an oxygen atom is $m_O = 16.0$ u. The distance between the atoms in a CO molecule is $d = 1.13 \cdot 10^{-10}$ m. Determine how far the center of mass, $X_{com}$, is from the carbon atom. Denote the position of the carbon atoms as $X_C$ and the position of the oxygen atom as $X_O$.

**SKETCH:**




**RESEARCH:** The center of mass of a system is given by $X_{com} = \dfrac{1}{M}\displaystyle\sum_{i=1}^{n} m_i x_i$.

The total mass of the system is $M = m_C + m_O$. It is convenient to assign the position of the oxygen atom to be at the origin, $X_O = 0$. Then the center of mass becomes

$$X_{com} = \frac{(0)m_O + m_C d}{m_O + m_C} = \frac{m_C d}{m_O + m_C}.$$

Once $X_{\text{com}}$ is determined, then the distance from it to the carbon atom can be determined using the equation $X_{\text{dC}} = X_{\text{C}} - X_{\text{com}}$, where $X_{\text{dC}}$ is the distance from the center of mass to the carbon atom.

**SIMPLIFY:** Substitute the expression $X_{\text{com}} = (m_{\text{C}} d)/(m_{\text{O}} + m_{\text{C}})$ into the expression for $X_{\text{dC}}$ to get $X_{\text{dC}} = X_{\text{C}} - \dfrac{m_{\text{C}} d}{m_{\text{O}} + m_{\text{C}}}$. Substitute $X_C = d$ to get $X_{\text{dC}} = d - \dfrac{m_{\text{C}} d}{m_{\text{O}} + m_{\text{C}}}$.

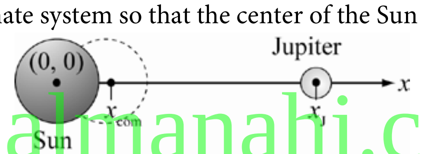
**CALCULATE:** $X_{\text{dC}} = (1.13\cdot 10^{-10}\text{ m}) - \left(\dfrac{12.0\text{ u}}{28.0\text{ u}}\right)(1.13\cdot 10^{-10}\text{ m}) = 6.4571\cdot 10^{-11}\text{ m}$

**ROUND:** Three significant figures were provided in the problem so the answer should be rounded to $X_{\text{dC}} = 6.46\cdot 10^{-11}$ m.

**DOUBLE-CHECK:** The center of mass of the system is closer to the more massive oxygen atom, as it should be.

**8.56.** **THINK:** The system to be considered consists of the Sun and Jupiter. Denote the position of the Sun's center of mass as $X_{\text{S}}$ and the mass as $m_{\text{S}}$. Denote the position of Jupiter's center of mass as $X_{\text{J}}$ and its mass as $m_{\text{J}}$. Determine the distance that the Sun wobbles due to its rotation about the center of mass. Also, determine how far the system's center of mass, $X_{\text{com}}$, is from the center of the Sun. The mass of the Sun is $m_{\text{S}} = 1.98892\cdot 10^{30}$ kg. The mass of Jupiter is $m_{\text{J}} = 1.8986\cdot 10^{27}$ kg. The distance from the center of the Sun to the center of Jupiter is $X_{\text{J}} = 7.78\cdot 10^{8}$ km.

**SKETCH:** Construct the coordinate system so that the center of the Sun is positioned at the origin.






**RESEARCH:** The system's center of mass is given by $X_{\text{com}} = \dfrac{1}{M}\sum_{i=1}^{n} m_i x_i$.

The total mass of the system is $M = m_{\text{S}} + m_{\text{J}}$. The dashed line in the sketch denotes the Sun's orbit about the system's center of mass. From the sketch it can be seen that the distance the sun wobbles is twice the distance from the Sun's center to the system's center of mass.

**SIMPLIFY:** $X_{\text{com}} = \dfrac{m_{\text{S}} X_{\text{S}} + m_{\text{J}} X_{\text{J}}}{m_{\text{S}} + m_{\text{J}}}$. The coordinate system was chosen in such a way that $X_{\text{S}} = 0$. The center of mass equation can be simplified to $X_{\text{com}} = \dfrac{m_{\text{J}} X_{\text{J}}}{m_{\text{S}} + m_{\text{J}}}$. Once $X_{\text{com}}$ is determined, it can be doubled to get the Sun's wobble.

**CALCULATE:** $X_{\text{com}} = \dfrac{(1.8986\cdot 10^{27}\text{ kg})(7.78\cdot 10^{8}\text{ km})}{1.98892\cdot 10^{30}\text{ kg} + 1.8986\cdot 10^{27}\text{ kg}} = 741961.5228\text{ km}$

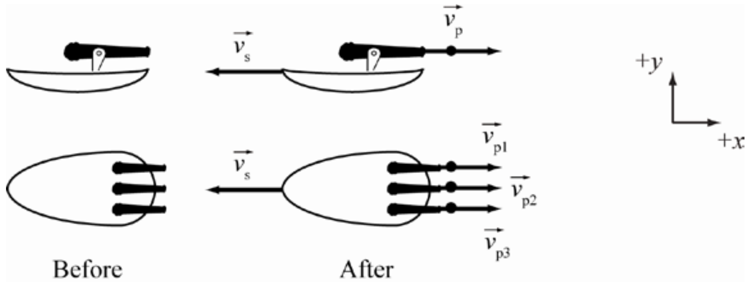
The Sun's wobble is $2(741961.5228\text{ km}) = 1483923.046$ km.

**ROUND:** Rounding the results to three figures, $X_{\text{com}} = 7.42\cdot 10^{5}$ km and the Sun's wobble is $1.49\cdot 10^{6}$ km.

**DOUBLE-CHECK:** It is expected that the system's center of mass is much closer to the Sun than it is to Jupiter, and the results are consistent with this.

**8.57.** **THINK:** The mass of the battleship is $m_{\text{s}} = 136{,}634{,}000$ lbs. The ship has twelve 16-inch guns and each gun is capable of firing projectiles of mass, $m_{\text{p}} = 2700.$ lb, at a speed of $v_{\text{p}} = 2300.$ ft/s. Three of the guns fire projectiles in the same direction. Determine the recoil velocity, $v_{\text{s}}$, of the ship. Assume the ship is initially stationary.

**SKETCH:**




**RESEARCH:** The total mass of the ship and projectile system is $M = m_s + \sum_{i=1}^{n} m_{pi}$.

All of the projectiles have the same mass and same speed when they are shot from the guns. This problem can be solved considering the conservation of momentum. The equation for the conservation of momentum is $\vec{P}_i = \vec{P}_f$. $\vec{P}_i$ is the initial momentum of the system and $\vec{P}_f$ is the final momentum of the system. Assume the ship carries one projectile per gun. $\vec{P}_i = 0$ because the battleship is initially at rest and $\vec{P}_f = -(m_s + 9m_p)v_s + 3m_p v_p$.

**SIMPLIFY:** $\vec{P}_i = \vec{P}_f \Rightarrow 0 = -(m_s + 9m_p)v_s + 3m_p v_p \Rightarrow v_s = \dfrac{3m_p v_p}{(m_s + 9m_p)}$

**CALCULATE:** $v_s = \dfrac{3(2700.\text{ lb})(2300.\text{ ft/s})}{(136{,}634{,}000\text{ lb} + 9(2700.\text{ lb}))} = 0.136325\text{ ft/s}$

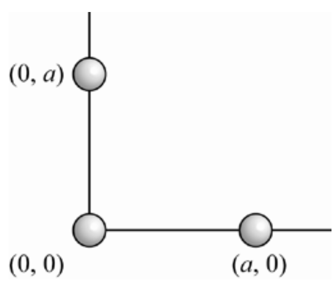
**ROUND:** The values for the mass and speed of the projectile that are given in the question have four significant figures, so the result should be rounded to $v_s = 0.1363$ ft/s. The recoil velocity is in opposite direction than the cannons fire.



**DOUBLE-CHECK:** The mass of the ship is much greater than the masses of the projectiles, so it reasonable that the recoil velocity is small because momentum depends on mass and velocity.

**8.58.** **THINK:** The system has three identical balls of mass *m*. The *x* and *y* coordinates of the balls are $\vec{r}_1 = (0\hat{x}, 0\hat{y})$, $\vec{r}_2 = (a\hat{x}, 0\hat{y})$ and $\vec{r}_3 = (0\hat{x}, a\hat{y})$. Determine the location of the system's center of mass, *R*.

**SKETCH:**




**RESEARCH:** The center of mass is a vector quantity, so the *x* and *y* components must be considered separately. The *x*- and *y*-components of the center of mass are given by

$$X_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \text{ and } Y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i.$$

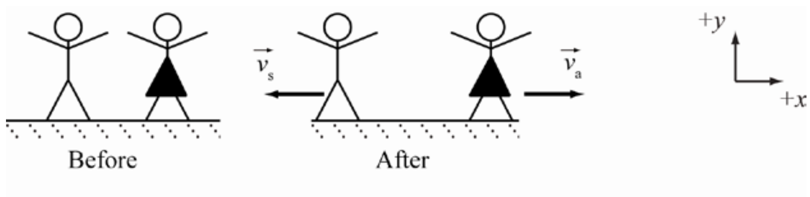
For this system, the equations can be rewritten as

$$X_{com} = \frac{m(0) + ma\hat{x} + m(0)}{3m} = \frac{a}{3}\hat{x} \text{ and } Y_{com} = \frac{m(0) + m(0) + ma\hat{y}}{3m} = \frac{a}{3}\hat{y}.$$

**SIMPLIFY:** The x and y components of the center of mass are known, so $\vec{R}_{com} = \frac{a}{3}\hat{x} + \frac{a}{3}\hat{y}.$

**CALCULATE:** This step is not necessary.
**ROUND:** This step is not necessary.
**DOUBLE-CHECK:** Considering the geometry of the system, the results are reasonable. In the *x*-direction we would expect the center of mass to be twice as far from the mass on the right as from the two on the left, and in the *y*-direction we would expect the center of mass to be twice as far from the upper mass as from the two lower ones.

**8.59.** **THINK:** Sam's mass is $m_S = 61.0$ kg and Alice's mass is $m_A = 44.0$ kg. They are standing on an ice rink with negligible friction. After Sam pushes Alice, she is moving away from him with a speed of $v_A = 1.20$ m/s with respect to the rink. Determine the speed of Sam's recoil, $v_S$. Also, determine the change in kinetic energy, $\Delta K$, of the Sam-Alice system.
**SKETCH:**




**RESEARCH:**
(a) To solve the problem, consider the conservation of momentum. The equation for conservation of momentum can be written $\vec{P}_i = \vec{P}_f$. $\vec{P}_i$ is the initial momentum of the system and $\vec{P}_f$ is the final momentum of the system. $\vec{P}_i = 0$ because Sam and Alice are initially stationary and $\vec{P}_f = -m_S\vec{v}_S + m_A\vec{v}_A$.
(b) The change in kinetic energy is $\Delta K = K_f - K_i = \left(m_S v_S^2\right)/2 + \left(m_A v_A^2\right)/2$.



**SIMPLIFY:**
(a) $\vec{P}_i = \vec{P}_f \Rightarrow 0 = -m_S\vec{v}_S + m_A\vec{v}_A \Rightarrow v_S = \frac{m_A\vec{v}_A}{m_S}$

(b) The expression determined for $v_S$ in part (a) can be substituted into the equation for $\Delta K$ to get

$$\Delta K = \frac{1}{2}m_S\left(\frac{m_A\vec{v}_A}{m_S}\right)^2 + \frac{1}{2}m_A v_A^2.$$

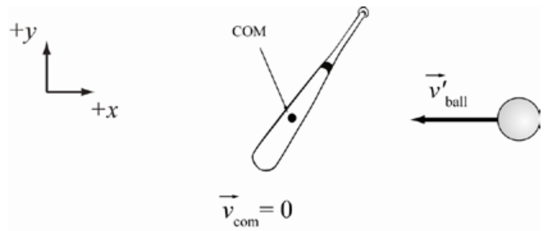
**CALCULATE:**
(a) $v_S = \frac{(44.0 \text{ kg})(1.20 \text{ m/s})}{61.0 \text{ kg}} = 0.8656 \text{ m/s}$

(b) $\Delta K = \frac{1}{2}(61.0 \text{ kg})\left(\frac{(44.0 \text{ kg})(1.20 \text{ m/s})}{61.0 \text{ kg}}\right)^2 + \frac{1}{2}(44.0 \text{ kg})(1.20 \text{ m/s})^2 = 54.53 \text{ J}$

(c) Sam did work on Alice when he pushed her. The work that Sam did was the source of the kinetic energy. Sam was able to do this work by converting chemical energy that was stored in his body into mechanical energy. The energy stored in Sam's body was provided by food that he ate and his body processed.
**ROUND:** Three significant figures were provided in the problem so the results should be rounded accordingly to $v_S = 0.866$ m/s and $\Delta K = 55$ J.
**DOUBLE-CHECK:** Sam's mass is greater than Alice's so it reasonable that his recoil speed is slower than her sliding speed. The change in kinetic energy is reasonable considering the masses and velocities given.

**8.60.** **THINK:** The mass of the bat is $m_{\text{bat}}$ and the mass of the ball is $m_{\text{ball}}$. Assume that the center of mass of the ball and bat system is essentially at the bat. The initial velocity of the ball is $\vec{v}_{\text{ball,i}} = -30.0$ m/s and the initial velocity of the bat is $\vec{v}_{\text{bat}} = 35.0$ m/s. The bat and ball undergo a one-dimensional elastic collision. Determine the speed of the ball after the collision.

**SKETCH:**




**RESEARCH:** In the center of mass frame, $\vec{v}_{\text{com}} = 0$. Since the collision is elastic, in the center of mass frame the final velocity of the ball, $\vec{v}_{\text{ball,f}}$, will be equal to the negative of the ball's initial velocity, $\vec{v}_{\text{ball,i}}$. This statement can be written mathematically as $\vec{v}_{\text{ball,i}} = -\vec{v}_{\text{ball,f}}$. Since the center of mass is in the bat, the $\vec{v}_{\text{com}}$ in the lab reference frame equals $\vec{v}_{\text{bat}}$. The following relationships can be written for this system: $\vec{v}'_{\text{ball,i}} = \vec{v}_{\text{ball,i}} - \vec{v}_{\text{com}}$ (1) and $\vec{v}'_{\text{ball,f}} = \vec{v}_{\text{ball,f}} - \vec{v}_{\text{com}}$ (2).

**SIMPLIFY:** Recall that $\vec{v}'_{\text{ball,i}} = -\vec{v}'_{\text{ball,f}}$. Therefore, the following equality can be written:

$\vec{v}_{\text{ball,i}} - \vec{v}_{\text{com}} = -\left(\vec{v}_{\text{ball,f}} - \vec{v}_{\text{com}}\right) \Rightarrow \vec{v}_{\text{ball,f}} = 2\vec{v}_{\text{com}} - \vec{v}_{\text{ball,i}}$.

Recall that $\vec{v}_{\text{com}}$ is equal to $\vec{v}_{\text{bat}}$, so the above expression can be rewritten as $\vec{v}_{\text{ball,f}} = 2\vec{v}_{\text{bat}} - \vec{v}_{\text{ball,i}}$.

**CALCULATE:** $\vec{v}_{\text{ball,f}} = 2(35.0 \text{ m/s}) - (-30.0 \text{ m/s}) = 100.0 \text{ m/s}$

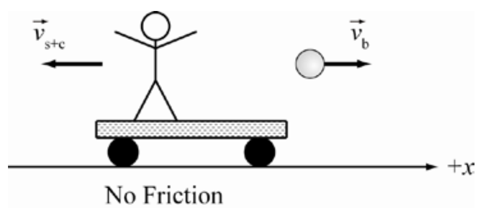


**ROUND:** Rounding to three significant figures: $\vec{v}_{\text{ball,f}} = 100.$ m/s

**DOUBLE-CHECK:** The initial velocities of the bat and ball are similar, but the bat is much more massive than the ball, so the speed of the ball after the collision is expected to be high.

**8.61.** **THINK:** The student's mass is $m_{\text{s}} = 40.0$ kg, the ball's mass is $m_{\text{b}} = 5.00$ kg and the cart's mass is $m_{\text{c}} = 10.0$ kg. The ball's relative speed is $v'_{\text{b}} = 10.0$ m/s and the student's initial speed is $v_{\text{si}} = 0$. Determine the ball's velocity with respect to the ground, $\vec{v}_{\text{b}}$, after it is thrown.

**SKETCH:**




**RESEARCH:** $\vec{v}_{\text{b}}$ can be determined by considering the conservation of momentum, $\vec{P}_{\text{i}} = \vec{P}_{\text{f}}$, where $p = mv$. Note the ball's relative speed is $\vec{v}'_{\text{b}} = \vec{v}_{\text{b}} - \vec{v}_{\text{s+c}}$, where $\vec{v}_{\text{b}}$ and $\vec{v}_{\text{s+c}}$ are measured relative to the ground.

**SIMPLIFY:** $\vec{P}_{\text{i}} = \vec{P}_{\text{f}} \Rightarrow 0 = \left(m_{\text{s}} + m_{\text{c}}\right)\vec{v}_{\text{s+c}} + m_{\text{b}}\vec{v}_{\text{b}} \Rightarrow 0 = \left(m_{\text{s}} + m_{\text{c}}\right)\left(\vec{v}_{\text{b}} - \vec{v}'_{\text{b}}\right) + m_{\text{b}}\vec{v}_{\text{b}} \Rightarrow \vec{v}_{\text{b}} = \dfrac{\vec{v}'_{\text{b}}\left(m_{\text{s}} + m_{\text{c}}\right)}{m_{\text{s}} + m_{\text{c}} + m_{\text{b}}}$

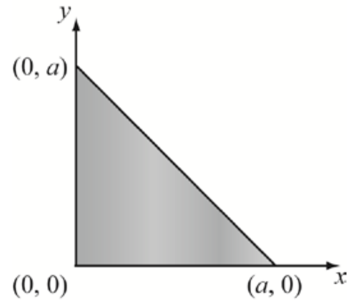
**CALCULATE:** $\vec{v}_{\text{b}} = \dfrac{(10.0 \text{ m/s})(40.0 \text{ kg} + 10.0 \text{ kg})}{(40.0 \text{ kg} + 10.0 \text{ kg} + 5.00 \text{ kg})} = 9.0909 \text{ m/s}$

**ROUND:** $\vec{v}_{\text{b}} = 9.09$ m/s in the direction of $\vec{v}'_{\text{b}}$ (horizontal)

**DOUBLE-CHECK:** It is expected that $v_{\text{b}} < v'_{\text{b}}$ since the student and cart move away from the ball when it is thrown.

**8.62.** **THINK:** Determine the center of mass of an isosceles triangle of constant density $\sigma$.

**SKETCH:**




**RESEARCH:** To determine the center of mass of a two-dimensional object of constant density $\sigma$, use $X = \frac{1}{A}\int_A \sigma x dA$ and $Y = \frac{1}{A}\int_A \sigma y dA$.

**SIMPLIFY:** Note the boundary condition on the hypotenuse of the triangle, $x + y = a$. First, determine $X$. As $x$ varies, take $dA = ydx$. Then the equation becomes $X = \frac{\sigma}{A}\int_0^a xydx$. From the boundary condition, $y = a - x$. Then the equation can be rewritten as $X = \frac{\sigma}{A}\int_0^a x(a-x)dx = \left[\frac{\sigma}{A}\left(\frac{1}{2}ax^2 - \frac{1}{3}x^3\right)\right]_0^a = \frac{a^3\sigma}{6A}$. Similarly for $Y$, take $dA = xdy$ and $x = a - y$ to get $Y = \frac{\sigma}{A}\int_0^a y(a-y)dy = \left[\frac{\sigma}{A}\left(\frac{1}{2}ay^2 - \frac{1}{3}y^3\right)\right]_0^a = \frac{a^3\sigma}{6A}$, with $A = \int \sigma dA = \sigma \cdot \frac{bh}{2} = \frac{a^2\sigma}{2}$ we get $X = Y = \frac{2}{a^2\sigma}\cdot\frac{a^3\sigma}{6A} = \frac{a}{3}$.



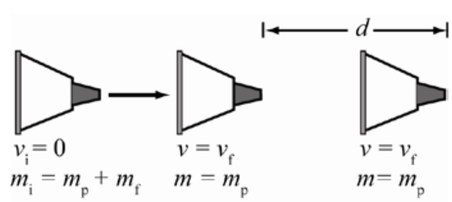
**CALCULATE:** This step is not applicable.

**ROUND:** This step is not applicable.

**DOUBLE-CHECK:** The center of mass coordinates that we obtained are contained within the isosceles triangle, as expected for a solid object.

**8.63.** **THINK:** The payload's mass is $m_p = 4390.0$ kg and the fuel mass is $m_f = 1.761\cdot 10^5$ kg. The initial velocity is $v_i = 0$. The distance traveled after achieving $v_f$ is $d = 3.82\cdot 10^8$ m. The trip time is $t = 7.00 \text{ h} = 2.52\cdot 10^4$ s. Determine the propellant expulsion speed, $v_c$.

**SKETCH:**




**RESEARCH:** $v_c$ can be determined from $v_f - v_i = v_c \ln(m_i / m_f)$. First, $v_f$ must be determined from the relationship $v = d/t$.

**SIMPLIFY:** First, determine $v_f$ from $v_f = d/t$. Substitute this expression and $v_i = 0$ into the above equation to determine $v_c$:

$$v_c = \frac{v_f}{\ln\left(\frac{m_i}{m_f}\right)} = \frac{d}{t\ln\left(\frac{m_i}{m_f}\right)} = \frac{d}{t\ln\left(\frac{m_p + m_f}{m_p}\right)}.$$

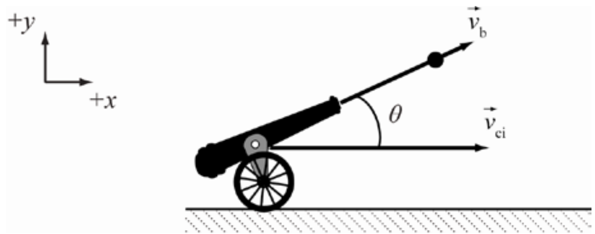
**CALCULATE:** $v_c = \dfrac{3.82\cdot 10^8 \text{ m}}{\left(2.52\cdot 10^4 \text{ s}\right)\ln\left(\dfrac{4390.0 \text{ kg}+1.761\cdot 10^5 \text{ kg}}{4390.0 \text{ kg}}\right)} = 4.079\cdot 10^3 \text{ m/s}$

**ROUND:** Since $t$ has three significant figures, the result should be rounded to $v_c = 4.08$ km/s.

**DOUBLE-CHECK:** This expulsion velocity is reasonable.

**8.64.** **THINK:** The cannon's mass is $M$ = 350 kg. The cannon's initial speed is $v_{ci} = 7.5$ m/s. The ball's mass is $m$ = 15 kg and the launch angle is $\theta = 55°$. The cannon's final velocity after the shot is $v_{cf} = 0$. Determine the velocity of the ball relative to the cannon, $\vec{v}_b'$.

**SKETCH:**




**RESEARCH:** Use conservation of momentum, $\vec{P}_i = \vec{P}_f$, where $\vec{P} = m\vec{v}$. To determine the relative velocity, $\vec{v}_b'$, with respect to the cannon, use $\vec{v}_b' = \vec{v}_b - \vec{v}_c$, where $\vec{v}_b$ is the ball's velocity in the lab frame. Finally, since the cannon moves only in the horizontal ($x$) direction, consider only momentum conservation in this dimension. Take $\vec{v}_{ci}$ to be along the positive $x$-direction, that is $v_{ci} = +7.5$ m/s. With $v_{bx}$ known, find $v_b$ from the expression $v_{bx} = v_b\cos\theta$ and then $v_b'$ can be determined.

**SIMPLIFY:** $P_{xi} = P_{xf} \Rightarrow (m_b + m_c)v_{ci} = m_c v_{cf} + m_b v_{bx}$. Note since $v_{cf}$ is zero, $v_{bx} = v_{bx}'$, that is, the ball's speed relative to the cannon is the same as its speed in the lab frame since the cannon has stopped moving.



Rearranging the above equation gives $v_{bx} = \dfrac{(m_b + m_c)v_{ci}}{m_b} \Rightarrow v_b = \dfrac{(m_b + m_c)v_{ci}}{m_b\cos\theta}$.

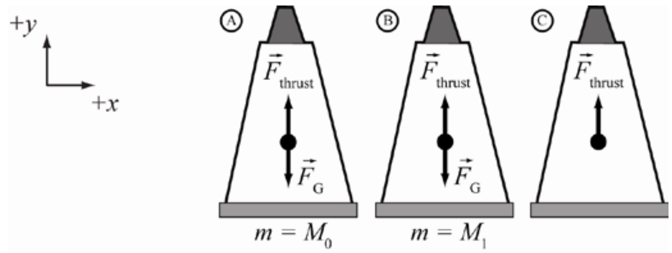
**CALCULATE:** $v_b = \dfrac{(15.0 \text{ kg} + 350 \text{ kg})(7.50 \text{ m/s})}{(15.0 \text{ kg})\cos(55.0°)} = 318.2$ m/s

**ROUND:** Each given value has three significant figures, so the result should be rounded to $v_b = 318$ m/s.

**DOUBLE-CHECK:** This is a reasonable speed at which to launch a cannonball. The component of the momentum of the cannon/cannon ball system in the $x$-direction before the ball is shot is $p_{x,\text{before}} = (350 \text{ kg} + 15 \text{ kg})(7.5 \text{ m/s}) = 2737.5$ kg m/s. The component of the momentum of the cannon/cannon ball system in the $x$-direction after the ball is shot is $p_{x,\text{after}} = (15 \text{ kg})(318.2 \text{ m/s})\cos(55°) = 2737.68$ kg m/s. These components agree to within three significant figures.

**8.65.** **THINK:** The rocket's initial mass is $M_0 = 2.80\cdot 10^6$ kg. Its final mass is $M_1 = 8.00\cdot 10^5$ kg. The time to burn all the fuel is $\Delta t = 160.$ s. The exhaust speed is $v = v_c = 2700.$ m/s. Determine (a) the upward acceleration, $a_0$, of the rocket as it lifts off, (b) its upward acceleration, $a_1$, when all the fuel has burned and (c) the net change in speed, $\Delta v$ in time $\Delta t$ in the absence of a gravitational force.

**SKETCH:**




**RESEARCH:** To determine the upward acceleration, all the vertical forces on the rocket must be balanced. Use the following equations: $\vec{F}_{\text{thrust}} = -\vec{v}_{\text{c}}\dfrac{dm}{dt}$, $\vec{F}_{\text{g}} = m\vec{g}$, $\dfrac{dm}{dt} = \dfrac{\Delta m}{\Delta t}$. The mass of the fuel used is $\Delta m = M_0 - M_1$. To determine $\Delta v$ in the absence of other forces (other than $\vec{F}_{\text{thrust}}$), use $v_{\text{f}} - v_{\text{i}} = v_{\text{c}} \ln\left(m_{\text{i}} / m_{\text{f}}\right)$.

**SIMPLIFY:**

(a) $\dfrac{dm}{dt} = \dfrac{M_0 - M_1}{\Delta t}$

Balancing the vertical forces on the rocket gives

$$F_{\text{net}} = F_{\text{thrust}} - F_{\text{g}} = ma \Rightarrow M_0 a_0 = v_{\text{c}}\frac{dm}{dt} - M_0 g \Rightarrow a_0 = \frac{v_{\text{c}}}{M_0}\left(\frac{M_0 - M_1}{\Delta t}\right) - g \Rightarrow a_0 = \frac{v_{\text{c}}}{\Delta t}\left(1 - \frac{M_1}{M_0}\right) - g.$$

(b) Similarly to part (a):

$$F_{\text{net}} = F_{\text{thrust}} - F_{\text{g}} = ma \Rightarrow M_1 a_1 = v_{\text{c}}\frac{dm}{dt} - M_1 g \Rightarrow a_1 = \frac{v_{\text{c}}}{M_1}\left(\frac{M_0 - M_1}{\Delta t}\right) - g \Rightarrow a_1 = \frac{v_{\text{c}}}{\Delta t}\left(\frac{M_0}{M_1} - 1\right) - g.$$



(c) In the absence of gravity, $F_{\text{net}} = F_{\text{thrust}}$. The change in velocity due to this thrust force is $\Delta v = v_{\text{c}} \ln\left(M_0 / M_1\right)$.

**CALCULATE:**

(a) $a_0 = \left(\dfrac{2700.\text{ m/s}}{160\text{ s}}\right)\left(1 - \dfrac{8.00 \cdot 10^5\text{ kg}}{2.80 \cdot 10^6\text{ kg}}\right) - 9.81\text{ m/s}^2 = 2.244\text{ m/s}^2$

(b) $a_1 = \left(\dfrac{2700.\text{ m/s}}{160.\text{ s}}\right)\left(\dfrac{2.80 \cdot 10^6\text{ kg}}{8.00 \cdot 10^5\text{ kg}} - 1\right) - 9.81\text{ m/s}^2 = 32.38\text{ m/s}^2$

(c) $\Delta v = \left(2700.\text{ m/s}\right)\ln\left(\dfrac{2.80 \cdot 10^6\text{ kg}}{8.00 \cdot 10^5\text{ kg}}\right) = 3382\text{ m/s}$

**ROUND:**

(a) $a_0 = 2.24\text{ m/s}^2$

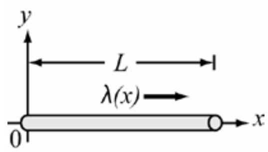
(b) $a_1 = 32.4\text{ m/s}^2$

(c) $\Delta v = 3380\text{ m/s}$

**DOUBLE-CHECK:** It can be seen that $a_1 > a_0$, as it should be since $M_1 < M_0$. It is not unusual for $\Delta v$ to be greater than $v_{\text{c}}$.

**8.66.** **THINK:** The rod has a length of $L$ and its linear density is $\lambda(x)=cx$, where $c$ is a constant. Determine the rod's center of mass.

**SKETCH:**




**RESEARCH:** To determine the center of mass, take a differentially small element of mass: $dm=\lambda dx$ and use $X=\frac{1}{M}\int_L x\cdot dm=\frac{1}{M}\int_L x\lambda(x)dx$, where $M=\int_L dm=\int_L \lambda(x)dx$.

**SIMPLIFY:** First, determine $M$ from $M=\int_0^L cxdx=\left[c\frac{1}{2}x^2\right]_0^L=\frac{1}{2}cL^2$. Then, the equation for the center of mass becomes:

$$X=\frac{1}{M}\int_0^L x(cx)dx=\frac{1}{M}\int_0^L cx^2dx=\frac{1}{M}c\left[\frac{1}{3}x^3\right]_0^L=\frac{1}{3M}cL^3.$$

Substituting the expression for $M$ into the above equation gives:

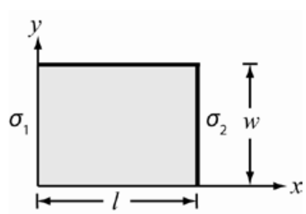
$$X=\frac{cL^3}{3\left(\frac{1}{2}cL^2\right)}=\frac{2}{3}L.$$

**CALCULATE:** This step is not applicable.
**ROUND:** This step is not applicable.
**DOUBLE-CHECK:** $X$ is a function of $L$. Also, as expected, $X$ is closer to the denser end of the rod.



**8.67.** **THINK:** The length and width of the plate are $l$ = 20.0 cm and $w$ = 10.0 cm, respectively. The mass density, $\sigma$, varies linearly along the length; at one end it is $\sigma_1=5.00\text{ g/cm}^2$ and at the other it is $\sigma_2=20.0\text{ g/cm}^2$. Determine the center of mass.

**SKETCH:**




**RESEARCH:** The mass density does not vary in width, i.e. along the $y$-axis. Therefore, the $Y$ coordinate is simply $w/2$. To determine the $X$ coordinate, use

$$X=\frac{1}{M}\int_A x\sigma(\vec{r})dA,\text{ where }M=\int_A \sigma(\vec{r})dA.$$

To obtain a functional form for $\sigma(\vec{r})$, consider that it varies linearly with $x$, and when the bottom left corner of the plate is at the origin of the coordinate system, $\sigma$ must be $\sigma_1$ when $x = 0$ and $\sigma_2$ when $x = l$. Then, the conditions are satisfied by $\sigma(\vec{r})=\sigma(x)=\frac{(\sigma_2-\sigma_1)}{l}x+\sigma_1$.

**SIMPLIFY:** First determine $M$ from $M=\int_A \sigma(\vec{r})dA=\int_0^l\int_0^w \sigma(x)dydx=\int_0^w dy\int_0^l\left(\frac{(\sigma_2-\sigma_1)}{l}x+\sigma_1\right)dx$. $y$ is not dependent on $x$ in this case, so

$$M=\left[y\right]_0^w\left[\frac{1}{2}\frac{(\sigma_2-\sigma_1)}{l}x^2+\sigma_1 x\right]_0^l=w\left(\frac{1}{2}\frac{(\sigma_2-\sigma_1)}{l}l^2+\sigma_1 l\right)=wl\left(\frac{1}{2}(\sigma_2-\sigma_1)+\sigma_1\right)=\frac{wl}{2}(\sigma_2+\sigma_1).$$

Now, reduce the equation for the center of mass:

$$X=\frac{1}{M}\int_A x\sigma(x)dA=\frac{1}{M}\int_0^l\int_0^w x\left(\frac{(\sigma_2-\sigma_1)}{l}x+\sigma_1\right)dydx=\frac{1}{M}\int_0^w dy\int_0^l\left(\frac{(\sigma_2-\sigma_1)}{l}x^2+\sigma_1 x\right)dx$$

$$=\frac{1}{M}\left[y\right]_0^w\left[\frac{(\sigma_2-\sigma_1)}{3l}x^3+\frac{1}{2}\sigma_1 x^2\right]_0^l=\frac{1}{M}w\left(\frac{(\sigma_2-\sigma_1)}{3l}l^3+\frac{1}{2}\sigma_1 l^2\right)=\frac{1}{M}wl^2\left(\frac{1}{3}(\sigma_2-\sigma_1)+\frac{1}{2}\sigma_1\right)$$

$$=\frac{1}{M}wl^2\left(\frac{1}{3}\sigma_2+\frac{1}{6}\sigma_1\right)$$

Substitute the expression for $M$ into the above equation to get

$$X=\frac{l\left(\frac{1}{3}\sigma_2+\frac{1}{6}\sigma_1\right)}{\frac{1}{2}(\sigma_2+\sigma_1)}=\frac{2l\left(\frac{1}{3}\sigma_2+\frac{1}{6}\sigma_1\right)}{\sigma_2+\sigma_1}.$$

**CALCULATE:** $X=\dfrac{2(20.0\text{ cm})\left(\frac{1}{3}(20.0\text{ g/cm}^2)+\frac{1}{6}(5.00\text{ g/cm}^2)\right)}{20.0\text{ g/cm}^2+5.00\text{ g/cm}^2}=12.00\text{ cm},\;\; Y=\frac{1}{2}(10.0\text{ cm})=5.00\text{ cm}$

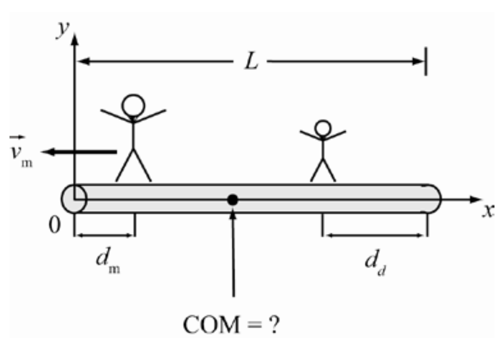
**ROUND:** The results should be written to three significant figures: $X=12.0$ cm and $Y=5.00$ cm. The center of mass is at (12.0 cm, 5.00 cm).

**DOUBLE-CHECK:** It is expected that the center of mass for the $x$ coordinate is closer to the denser end of the rectangle (before rounding).



**8.68.** **THINK:** The log's length and mass are $L=2.50$ m and $m_1=91.0$ kg, respectively. The man's mass is $m_\text{m}=72$ kg and his location is $d_\text{m}=0.220$ m from one end of the log. His daughter's mass is $m_\text{d}=20.0$ kg and her location is $d_\text{d}=1.00$ m from the other end of the log. Determine (a) the system's center of mass and (b) the initial speed of the log and daughter, $v_{1+\text{d}}$, when the man jumps off the log at a speed of $v_\text{m}=3.14$ m/s.

**SKETCH:**




**RESEARCH:** In one dimension, the center of mass location is given by $X=\frac{1}{M}\sum_{i=1}^{n}x_i m_i$. Take the origin of the coordinate system to be at the end of log near the father. To determine the initial velocity of the log and girl system, consider the conservation of momentum, $\vec{p}_\text{i}=\vec{p}_\text{f}$, where $\vec{p}=m\vec{v}$. Note that the man's velocity is away from the daughter. Take this direction to be along the $-\hat{x}$ direction, so that $\vec{v}_\text{m}=-3.14\text{ m/s }\hat{x}$ .

**SIMPLIFY:**

(a) $X=\frac{1}{M}\left(x_{\text{m}}m_{\text{m}}+x_{\text{d}}m_{\text{d}}+x_{\text{l}}m_{\text{l}}\right)=\frac{\left(d_{\text{m}}m_{\text{m}}+\left(L-d_{\text{d}}\right)m_{\text{d}}+\frac{1}{2}Lm_{\text{l}}\right)}{m_{\text{m}}+m_{\text{d}}+m_{\text{l}}}$

(b) $\vec{p}_{\text{i}}=\vec{p}_{\text{f}}\;\Rightarrow\;0=m_{\text{m}}\vec{v}_{\text{m}}+\left(m_{\text{d}}+m_{\text{l}}\right)\vec{v}_{\text{d+l}}\;\Rightarrow\;\vec{v}_{\text{d+l}}=-\frac{m_{\text{m}}\vec{v}_{\text{m}}}{\left(m_{\text{d}}+m_{\text{l}}\right)}$

**CALCULATE:**

(a) $X=\frac{\left((0.220\text{ m})(72.0\text{ kg})+(2.50\text{ m}-1.00\text{ m})(20.0\text{ kg})+\frac{1}{2}(2.50\text{ m})(91.0\text{ kg})\right)}{72.0\text{ kg}+20.0\text{ kg}+91.0\text{ kg}}=0.8721\text{ m}$

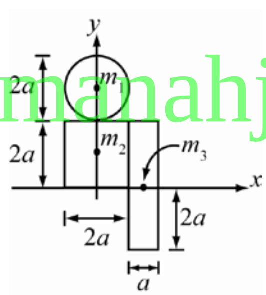
(b) $\vec{v}_{\text{d+l}}=-\frac{(72.0\text{ kg})(-3.14\text{ m/s }\hat{x})}{(20.0\text{ kg}+91.0\text{ kg})}=2.0368\text{ m/s }\hat{x}$

**ROUND:** To three significant figures, the center of mass of the system is $X$ = 0.872 m from the end of the log near the man, and the speed of the log and child is $v_{\text{d+l}}=2.04$ m/s.

**DOUBLE-CHECK:** As it should be, the center of mass is between the man and his daughter, and $v_{\text{d+l}}$ is less than $v_{\text{m}}$ (since the mass of the log and child is larger than the mass of the man).

**8.69.** **THINK:** Determine the center of mass of an object which consists of regularly shaped metal of uniform thickness and density. Assume that the density of the object is $\rho$.

**SKETCH:**






**RESEARCH:** First, as shown in the figure above, divide the object into three parts, $m_1$, $m_2$ and $m_3$. Determine the center of mass by using $\vec{R}=\frac{1}{M}\sum_{i=1}^{3}m_i\vec{r}_i$, or in component form $X=\frac{1}{M}\sum_{i=1}^{3}m_ix_i$ and $Y=\frac{1}{M}\sum_{i=1}^{3}m_iy_i$. Also, use $m=\rho At$ for the mass, where $A$ is the area and $t$ is the thickness.

**SIMPLIFY:** The center of mass components are given by:

$$X=\frac{m_1x_1+m_2x_2+m_3x_3}{M}\text{ and }Y=\frac{m_1y_1+m_2y_2+m_3y_3}{M}$$

The masses of the three parts are $m_1=\rho\pi a^2t$, $m_2=\rho(2a)^2t$ and $m_3=\rho4a^2t$. The center of mass of the three parts are $x_1=0$, $y_1=3a$, $x_2=0$, $y_2=a$, $x_3=3a/2$ and $y_3=0$. The total mass of the object is $M=m_1+m_2+m_3=\rho\pi a^2t+4\rho a^2t+4\rho a^2t=\rho a^2t(8+\pi)$.

**CALCULATE:** The center of mass of the object is given by the following equations:

$$X=\frac{0+0+4\rho a^2t(3a/2)}{\rho a^2t(8+\pi)}=\left(\frac{6}{8+\pi}\right)a;$$

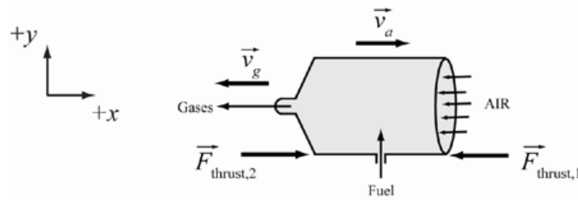
$$Y=\frac{\rho\pi a^2t(3a)+4\rho a^2t(a)+0}{\rho a^2t(8+\pi)}=\left(\frac{4+3\pi}{8+\pi}\right)a.$$

**ROUND:** Rounding is not required.

**DOUBLE-CHECK:** The center of mass of the object is located in the area of $m_2$. By inspection of the figure this is reasonable.

**8.70.** **THINK:** A jet aircraft has a speed of 223 m/s. The rate of change of the mass of the aircraft is $(dM/dt)_{\text{air}} = 80.0 \text{ kg/s}$ (due to the engine taking in air) and $(dM/dt)_{\text{fuel}} = 3.00 \text{ kg/s}$ (due to the engine taking in and burning fuel). The speed of the exhaust gases is 600. m/s. Determine the thrust of the jet engine.

**SKETCH:**




**RESEARCH:** The thrust is calculated by using $\vec{F}_{\text{thrust}} = -\vec{v}\,dM/dt$, where $\vec{v}$ is the velocity of the gases or air, relative to the engine. There are two forces on the engine. The first force, $F_{\text{thrust,1}}$, is the thrust due to the engine taking in air and the second force, $F_{\text{thrust,2}}$, is the thrust due to the engine ejecting gases.

$$\vec{F}_{\text{thrust,1}} = -\vec{v}_{\text{a}}\left(\frac{dM}{dt}\right)_{\text{air}}, \quad \vec{F}_{\text{thrust,2}} = -\vec{v}_{\text{g}}\left[\left(\frac{dM}{dt}\right)_{\text{air}} + \left(\frac{dM}{dt}\right)_{\text{fuel}}\right]$$

The net thrust is given by $\vec{F}_{\text{thrust}} = \vec{F}_{\text{thrust,1}} + \vec{F}_{\text{thrust,2}}$.

**SIMPLIFY:** Simplification is not required.

**CALCULATE:** $\vec{F}_{\text{thrust,1}} = -(223 \text{ m/s}\,\hat{x})(80.0 \text{ kg/s}) = -17840 \text{ N}\,\hat{x}$,



$\vec{F}_{\text{thrust,2}} = -(600. \text{ m/s}\,(-\hat{x}))[80.0 \text{ kg/s} + 3.00 \text{ kg/s}] = 49800 \text{ N}\,\hat{x}$,

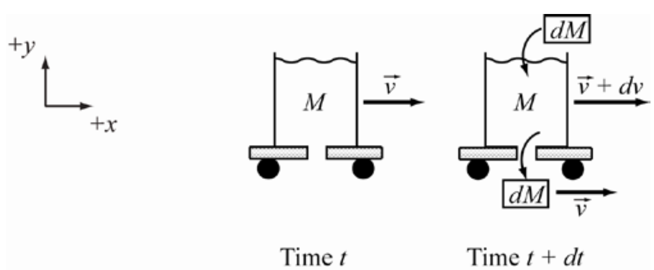
$\vec{F}_{\text{thrust}} = -17840 \text{ N}\,\hat{x} + 49800 \text{ N}\,\hat{x} = 31960 \text{ N}\,\hat{x}$

**ROUND:** To three significant figures, the thrust of the jet engine is $\vec{F}_{\text{thrust}} = 32.0 \text{ kN}\,\hat{x}$.

**DOUBLE-CHECK:** Since the $\hat{x}$ direction is in the forward direction of the aircraft, the plane moves forward, which it must. A jet engine is very powerful, so the large magnitude of the result is reasonable.

**8.71.** **THINK:** The solution to this problem is similar to a rocket system. Here the system consists of a bucket, a skateboard and water. The total mass of the system is $M$ = 10.0 kg. The total mass of the bucket, skateboard and water remains constant at $\lambda = dM/dt = 0.100 \text{ kg/s}$ since rain water enters the top of the bucket at the same rate that it exits the bottom. Determine the time required for the bucket and the skateboard to reach a speed of half the initial speed.

**SKETCH:**




**RESEARCH:** To solve this problem, consider the conservation of momentum, $\vec{p}_{\text{i}} = \vec{p}_{\text{f}}$. The initial momentum of the system at time $t$ is $p_{\text{i}} = Mv$. After time $t + dt$, the momentum of the system is $p_{\text{f}} = v\,dM + M(v + dv)$.

**SIMPLIFY:** $p_{\text{i}} = p_{\text{f}} \Rightarrow Mv = vdM + Mv + Mdv \Rightarrow Mdv = -vdM$

Dividing both sides by $dt$ gives

$$M\frac{dv}{dt}=-v\frac{dM}{dt}=-v\lambda \text{ or } \frac{1}{v}\frac{dv}{dt}=-\frac{\lambda}{M} \Rightarrow \frac{1}{v}dv=-\frac{\lambda}{M}dt.$$

Integrate both sides to get

$$\int_{v=v_0}^{v}\frac{1}{v}dv=\int_{t=0}^{t}-\frac{\lambda}{M}dt \Rightarrow \ln v-\ln v_0=-\frac{\lambda}{M}t \Rightarrow \ln\left(\frac{v}{v_0}\right)=-\frac{\lambda}{M}t.$$

Determine the time such that $v=v_0/2$. Substituting $v=v_0/2$ into the above equation gives

$$\ln\left(\frac{v_0/2}{v_0}\right)=-\frac{\lambda}{M}t \Rightarrow t=-\frac{M}{\lambda}\ln\left(\frac{1}{2}\right)=\frac{M}{\lambda}\ln(2).$$

**CALCULATE:** $t=\frac{(10.0 \text{ kg})\ln(2)}{0.100 \text{ kg/s}}=69.3147 \text{ s}$

**ROUND:** To three significant figures, the time for the system to reach half of its initial speed is $t = 69.3$ s.

**DOUBLE-CHECK:** It is reasonable that the time required to reduce the speed of the system to half its original value is near one minute.

**8.72.** **THINK:** The mass of a cannon is $M = 1000.$ kg and the mass of a shell is $m = 30.0$ kg. The shell is shot at an angle of $\theta=25.0°$ above the horizontal with a speed of $v_s=500.$ m/s. Determine the recoil velocity of the cannon.

**SKETCH:**




**RESEARCH:** The momentum of the system is conserved, $p_i=p_f$, or in component form, $p_{xi}=p_{xf}$ and $p_{yi}=p_{yf}$. Use only the *x* component of the momentum.

**SIMPLIFY:** $p_{xi}$ is equal to zero since both the cannon and the shell are initially at rest. Therefore,

$$p_{xi}=p_{xf} \Rightarrow mv_s\cos\theta+Mv_c=0 \Rightarrow v_c=-\frac{m}{M}v_s\cos\theta.$$

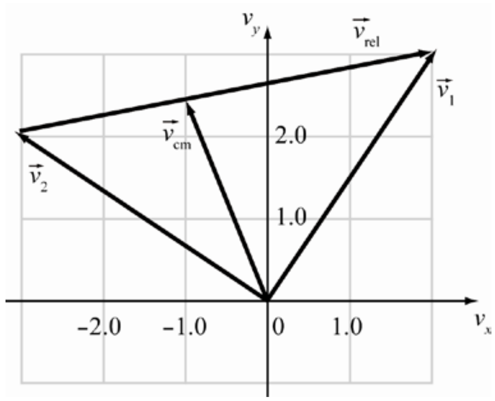
**CALCULATE:** $v_c=-\frac{(30.0 \text{ kg})(500. \text{ m/s})\cos(25.0°)}{1000. \text{ kg}}=-13.595 \text{ m/s}$

**ROUND:** To three significant figures: $v_c=-13.6$ m/s

**DOUBLE-CHECK:** The direction of the recoil is expected to be in the opposite direction to the horizontal component of the velocity of the shell. This is why the result is negative.

**8.73.** **THINK:** There are two masses, $m_1=2.0$ kg and $m_2=3.0$ kg. The velocity of their center of mass and the velocity of mass 1 relative to mass 2 are $\vec{v}_{cm}=(-1.00\hat{x}+2.40\hat{y})$ m/s and $\vec{v}_{rel}=(5.00\hat{x}+1.00\hat{y})$ m/s. Determine the total momentum of the system and the momenta of mass 1 and mass 2.

**SKETCH:**




**RESEARCH:** The total momentum of the system is $\vec{p}_{\text{cm}} = M\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2$. The velocity of mass 1 relative to mass 2 is $\vec{v}_{\text{rel}} = \vec{v}_1 - \vec{v}_2$.

**SIMPLIFY:** The total mass $M$ of the system is $M = m_1 + m_2$. The total momentum of the system is given by $\vec{p}_{\text{cm}} = M\vec{v}_{\text{cm}} = (m_1 + m_2)\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2$. Substitute $\vec{v}_2 = \vec{v}_1 - \vec{v}_{\text{rel}}$ into the equation for the total momentum of the system to get $M\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2(\vec{v}_1 - \vec{v}_{\text{rel}}) = (m_1 + m_2)\vec{v}_1 - m_2\vec{v}_{\text{rel}}$. Therefore, $\vec{v}_1 = \vec{v}_{\text{cm}} + \dfrac{m_2}{M}\vec{v}_{\text{rel}}$. Similarly, substitute $\vec{v}_1 = \vec{v}_2 + \vec{v}_{\text{rel}}$ into the equation for the total momentum of the system to get $M\vec{v}_{\text{cm}} = m_1\vec{v}_{\text{rel}} + (m_1 + m_2)\vec{v}_2$ or $\vec{v}_2 = \vec{v}_{\text{cm}} - \dfrac{m_1}{M}\vec{v}_{\text{rel}}$. Therefore, the momentums of mass 1 and mass 2 are $\vec{p}_1 = m_1\vec{v}_1 = m_1\vec{v}_{\text{cm}} + \dfrac{m_1 m_2}{M}\vec{v}_{\text{rel}}$ and $\vec{p}_2 = m_2\vec{v}_2 = m_2\vec{v}_{\text{cm}} - \dfrac{m_1 m_2}{M}\vec{v}_{\text{rel}}$.



**CALCULATE:**

(a)

$$\vec{p}_{\text{cm}} = (2.00 \text{ kg} + 3.00 \text{ kg})(-1.00\hat{x} + 2.40\hat{y})\text{ m/s} = (-5.00\hat{x} + 12.0\hat{y}) \text{ kg m/s}$$

$$\vec{p}_{\text{cm}} = (2.0 \text{ kg} + 3.0 \text{ kg})(-1.0\hat{x} + 2.4\hat{y})\text{ m/s} = (-5.0\hat{x} + 12\hat{y}) \text{ kg m/s}$$

(b) $$\vec{p}_1 = (2.00 \text{ kg})(-1.00\hat{x} + 2.40\hat{y})\text{ m/s} + \frac{(2.00 \text{ kg})(3.00 \text{ kg})}{2.00 \text{ kg} + 3.00 \text{ kg}}(5.00\hat{x} + 1.00\hat{y}) \text{ m/s}$$

$$= (-2.00\hat{x} + 4.80\hat{y}) \text{ kg m/s} + (6.00\hat{x} + 1.20\hat{y}) \text{ kg m/s} = (4.00\hat{x} + 6.00\hat{y}) \text{ kg m/s}$$

(c) $$\vec{p}_2 = (3.00 \text{ kg})(-1.00\hat{x} + 2.40\hat{y})\text{ m/s} - \frac{(2.00 \text{ kg})(3.00 \text{ kg})}{2.00 \text{ kg} + 3.00 \text{ kg}}(5.00\hat{x} + 1.00\hat{y}) \text{ m/s}$$

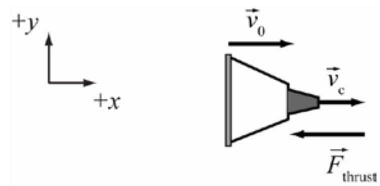
$$= (-3.00\hat{x} + 7.20\hat{y}) \text{ kg m/s} - (6.00\hat{x} + 1.20\hat{y}) \text{ kg m/s} = (-9.00\hat{x} + 6.00\hat{y}) \text{ kg m/s}$$

**ROUND:** The answers have already been rounded to three significant figures.

**DOUBLE-CHECK:** It is clear from the results of (a), (b) and (c) that $\vec{p}_{\text{cm}} = \vec{p}_1 + \vec{p}_2$.

**8.74.** **THINK:** A spacecraft with a total initial mass of $m_s = 1000.$ kg and an initial speed of $v_0 = 1.00$ m/s must be docked. The mass of the fuel decreases from 20.0 kg. Since the mass of the fuel is small compared to the mass of the spacecraft, we can ignore it. To reduce the speed of the spacecraft, a small retro-rocket is used which can burn fuel at a rate of $dM/dt = 1.00$ kg/s and with an exhaust speed of $v_E = 100.$ m/s.

**SKETCH:**




**RESEARCH:**

(a) The thrust of the retro-rocket is determined using $F_{thrust} = v_c dM/dt$.

(b) In order to determine the amount of fuel needed, first determine the time to reach a speed of $v = 0.0200$ m/s. Use $v = v_0 - at$. By Newton's Second Law the thrust is also given by $\vec{F}_{thrust} = m_s\vec{a}$.

(c) The burn of the retro-rocket must be sustained for a time sufficient to reduce the speed to 0.0200 m/s, found in part (b).

(d) Use the conservation of momentum, $\vec{p}_i = \vec{p}_f$.

**SIMPLIFY:**

(a) $\vec{F}_{thrust} = -\vec{v}_c \dfrac{dM}{dt}$

(b) $t = \dfrac{v_0 - v}{a}$

The acceleration is given by $a = F_{thrust}/m_s$. Substitute this expression into the equation for $t$ above to get $t = \dfrac{(v_0 - v)m_s}{F_{thrust}}$. Therefore, the mass of fuel needed is $m_F = \left(\dfrac{dM}{dt}\right)t = \left(\dfrac{dM}{dt}\right)\dfrac{(v_0 - v)m_s}{F_{thrust}}$.

(c) $t = \dfrac{(v_0 - v)m_s}{F_{thrust}}$



(d) $m_s\vec{v} = (M + m_s)\vec{v}_f \Rightarrow \vec{v}_f = \dfrac{m_s}{M + m_s}\vec{v}$, where $M$ is the mass of the space station.

**CALCULATE:**

(a) The thrust is $\vec{F}_{thrust} = -(100.\text{ m/s})(1.00\text{ kg/s})\hat{v}_E = -100.0\text{ N }\hat{v}_c$, or 100.0 N in the opposite direction to the velocity of the spacecraft.

(b) $m_F = (1.00\text{ kg/s})\dfrac{(1.00\text{ m/s} - 0.0200\text{ m/s})1000.\text{ kg}}{100.0\text{ N}} = 9.800\text{ kg}$

(c) $t = \dfrac{(1.00\text{ m/s} - 0.0200\text{ m/s})1000.\text{ kg}}{100.0\text{ N}} = 9.800\text{ s}$

(d) $\vec{v}_f = \dfrac{1000.\text{ kg}(0.0200\text{ m/s})}{5.00 \cdot 10^5\text{ kg} + 1000.\text{ kg}}\hat{v} = 3.992 \cdot 10^{-5}\text{ m/s }\hat{v}$; that is, in the same direction as the spacecraft is moving.

**ROUND:** The answers should be expressed to three significant figures:

(a) $\vec{F}_{thrust} = -100.\text{ N }\hat{v}_c$

(b) $m_F = 9.80\text{ kg}$

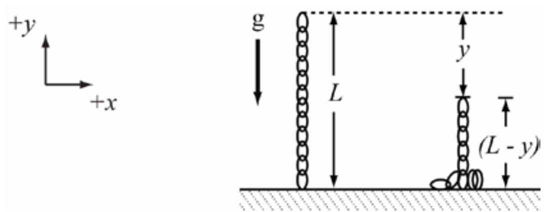
(c) $t = 9.80\text{ s}$

(d) $\vec{v}_f = 3.99 \cdot 10^{-5}\text{ m/s }\hat{v}$

**DOUBLE-CHECK:** It is expected that the speed of the combined mass will be very small since its mass is very large.

**8.75.** **THINK:** A chain has a mass of 3.00 kg and a length of 5.00 m. Determine the force exerted by the chain on the floor. Assume that each link in the chain comes to rest when it reaches the floor.

**SKETCH:**




**RESEARCH:** Assume the mass per unit length of the chain is $\rho = M/L$. A small length of the chain, $dy$ has a mass of $dm$, where $dm = Mdy/L$. At an interval of time $dt$, the small element of mass $dm$ has reached the floor. The impulse caused by the chain is given by $J = F_J dt = \Delta p = vdm$. Therefore, the force $F_J$ is given by $F_J = v\dfrac{dm}{dt} = v\dfrac{dm}{dy}\dfrac{dy}{dt}$.

**SIMPLIFY:** Using $dm/dy = M/L$ and $v = dy/dt$, the expression for force, $F_J$ is

$$F_J = v^2 \frac{M}{L}.$$

For a body in free fall motion, $v^2 = 2gy$. Thus, $F_J = 2Mgy/L$. There is another force which is due to gravity. The gravitational force exerted by the chain on the floor when the chain has fallen a distance $y$ is given by $F_g = Mgy/L$ (the links of length $y$ are on the floor). The total force is given by

$$F = F_J + F_g = \frac{2Mgy}{L} + \frac{Mgy}{L} = \frac{3Mgy}{L}.$$



When the last link of the chain lands on the floor, the force exerted by the chain is obtained by substituting $y = L$, that is, $F = \dfrac{3Mgy}{L} = 3Mg$.

**CALCULATE:** $F = 3(3.0\text{ kg})\left(9.81\text{ m/s}^2\right) = 88.29\text{ N}$

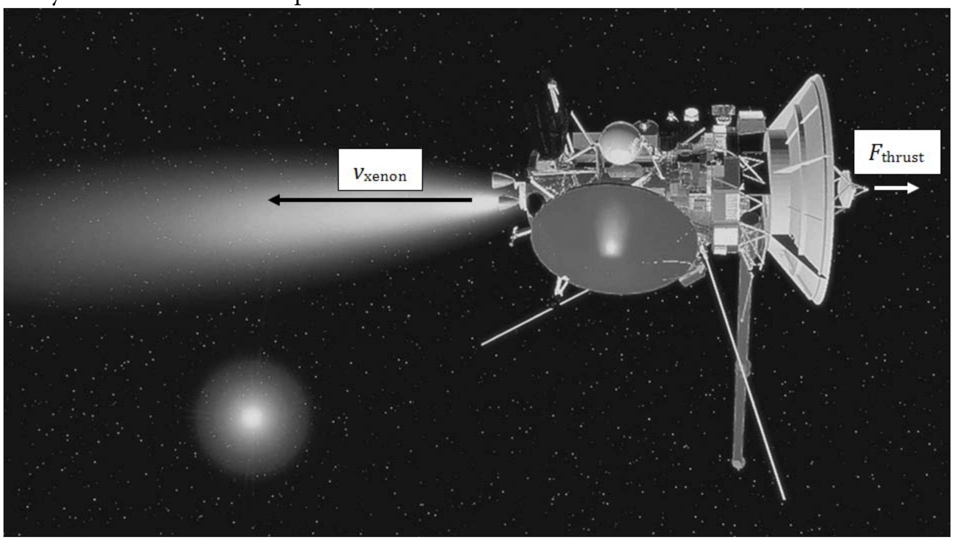
**ROUND:** To three significant figures, the force exerted by the chain on the floor as the last link of chain lands on the floor is $F = 88.3$ N.

**DOUBLE-CHECK:** $F$ is expected to be larger than $Mg$ due to the impulse caused by the chain as it falls.

## Multi-Version Exercises

**8.76.** **THINK:** This question asks about the fuel consumption of a satellite. This is an example of rocket motion, where the mass of the satellite (including thruster) decreases as the fuel is ejected.

**SKETCH:** The direction in which the xenon ions are ejected is opposite to the direction of the thrust. The velocity of the xenon with respect to the satellite and the thrust force are shown.




**RESEARCH:** The equation of motion for a rocket in interstellar space is given by $\vec{F}_{\text{thrust}} = -\vec{v}_c \dfrac{dm}{dt}$. The velocity of the xenon ions with respect to the shuttle is given in km/s and the force is given in Newtons, or $\text{kg} \cdot \text{m} / \text{s}^2$. The conversion factor for the velocity is given by $\dfrac{1000 \text{ m/s}}{1 \text{ km/s}}$.



**SIMPLIFY:** Since the thrust and velocity act along a single axis, it is possible to use the scalar form of the equation, $F_{\text{thrust}} = -v_c \dfrac{dm}{dt}$. The rate of fuel consumption equals the change in mass (the loss of mass is due to xenon ejected from the satellite), so solve for $\dfrac{dm}{dt}$ to get $\dfrac{dm}{dt} = -\dfrac{F_{\text{thrust}}}{v_c}$.

**CALCULATE:** The question states that the speed of the xenon ions with respect to the rocket is $v_c = v_{\text{xenon}} = 21.45$ km/s. The thrust produced is $F_{\text{thrust}} = 1.187 \cdot 10^{-2}$ N. Thus the rate of fuel consumption is:

$$\begin{aligned}\frac{dm}{dt} &= -\frac{F_{\text{thrust}}}{v_c} \\ &= -\frac{1.187 \cdot 10^{-2} \text{ N}}{21.45 \text{ km/s} \cdot \dfrac{1000 \text{ m/s}}{1 \text{ km/s}}} \\ &= -5.533799534 \cdot 10^{-7} \text{ kg/s} \\ &= -1.992167832 \text{ g/hr}\end{aligned}$$

**ROUND:** The measured values are all given to four significant figures, and the final answer should also have four significant figures. The thruster consumes fuel at a rate of $5.534 \cdot 10^{-7}$ kg/s or 1.992 g/hr.

**DOUBLE-CHECK:** Because of the cost of sending a satellite into space, the weight of the fuel consumed per hour should be pretty small; a fuel consumption rate of 1.992 g/hr is reasonable for a satellite launched from earth. Working backwards, if the rocket consumes fuel at a rate of $5.534 \cdot 10^{-4}$ g/s, then the thrust is

$$-21.45 \text{ km/s} \cdot \left(-5.534 \cdot 10^{-4} \text{ g/s}\right) = 0.01187 \text{ km} \cdot \text{g/s}^2 = 1.187 \cdot 10^{-2} \text{ N}$$

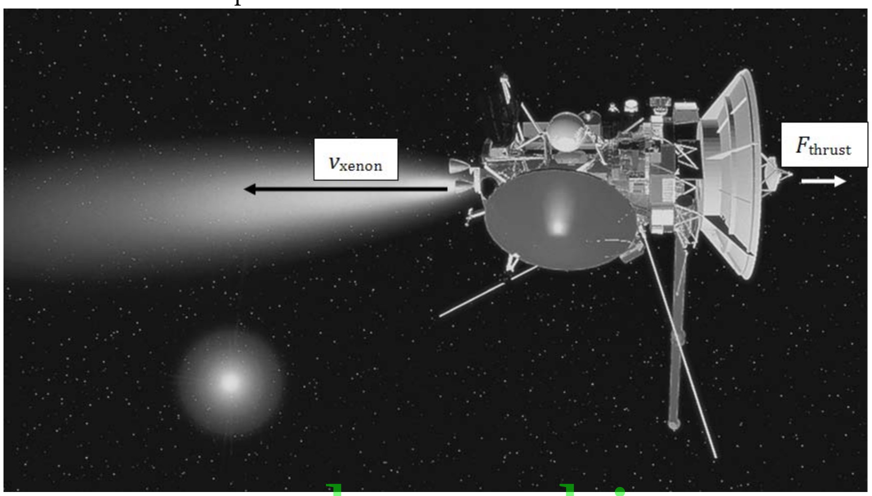
(the conversion factor is $1 \text{ km} \cdot \text{g/s}^2 = 1 \text{ kg} \cdot \text{m/s}^2$). So, this agrees with the given thrust force of $1.187 \cdot 10^{-2}$ N.

**8.77.** $F = v_c \dfrac{dm}{dt} = \left(23.75 \cdot 10^3 \text{ m/s}\right)\left(5.082 \cdot 10^{-7} \text{ kg/s}\right) = 1.207 \cdot 10^{-2} \text{ N}$

**8.78.** $v_c = \dfrac{F}{dm/dt} = \dfrac{1.299 \cdot 10^{-2} \text{ N}}{4.718 \cdot 10^{-7} \text{ kg/s}} = 26.05 \text{ km/s}$

**8.79.** **THINK:** This question asks about the speed of a satellite. This is an example of rocket motion, where the mass of the satellite (including thruster) decreases as the fuel is ejected.

**SKETCH:** The direction in which the xenon ions are ejected is opposite to the direction of the thrust. The velocity of the xenon with respect to the satellite and the thrust force are shown.






**RESEARCH:** Initially, the mass of the system is the total mass of the satellite, including the mass of the fuel: $m_\text{i} = m_\text{satellite}$ After all of the fuel is consumed, the mass of the system is equal to the mass of the satellite minus the mass of the fuel consumed: $m_\text{f} = m_\text{satellite} - m_\text{fuel}$. The change in speed of the satellite is given by the equation $v_\text{f} - v_\text{i} = v_c \ln\left(m_\text{i}/m_\text{f}\right)$, where $v_c$ is the speed of the xenon with relative to the satellite.

**SIMPLIFY:** To make the problem easier, choose a reference frame where the initial speed of the satellite equals zero. Then $v_\text{f} - v_\text{i} = v_\text{f} - 0 = v_\text{f}$, so it is necessary to find $v_\text{f} = v_c \ln\left(m_\text{i}/m_\text{f}\right)$. Substituting in the masses of the satellite and fuel, this becomes $v_\text{f} = v_c \ln\left(m_\text{satellite}/\left[m_\text{satellite} - m_\text{fuel}\right]\right)$.

**CALCULATE:** The initial mass of the satellite (including fuel) is 2149 kg, and the mass of the fuel consumed is 23.37 kg. The speed of the ions with respect to the satellite is 28.33 km/s, so the final velocity of the satellite is:

$$\begin{aligned} v_\text{f} &= v_c \ln\left(m_\text{satellite}/\left[m_\text{satellite} - m_\text{fuel}\right]\right) \\ &= \left(28.33 \text{ km/s}\right)\ln\left(\frac{2149 \text{ kg}}{2149 \text{ kg} - 23.37 \text{ kg}}\right) \\ &= 3.0977123 \cdot 10^{-1} \text{ km/s} \end{aligned}$$

**ROUND:** The measured values are all given to four significant figures, and the weight of the satellite minus the weight of the fuel consumed also has four significant figures, so the final answer will have four figures. The change in the speed of the satellite is $3.098 \cdot 10^{-1}$ km/s or 309.8 m/s.

**DOUBLE-CHECK:** Alhough the satellite is moving quickly after burning all of its fuel, this is not an unreasonable speed for space travel. Working backwards, if the change in speed was $3.098 \cdot 10^{-1}$ km/s, then the velocity of the xenon particles was $v_c = \dfrac{\Delta v_\text{satellite}}{\ln(m_\text{i}/m_\text{f})}$, or

$$v_c = \frac{3.098 \cdot 10^{-1} \text{km/s}}{\ln(2149 \text{ kg}/\left[2149 \text{ kg} - 23.37 \text{ kg}\right])} = 28.33 \text{ km/s}.$$

This agrees with the number given in the question, confirming that the calculations are correct.

**8.80.** $\Delta v = v_c \ln\left(\frac{m_i}{m_f}\right)$

$$\frac{\Delta v}{v_c} = \ln\left(\frac{m_i}{m_f}\right)$$

$$e^{\frac{\Delta v}{v_c}} = \frac{m_i}{m_f}$$

$$m_f = m_i e^{-\frac{\Delta v}{v_c}}$$

$$m_{fuel} = m_i - m_f = m_i - m_i e^{-\frac{\Delta v}{v_c}} = m_i\left(1 - e^{-\frac{\Delta v}{v_c}}\right)$$

$$m_{fuel} = (2161\text{ kg})\left(1 - e^{-\frac{236.4\text{ m/s}}{20.61\cdot 10^3\text{ m/s}}}\right) = 24.65\text{ kg}$$

**8.81.** $\Delta v = v_c \ln\left(\frac{m_i}{m_f}\right)$

$$\frac{\Delta v}{v_c} = \ln\left(\frac{m_i}{m_f}\right)$$

$$e^{\frac{\Delta v}{v_c}} = \frac{m_i}{m_f}$$



$$m_i = m_f e^{\frac{\Delta v}{v_c}}$$

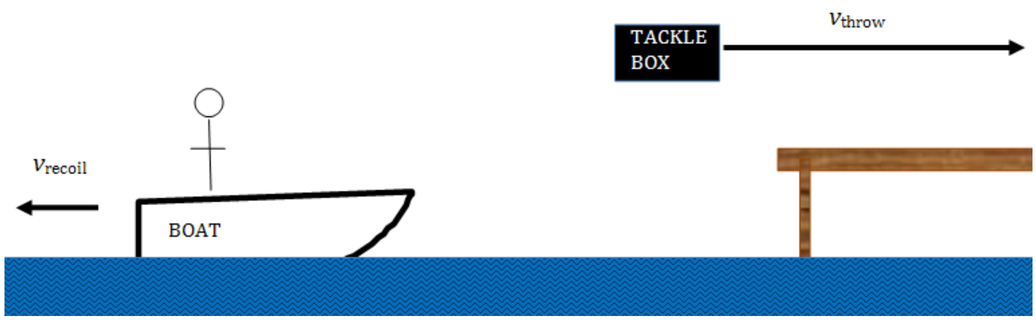
$$m_f = m_i - m_{fuel}$$

$$m_i = (m_i - m_{fuel})e^{\frac{\Delta v}{v_c}} = m_i e^{\frac{\Delta v}{v_c}} - m_{fuel} e^{\frac{\Delta v}{v_c}}$$

$$m_i e^{\frac{\Delta v}{v_c}} - m_i = m_{fuel} e^{\frac{\Delta v}{v_c}}$$

$$m_i = \frac{m_{fuel} e^{\frac{\Delta v}{v_c}}}{e^{\frac{\Delta v}{v_c}} - 1} = m_{fuel}\frac{1}{1 - e^{-\frac{\Delta v}{v_c}}} = (25.95\text{ kg})\frac{1}{1 - e^{-\frac{275.0\text{ m/s}}{22.91\cdot 10^3\text{ m/s}}}} = 2175\text{ kg}$$

**8.82.** **THINK:** The fisherman, boat, and tackle box are at rest at the beginning of this problem, so the total momentum of the fisherman, boat, and tackle box before and after the fisherman throws the tackle box must be zero. Using the principle of conservation of momentum and the fact that the momentum of the tackle box must cancel out the momentum of the fisherman and boat, it is possible to find the speed of the fisherman and boat after the tackle box has been thrown.
**SKETCH:** The sketch shows the motion of the tackle box, boat, and fisherman after the throw:



**RESEARCH:** The total initial momentum is zero, because there is no motion with respect to the dock. After the fisherman throws the tackle box, the momentum of the tackle box is $p_{\text{box}} = m_{\text{box}} v_{\text{box}} = m_{\text{box}} v_{\text{throw}}$ towards the dock. The total momentum after the throw must equal the total momentum before the throw, so the sum of the momentum of the box, the momentum of the boat, and the momentum of the fisherman must be zero: $p_{\text{box}} + p_{\text{fisherman}} + p_{\text{boat}} = 0.$ The fisherman and boat both have the same velocity, so $p_{\text{fisherman}} = m_{\text{fisherman}} v_{\text{fisherman}} = m_{\text{fisherman}} v_{\text{recoil}}$ away from the dock and $p_{\text{boat}} = m_{\text{boat}} v_{\text{boat}} = m_{\text{boat}} v_{\text{recoil}}$ away from the dock.

**SIMPLIFY:** The goal is to find the recoil velocity of the fisherman and boat. Using the equation for momentum after the tackle box has been thrown, $p_{\text{box}} + p_{\text{fisherman}} + p_{\text{boat}} = 0$, substitute in the formula for the momenta of the tackle box, boat, and fisherman: $0 = m_{\text{box}} v_{\text{throw}} + m_{\text{fisherman}} v_{\text{recoil}} + m_{\text{boat}} v_{\text{recoil}}$. Solve for the recoil velocity:

$$m_{\text{box}} v_{\text{throw}} + m_{\text{fisherman}} v_{\text{recoil}} + m_{\text{boat}} v_{\text{recoil}} = 0$$
$$m_{\text{fisherman}} v_{\text{recoil}} + m_{\text{boat}} v_{\text{recoil}} = -m_{\text{box}} v_{\text{throw}}$$
$$v_{\text{recoil}} \left( m_{\text{fisherman}} + m_{\text{boat}} \right) = -m_{\text{box}} v_{\text{throw}}$$
$$v_{\text{recoil}} = -\frac{m_{\text{box}} v_{\text{throw}}}{m_{\text{fisherman}} + m_{\text{boat}}}$$

**CALCULATE:** The mass of the tackle box, fisherman, and boat, as well as the velocity of the throw (with respect to the dock) are given in the question. Using these values gives:

$$\begin{aligned} v_{\text{recoil}} &= -\frac{m_{\text{box}} v_{\text{throw}}}{m_{\text{fisherman}} + m_{\text{boat}}} \\ &= -\frac{13.63 \text{ kg} \cdot 2.911 \text{ m/s}}{75.19 \text{ kg} + 28.09 \text{ kg}} \\ &= -0.3841685709 \text{ m/s} \end{aligned}$$



**ROUND:** The masses and velocity given in the question all have four significant figures, and the sum of the mass of the fisherman and the mass of the boat has five significant figures, so the final answer should have four significant figures. The final speed of the fisherman and boat is –0.3842 m/s towards the dock, or 0.3842 m/s away from the dock.

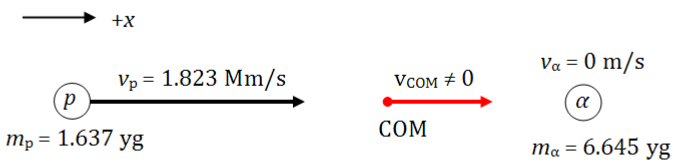
**DOUBLE-CHECK:** It makes intuitive sense that the much more massive boat and fisherman will have a lower speed than the less massive tackle box. Their momenta should be equal and opposite, so a quick way to check this problem is to see if the magnitude of the tackle box's momentum equals the magnitude of the man and boat. The tackle box has a momentum of magnitude 13.63 kg · 2.911 m/s = 39.68 kg·m/s after it is thrown. The boat and fisherman have a combined mass of 103.28 kg, so their final momentum has a magnitude of 103.28 kg · 0.3842 m/s = 39.68 kg·m/s. This confirms that the calculations were correct.

**8.83.** $$v_{\text{box}} = \frac{m_{\text{man}} + m_{\text{boat}}}{m_{\text{box}}} v_{\text{boat}} = \frac{77.49 \text{ kg} + 28.31 \text{ kg}}{14.27 \text{ kg}} (0.3516 \text{ m/s}) = 2.607 \text{ m/s}$$

**8.84.** $$\left( m_{\text{man}} + m_{\text{boat}} \right) v_{\text{boat}} = m_{\text{box}} v_{\text{box}}$$
$$m_{\text{man}} v_{\text{boat}} + m_{\text{boat}} v_{\text{boat}} = m_{\text{box}} v_{\text{box}}$$
$$m_{\text{man}} = \frac{m_{\text{box}} v_{\text{box}} - m_{\text{boat}} v_{\text{boat}}}{v_{\text{boat}}} = m_{\text{box}} \frac{v_{\text{box}}}{v_{\text{boat}}} - m_{\text{boat}}$$
$$m_{\text{man}} = (14.91 \text{ kg}) \frac{3.303 \text{ m/s}}{0.4547 \text{ m/s}} - 28.51 \text{ kg} = 79.80 \text{ kg}$$

**8.85.** **THINK:** The masses and initial speeds of both particles are known, so the momentum of the center of mass can be calculated. The total mass of the system is known, so the momentum can be used to find the speed of the center of mass.

**SKETCH:** To simplify the problem, choose the location of the particle at rest to be the origin, with the proton moving in the $+x$ direction. All of the motion is along a single axis, with the center of mass (COM) between the proton and the alpha particle.




**RESEARCH:** The masses and velocities of the particles are given, so the momenta of the particles can be calculated as the product of the mass and the speed $p_\alpha = m_\alpha v_\alpha$ and $p_p = m_p v_p$ towards the alpha particle. The center-of-mass momentum can be calculated in two ways, either by taking the sum of the momenta of each particle ( $P_{COM} = \sum_{i=0}^{n} p_i$ ) or as the product of the total mass of the system times the speed of the center of mass ( $P_{COM} = M \cdot v_{COM}$ ).

**SIMPLIFY:** The masses of both particles are given in the problem, and the total mass of the system $M$ is the sum of the masses of each particle, $M = m_{\text{p}} + m_\alpha$. The total momentum $P_{COM} = \sum_{i=0}^{n} p_i = p_\alpha + p_p$ and $P_{COM} = M \cdot v_{COM}$, so $M \cdot v_{COM} = p_\alpha + p_p$. Substitute for the momenta of the proton and alpha particle (since the alpha particle is not moving, it has zero momentum), substitute for the total mass, and solve for the velocity of the center of mass:

$$M \cdot v_{COM} = p_\alpha + p_p \Rightarrow$$
$$v_{COM} = \frac{p_\alpha + p_p}{M} = \frac{m_\alpha v_\alpha + m_p v_p}{m_\alpha + m_p} = \frac{m_\alpha \cdot 0 + m_p v_p}{m_\alpha + m_p} = \frac{m_p v_p}{m_\alpha + m_p}$$



**CALCULATE:** The problem states that the proton has a mass of $1.673 \cdot 10^{-27}$ kg and moves at a speed of $1.823 \cdot 10^{6}$ m/s towards the alpha particle, which is at rest and has a mass of $6.645 \cdot 10^{-27}$ kg. So the center of mass has a speed of

$$v_{\text{COM}} = \frac{m_{\text{p}} v_{\text{p}}}{m_{\text{p}\alpha} + m} = \frac{(1.823 \cdot 10^{6} \text{ m/s})(1.673 \cdot 10^{-27} \text{ kg})}{1.673 \cdot 10^{-27} \text{ kg} + 6.645 \cdot 10^{-27} \text{ kg}} = 3.666601346 \cdot 10^{5} \text{ m/s}$$

**ROUND:** The masses of the proton and alpha particle, as well as their sum, have four significant figures. The speed of the proton also has four significant figures. The alpha particle is at rest, so its speed is not a calculated value, and the zero speed does not change the number of figures in the answer. Thus, the speed of the center of mass is $3.667 \cdot 10^{5}$ m/s, and the center of mass is moving towards the alpha particle.

**DOUBLE-CHECK:** To double check, find the location of the center of mass as a function of time, and take the time derivative to find the velocity. The distance between the particles is not given in the problem, so call the distance between the particles at an arbitrary starting time $t = 0$ to be $d_0$. The positions of each particle can be described by their location along the axis of motion, $r_\alpha = 0$ and $r_{\text{p}} = d_0 + v_{\text{p}} t$.

Using this, the location of the center of mass is

$$R_{\text{COM}} = \frac{1}{m_{\text{p}} + m_\alpha}\left(r_{\text{p}} m_{\text{p}} + r_\alpha m_\alpha\right).$$

Take the time derivative to find the velocity:

$$\begin{aligned}
\frac{d}{dt}R_{\text{COM}} &= \frac{d}{dt}\left[\frac{1}{m_{\text{p}} + m_\alpha}\left(r_{\text{p}} m_{\text{p}} + r_\alpha m_\alpha\right)\right] \\
&= \frac{1}{m_{\text{p}} + m_\alpha}\frac{d}{dt}\left[\left(d_0 + v_{\text{p}} t\right) m_{\text{p}} + 0 \cdot m_\alpha\right] \\
&= \frac{1}{m_{\text{p}} + m_\alpha}\frac{d}{dt}\left(d_0 m_{\text{p}} + v_{\text{p}} m_{\text{p}} t + 0\right) \\
&= \frac{1}{m_{\text{p}} + m_\alpha}\frac{d}{dt}\left(d_0 m_{\text{p}} + v_{\text{p}} m_{\text{p}} t\right) \\
&= \frac{1}{m_{\text{p}} + m_\alpha}\left(0 + v_{\text{p}} m_{\text{p}}\right) \\
&= \frac{v_{\text{p}} m_{\text{p}}}{m_{\text{p}} + m_\alpha} \\
&= \frac{\left(1.823 \cdot 10^6 \text{ m/s}\right)\left(1.673 \cdot 10^{-27} \text{ kg}\right)}{1.673 \cdot 10^{-27} \text{ kg} + 6.645 \cdot 10^{-27} \text{ kg}} \\
&= 3.666601346 \cdot 10^5 \text{ m/s}
\end{aligned}$$

This agrees with the earlier result.

**8.86.** $(m_p + m_\alpha) v_{\text{cm}} = m_p v_p + m_\alpha v_\alpha$



Since $v_\alpha = 0$,

$$v_p = \frac{m_p + m_\alpha}{m_p} v_{\text{cm}} = \frac{1.673 \cdot 10^{-27} \text{ kg} + 6.645 \cdot 10^{-27} \text{ kg}}{\left(1.673 \cdot 10^{-27} \text{ kg}\right)}\left(5.509 \cdot 10^5 \text{ m/s}\right) = 2.739 \cdot 10^6 \text{ m/s}$$